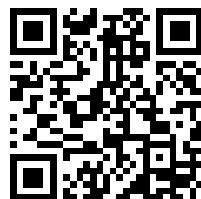

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ADMIRALTY MANUAL



DEVIATIONS OF THE COMPASS

1869.



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ADMIRALTY MANUAL

FOR THE

DEVIATIONS OF THE COMPASS.

EDITED BY

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AND

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THIRD EDITION.

PUBLISHED BY ORDER OF THE LORDS COMMISSIONERS OF THE ADMIRALTY.

LONDON:

PRINTED FOR THE HYDROGRAPHIC OFFICE, ADMIRALTY.

AND SOLD BY

J. D. POTTER, *Agent for Admiralty Charts.*
81 POULTRY, AND 11 KING STREET, TOWER HILL.
1869.

Price 4s. 6d.

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ADVERTISEMENT

To First Edition.

THE increased use of iron in the construction and armature of ships of war has led to a revision of the various instructions relating to the deviation of the Compass heretofore published by order of the Lords Commissioners of the Admiralty.

The results of much patient investigation, both experimental and theoretical, and of lengthened practical experience, are now embodied in one volume for the seaman's information and guidance. The manual is divided into four parts, each part being complete as a distinct branch of the subject, but to be considered subsidiary to the whole as collected.

Part I. contains the well-known Practical Rules drawn up originally in 1842 by a committee consisting of the late Admirals Sir F. Beaufort and Sir James C. Ross, Captain Johnson, R.N., Mr. Christie, and General Sabine, now President of the Royal Society. These rules have undergone additions and revision from time to time as theory and practice progressed. They are now revised to meet the great changes in naval architecture.

Part II. originally appeared as a supplement to the Practical Rules, and is now given in more detail, experience having proved the great value of the free use of Napier's diagram in ascertaining and applying the deviations of the compass, especially when dealing with the larger deviations of iron ships.

Part III. was published separately in the year 1851, and afterwards as a supplement to the Practical Rules in 1855. The large deviations found in iron-plated ships of war having rendered necessary the use of the exact instead of the approximate formulæ, this part has been re-written by its accomplished author, Archibald Smith, Esq., F.R.S., of Lincoln's Inn. Although the mathematical investigations from which these formulæ are derived can only be followed by a skilled mathematician, and have therefore been reserved for an Appendix, yet the *use* of the formulæ, and particularly of the tabular forms for deducing the coefficients A, B, C, D, E, for any ship, may easily be acquired by any one accustomed to make the computations which usually occur in the practice of navigation.

Part IV.* is introduced for the first time in illustration of the three magnetic charts which are now given. These are (1), a

* Now Part V. (1869).

chart of the variation of the compass. This will be found useful, as well in the ordinary practice of navigation as in giving the means of ascertaining the deviation of the compass by astronomical observations made at sea. (2) and (3), charts of the dip and horizontal force. These will be found useful, as throwing light on the changes which the deviations of the compass undergo on a lengthened voyage, and as enabling the navigator to anticipate the changes which will take place on a change of geographical position. The compilation of these charts for a given epoch has been a work of much labour, and has entailed the examination of several thousand observations made by observers of all nations in all parts of the globe. This has been performed by the Superintendent of the Compass Department, F. J. Evans, Esq., R.N., F.R.S.

The Appendix contains such parts as are less generally required by the seaman, but which are necessary in a complete treatise on this subject. No. 1* is a description of the Admiralty standard compass; No. 2,† a brief account of the mechanical modes of correcting the deviation of the compass by fixed magnets and soft iron; No. 3,‡ the mathematical theory of the deviation of the compass. This last is not intended for the use of any except the skilled mathematician.

For further information on the subject, the reader who desires to acquire a complete knowledge is referred to the following:—Papers in the Philosophical Transactions of the Royal Society, Airy, 1839 and 1855; Sabine, Contributions to Terrestrial Magnetism, 1845 to 1861; Evans, 1860; Smith and Evans, 1861. Also the published works by Captain Johnson, R.N., on the Deviations of the Compass, 2nd edit., 1852; Scoresby's Voyage and Introduction, 1857; Liverpool Compass Committee Reports, 1856–61.

I. W.

Hydrographic Office, Admiralty, July 1862.

Advertisement to Second Edition.

A new edition of the Manual being called for, the opportunity is taken to include the recent advances in the theory and practice of finding the correction of the Deviation.

In particular, new methods are introduced of determining the coefficients from observations of deviation and horizontal force made with the ship's head on *one* point; these furnish the means of approximately determining the deviations of a vessel without swinging her, and while she is still on the stocks or in dock.

* Now Appendix III. (1869). † Now Part IV. ‡ Now Appendix I. and II.

Considerable extension is also given to the new graphic methods briefly described in the first edition, these having been found convenient in practice; and in Appendix No. 4 * an endeavour is made to render them available for persons who are not mathematicians.

An Appendix (No. 6)† is also added, which it is hoped will prove interesting as well as useful to the seaman, giving an account of the history and present state of this branch of nautical science.

I. W.

Hydrographic Office, Admiralty, August 1863.

Advertisement to Third Edition.

The first edition of this work was published in 1862. The second, which was little more than a reprint of the first, in 1863. Immediately after its publication it was translated, with more or less modification in the arrangement of its parts, into Russian by Captain Belavenetz of the Imperial Russian Navy; into French by the late M. Darondeau in his *Cours de la Regulation des Compas de l'École du Genie Maritime*; into German by Dr. Schaub, Director of the Hydrographical department of the Austrian Imperial Marine; and more recently into Portuguese by Senhôr Capello, Director of the Nautical Observatory at Lisbon. It has also been adopted in the United States Navy, for the use of which navy the various memoirs on the deviation of the Compass which have appeared in England, and also a translation of Poisson's memoir on the deviation of the Compass, which is the foundation of the present theory, have been published in a convenient form by the Bureau of Navigation.

Since the publication of the first edition no addition has been made to the theory, except a remarkable extension of the Graphic method known as the Dygogram, which is due to Lieut. Colongue of the Imperial Russian Navy, the most useful parts of which have been introduced into the second Appendix to this edition; and an explanation of the differences in the deviation observed according to the direction in which a ship is swinging, due to M. Gaussin,—who as successor to M. Darondeau is charged with the instruction in this matter of the *École du Genie Maritime*,—which has also been introduced into this edition.

A new edition of the work being called for, advantage has been taken of the experience gained since the publication of the last edition, and of the longer time which could be devoted to the preparation of this edition, by improving the arrangement of the parts, and by adding examples to adapt the work to the several

* Now Appendix II. (1869).

† Now omitted.

classes of persons to whom it is believed it will be useful. Advantage has also been taken of this opportunity to give in Part IV., with the detail it is hoped adequate to the increased importance of the subject, methods of correcting or reducing the deviation mechanically, and to add a practical mode of correcting the heeling deviation, which it is believed will be found useful by persons professionally engaged in the adjustment of the compasses of iron ships.

It is considered that Part I. will be intelligible to every seaman competent to command the smallest coasting vessel. Part II. to all seamen competent to use charts and work dead reckoning. Part III. to those who have mastered the parts of Nautical astronomy which are used in navigation. Parts IV. and V. will, it is hoped, be found generally useful. The Appendixes I. and II. are only intended for the use of the skilled mathematician.

G. H. R.

Hydrographic Office, Admiralty,
July 1869.

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NOTE.—*Some portions of the former editions it has not been considered necessary to reprint in the present edition; these are—*

1. Comparison between the results of Poisson's Theory and Mr. Airy's Hypothesis.
1st ed., p. 70. 2nd ed., p. 86.
2. Direct determination of the exact coefficients from observations on 16 or 32 points by disturbed compass.
1st ed., p. 90. 2nd ed., p. 99.
3. Formulæ showing the result of deriving the quadrantal coefficient from $\frac{1}{3}$ of the sum of the seven deviations in each quadrant.
1st ed., p. 92.
4. Formulæ showing the effect of an open hatch on a compass above it.
1st ed., p. 94. 2nd ed., p. 115.
5. Correction of one erroneous observation.
1st ed., p. 103.
6. Theory of the arrangement of the needles in the Admiralty Compass card.
1st ed., p. 104. 2nd ed., p. 140.
7. Comparison of Mr. Airy's notation with that of the Manual.
1st ed., p. 106. 2nd ed., p. 142.
8. Extract from the introduction to Scoresby's Voyage of Magnetic Research.
2nd ed., p. 150.
9. Report to the British Association on the three reports of the Liverpool Compass Committee.
2nd ed., p. 153.

PART I.

PRACTICAL RULES FOR ASCERTAINING AND APPLYING THE DEVIATIONS OF THE COMPASS, CAUSED BY THE IRON IN A SHIP.

1. EVERY ship should be provided with a good Azimuth Compass, which may be called the STANDARD COMPASS; with it the Binnacle Compasses are to be frequently compared, and by it all bearings ought to be taken. It must be placed on the midship line of the quarter-deck or poop, and as far as possible from any considerable mass of iron, and should not be near the extremity of any elongated mass of iron, *especially if vertical*, such as the spindle of the capstan, iron stanchions, the iron quarter-davits, iron funnels, &c. It should be fixed on a permanent support or pillar, and at such a height as will permit amplitudes and bearings to be observed with it over the hammock-nettings, or bulwarks. No stand of arms or other iron, *subject to occasional removal*, should be placed within at least fourteen feet of this Standard Compass, whether on the same deck, or immediately below it.

2. When the ship is ready for sea, with her guns, shot, armstands, and all her iron stores on board and stowed in their proper places, and with the davits and other iron work secured in the positions in which it is intended they shall remain at sea,—then the deviation of the Standard Compass from the magnetic meridian is to be ascertained by one of the following means,—either of which, if the ship be in a basin,* can be executed at any time, but if riding in a tide-way at slack water only.

Process by bearing of a distant Object.

3. The requisite warps being prepared, the ship is to be gradually swung round, so as to bring her head successively upon each of the 32 points of the compass; and as her head approaches each of these points, so gently to check her motion as to prevent any continued swing of the card. When the ship is quite steady, and her head exactly on any one point, the Direct Bearing of some distant but well-defined object is to be observed

* In which there are no iron vessels.

PART I. with the Standard Compass, and registered, according to the Form shown in the first two columns of Table I., page 9.

4. The ship's head is then to be gently warped round in the same manner to the next point, and when duly stopped and steadied there, the bearing of the same object is to be again set, and again recorded; and so on, point after point, till the exact bearing of the one object has been ascertained with the ship's head on every separate point of the compass.

5. The object selected for that purpose should be at such a distance from the ship, that the diameter of the space through which she revolves shall made no sensible difference in its real bearing. If she be at anchor in a tideway; a steeple or a detached house, or a large single tree, at not less than from 6 to 8 miles distance, will well answer the purpose; but if in a basin, a less distance will suffice.

6. The next step is to determine the *real* or *correct* magnetic bearing of the selected object from the ship; or, in other words, the compass bearing which it would have from on board, if the compass were not disturbed by the attraction of the iron in the ship. This may generally be effected by taking the mean of all the observed bearings, if observed on equidistant points; or of four or more bearings, if observed also on equidistant points; but a surer result will be obtained by carrying the Standard Compass to some place on the adjacent wharf or shore, from whence the ship (that is to say, the part of the ship where the compass stood) and the distant object, of which the bearings had been observed, shall both be exactly in one line with the observer's eye.* The bearing of the object from that point will evidently be the same as its *correct* magnetic bearing from the ship by that compass. The difference between this *correct* magnetic bearing of the object, and the bearing observed with the Standard Compass on board, when the ship's head is on any particular point, will show the error on that point which was caused by the ship's iron,—or, in other words, the *Deviation* of the Standard Compass, according to the direction in which the ship's head was placed.

7. These several differences are now to be inscribed opposite to each point, in the third column of Table I.

8. The deviations thus found are named *East* when the north end of the needle is drawn to the eastward, or right hand, by the attraction of

* In some cases it may be more convenient to determine the correct magnetic bearing of the distant object previous to the operation of swinging the vessel; but in all cases, when a compass is landed, great care must be taken that the place in which it is to be used should not be near volcanic rocks or on a granite wharf, nor exposed to the influence of large masses of iron, or of anchors buried in the beach, or of any collection of iron articles in the neighbouring storehouses. The islands of St. Helena and Ascension are familiar examples of volcanic rocks affecting the compass when landed.

the ship's iron; and *West* when it is drawn to the westward, or to the left hand of the magnetic meridian. The employment of the terms *East* and *West Deviation*, being precisely analogous to that of *East* and *West Variation*, must be familiarly understood by every seaman.

And, therefore, in applying the correction for the deviation to the ship's *courses*, or to her compass bearings, he is to adopt the same method exactly as when applying the variation; that is to say, he is to suppose his eye in the centre of the card, and from thence, looking along the point in question, he is to apply the East deviation to the right hand, and the West deviation to the left hand.

Example. If the ship's course by the Standard Compass be N.N.E., and the deviation for that rhumb be $8^{\circ} 10'$ E., as in Table I., then the magnetic direction of her head will be obtained by applying the $8^{\circ} 10'$ to the *right* hand of N.N.E., giving N. $30^{\circ} 40'$ E., or about N.N.E. $\frac{3}{4}$ E.*

9. All *bearings* are in like manner to be corrected by the deviation which is due to the *direction of the ship's head* at the moment that they were taken; and the seaman will from thence perceive the necessity of stating that direction in his log-book, whenever he has any bearings to insert therein.

Example. If, with the ship's head on the same point as in the above example, the bearings of two islands be S.E. and W. by S. by the Standard Compass, then by applying the same correction $8^{\circ} 10'$ E., and in the same way, to the *right* hand, the correct magnetic bearings of those two islands will be S. $36^{\circ} 50'$ E., and S. $86^{\circ} 55'$ W., or roughly S.E. $\frac{3}{4}$ S. and W. $\frac{1}{4}$ S.

10. If it be required to steer the vessel on a certain magnetic course and for that purpose to determine what will be the corresponding course by the Standard Compass, he must recollect that this is the *reverse* of the operation given above, and therefore that the rule must also be *reversed*, taking care that the deviation applied is that of the *correct* magnetic course, see Article 18.

* It has been suggested that as the courses will afterwards have to be corrected for the variation, it would be a more simple process to combine the deviation and variation together, so as to employ them in a single correction. The rule for so doing would be: If they are both of the same name, i.e. both E. or both W., add one to the other, and apply the sum according to their joint name. *Example.* The deviation being as above, $8^{\circ} 10'$ E., and the variation $20^{\circ} 10'$ E., the sum thereof $28^{\circ} 20'$ E., being applied to the *right* hand of N.N.E., gives the true course, N. $50^{\circ} 50'$ E. But if one be E. and the other W., take their difference, give it the name of the greater, and apply it according to that name. *Example.* Suppose the deviation be still $8^{\circ} 10'$ E., but with the variation of $20^{\circ} 10'$ W., their difference $12^{\circ} 0'$ West, applied to the left hand, will give N. $10^{\circ} 30'$ E. for the true course.

PART I.

Process by Reciprocal Bearings.

11. Should there be no suitable object visible from the ship, and at the requisite distance as stated in Art. 5, the deviations must be ascertained by the process of Reciprocal Bearings. A careful observer must go on shore with a second compass, and place its tripod in some open spot, but strictly under the conditions enumerated in the note to Art. 6, and where it may be distinctly seen from the Standard Compass on board. Then, by means of preconcerted signals, the mutual bearings of those two compasses from each other are to be observed at the moment when the ship's head is quietly steady on each of the 32 points successively, as before directed. The mode of registering the observations, when this process with the two compasses has been adopted, is shown in Table II., page 10.

12. To ensure the success of this operation, the compass on shore should not be more distant from the ship than is consistent with the most distinct visibility with the naked eye of both compasses from each other. The observations should be made as strictly simultaneously as possible. And, to guard against any mistake, such as might be occasioned by a signal being misinterpreted, the time at which each bearing is taken should be noted, both on shore and on board, by compared watches.

13. Before this process is complete, the Standard Compass should be carried on shore, in order to be compared with the compass which had been employed there, by means of the bearing of some distant object; and the difference, if any, is to be recorded. This comparison of the two compasses may as well be made previously to the observations; and in all cases when compasses are compared, the caps, pivots, &c., should be first carefully examined.

14. It has been found a very convenient practice for the observer on shore to chalk each observation upon a black board, so that it may be read at once by the observer on board; by which means, if there should be any apparent inconsistency, the observation can be immediately repeated, and the necessity for again swinging the ship thereby prevented.

General Observations.

15. In the foregoing processes for obtaining the deviations of the compass, it has been assumed that every facility for swinging the ship has been available; but as circumstances may render inconvenient the employment of hawsers, and the making observations in detail from point to point of the compass, there will be found annexed (Appendix), the description of a graphic method whereby from the deviations determined,—as the ship swings round at anchor in a tide-way, or to the wind,—on *any* direction of

the ship's head, a curve of deviations for the 32 points may be constructed, and the deviations determined for each point of the compass. This method has the further advantage that any number of observations made in the complete circuit of the ship may be made use of, and the accuracy of the curve and the table proportionally increased.

16. The seaman is also reminded that by observing azimuths of the sun, or other heavenly body, and comparing the variation of his compass so determined with the known variation of the place of observation,* the deviation on that course is at once determined; and thus if a series of observations be made on various directions of the ship's head, as described in the preceding Article, whilst swinging at anchor in a roadstead, or whilst under sail or steam at sea, the graphic method affords him the means of preparing with these results a complete Table of deviations.

17. From the observations thus made, or by either of the processes which have been described, a Table of the Results should be forthwith constructed for general use, and copied by every person on board who keeps a reckoning. The best form for it is given in Table III., page 11, where the first column contains the courses shown by the Standard Compass, and the third column the corresponding correct magnetic courses. The second column, the deviation which is to be applied as a correction to all the *courses* and *bearings* taken by the Standard Compass. See Art. 9.

18. By comparing the first and third columns of this Table III. the seaman may also by inspection, or a simple interpolation, determine what course he will have to steer by the Standard Compass, in order to take up any given magnetic course, as referred to in Art. 10. For example, let the given or correct magnetic course be N.N.E., or N. 22° 30' E.; on referring to column 3 he will find that N. 22° 30' E. lies nearly midway between N. 13° 50' E., and N. 30° 40' E.; the Standard Compass courses corresponding to which are N. by E. and N.N.E.; the course to be steered is consequently N. by E. $\frac{1}{2}$ E. If great accuracy be required, it will be necessary to find the exact proportion between the actual and apparent angular changes of the ship's head with reference to the horizon; referring to the same example he will find that with the ship's head by Standard Compass between N. by E. and N.N.E., the actual angular change is 16° 50', represented by 11° 15' of the compass; in shaping the course,

* A chart of the curves of equal magnetic variation, embracing the whole navigable portions of the globe, is supplied to each of Her Majesty's ships for this purpose.

Azimuth Tables, computed for intervals of four minutes between the parallels of latitude 30° and 60° inclusive, are also furnished to Her Majesty's ships.

PART I. therefore, between these points, the value of the half point is about $8^{\circ} 25'$, the quarter point $4^{\circ} 12'$, and similarly for smaller divisions of the rhumbs.

To prevent, however, the possibility of error in such an important operation as that of shaping a course, a separate Table may be advantageously constructed expressly for that object.* See Table IV., page 12, where the desired course being sought in the first column, is immediately followed by the course to be steered by the Standard Compass; and given in degrees and minutes, as well as in points and fractional parts.

19. These Tables show also the points of *no* deviation, or the positions in which the ship's head must be placed, in order that her local attraction may produce no deviation in the direction of the needle of the Standard Compass. In most wood-built sailing ships these positions will be found not far from North and South, and nearly opposite to each other; but in those that carry steam-engines, they vary on either side of this meridional direction, and are not necessarily opposite each other. In iron-built vessels the direction of the points of *no* deviation are mainly dependent on the direction of the ship's head while building, and are not generally opposite to each other.

20. Experience, as well as theory, shows that not only the amount of deviation, but the ratio of its distribution among the several points, differs in places distant from each other in latitude;† so that in the southern hemisphere the deviation sometimes becomes westerly on the points on which it was easterly in the northern hemisphere, and easterly where it was westerly. It is indispensable, therefore, when a ship changes her geographic position *considerably*, and especially when she has passed from one hemisphere into the other, that new Tables should be formed by again swinging the ship, and thus ascertaining the actual deviation. The new Tables derived therefrom will of course continue in use until again superseded by others resulting from subsequent change of place, and from subsequent observations.

21. The deviations on the different points may be at any time examined at sea, by observing a few careful azimuths with the ship's head on different points, and comparing the variations so determined with that of the known variation at the place of observation, or with the mean of the variations so determined on four or more equidistant points. The changes due to

* It will be found that this can be performed expeditiously and accurately by means of the Graphic method described in Part II.

† In strictness there is a change whenever the dip or the horizontal force changes, and therefore the deviations may change from a change of longitude as well as from a change of latitude; but speaking generally the changes take place principally on a change of latitude.

alteration in the arrangement of iron near the compass, or to geographic position, may thus be daily ascertained and recorded for the points on which the ship may be steering, and ample warning afforded for any necessity of the reconstruction of the Deviation Table of the ship.

22. In iron-built vessels, it is desirable that the Standard Compass should be raised much higher than usual above the deck ; but in all other respects the processes and rules above given will apply to iron as well as to wood-built vessels. It is, however, to be remembered that more than common watchfulness over the compasses of iron-built vessels should be exercised, to see that they are traversing freely, to remedy every defect in their movements, and to verify their deviations as frequently as practicable. *Should an arrangement of magnets be employed to neutralize those large deviations occasionally found, and caused by the iron-ship's magnetism, the compass so corrected can never be considered as entirely compensated, and the deviation must be expected to change, on change of latitude and from other causes.* It will thus be seen that the mariner can have no absolutely safe guide, except in the system of actual and unceasing observation which has been enjoined in the foregoing pages.

23. Experience has also shown that the deviations in iron vessels (also wood-built vessels with iron beams) are affected by the heeling of the ship; the magnitude of the error so resulting being proportional to the amount of heel, and the *maximum* disturbance when heeling being found when the ship's head is *North* or *South* by the observing compass, the disturbance vanishing when the ship's head is *East* or *West* by the observing compass.* It becomes, therefore, an early duty of those in charge of the navigation of iron ships to ascertain by repeated observations at sea the degrees of error arising from the various conditions of heeling both to starboard and to port.

24. In steam vessels fitted with telescopic Funnels, and where, from necessity, the Standard Compass is placed near to the steam machinery,

* In a ship built in England and while she remains in the northern hemisphere the effect of the heeling error will in *general* be, that if the ship is kept steady on the same compass course, she will be found to windward of her supposed position when on northerly courses, to leeward when on southerly courses ; conversely, if she is steered steadily for a fixed point on the horizon she will *appear* by compass to fall off on northerly courses, to come up on southerly. Therefore, in steering by *compass*, in order to make a straight course as the ship heels over, keep away on either tack on northerly courses, and keep closer to the wind on either tack on southerly courses.

In some (but very few ships (chiefly in ships built head South, and which have a compass placed far aft) the effect is the reverse.

Note.—In the southern hemisphere the effect will frequently be the converse of that in the northern hemisphere, but this is a point which can be only ascertained by actual observation for each ship.

PART I. — the deviation is materially altered by the elongated or compressed state of the funnel. For instance, with her head east, H.M.S. *Blenheim* has 9° deviation with the funnel *up*, and $15\frac{3}{4}^{\circ}$ when *down*. In H.M.S. *Vulcan* (of iron) there is a contrary result. In swinging vessels, therefore, so fitted, and when the funnel is within 30 or 35 feet of the compass, it is necessary that on the eight principal points of the compass at least, the deviations should be ascertained in both conditions of the funnel.

25. The mariner must remember that the corrections found for the Standard Compass belong to that compass alone, and to that compass only while it is in its proper place; and that those corrections will furnish no guide whatever to the effects of the ship's iron on a compass placed in any other part of the ship. It is essential, therefore, that the ship's course should not only be *invariably* directed by the Standard Compass, but that all the courses and bearings inserted in the log should be those shown by that compass alone;* the Binnacle Compass being regarded solely as a guide to the helmsman. When the ship has been placed on her proper course by the Standard Compass, the helmsman will notice the point shown by the Binnacle Compass as being that to which *he* has to attend; and a comparison of the two compasses should be frequently repeated by the officer of the watch or the quartermaster, and should always be made when any alteration occurs in the direction of the course, or when the ship heels over. When the ship is by the wind, the apparent course by the Standard Compass is that which must be inserted in the log.

26. In ships where a single Binnacle Compass is made to traverse, so as to be in front of the helmsman on each tack, it must be compared afresh with the Standard Compass at every such traverse, because the effect of the local attraction on the Binnacle Compass will have been altered by its change of place. In ships which have two permanently-fixed Binnacle Compasses the comparison with the Standard must likewise be frequently repeated, both when the helmsman changes sides, and when the course is changed, for there will necessarily be some small difference in the deviations of these two compasses; the only safe or practical remedy is, therefore, *direct comparison on all occasions with the Standard Compass*.

27. During either of the processes for ascertaining the deviations of the Standard Compass, when the ship's head pauses at each point, it is desirable to note, in a separate Table, the corresponding directions of her head by the Binnacle Compasses.

28. It would also be a useful practice at sea to compare every day, at *noon*, the direction of the ship's head by the Binnacle Compasses with that

* If an iron ship, the degree of heel should also be noted, and whether to starboard or to port.

GENERAL OBSERVATIONS ON DEVIATIONS OF COMPASS. 9

shown by the Standard, on whatever course the ship may then be steering. PART I.
This daily comparison would serve as a convenient check in working the reckoning, and circumstances might render it an important record if entered in the log-book.

29. Finally. Should any temporary accident happen at sea to the Standard Compass, it should be immediately and accurately stated in the log-book.

OBSERVATIONS for determining the Deviations of the Standard Compass in H.M. Ship *Trident*, 23rd December 1856, at *Greenhithe*; Correct Magnetic Bearing of *Severndroog Tower* from the Ship, N. 63° 0' W., distant 11 miles.

TABLE I. *Form for registering the Process by Direct Bearing.*—
See Art. 7.

Ship's Head by the Standard Compass.	Bearing of Tower by the Standard Compass.	Deviation of the Standard Compass.
	° ' "	° ' "
North	N. 59 50 W.	3 10 W.
N. by E.	N. 65 35 W.	2 35 E.
N.N.E.	N. 71 10 W.	8 10 E.
N.E. by N.	N. 76 10 W.	13 10 E.
N.E.	N. 79 50 W.	16 50 E.
N.E. by E.	N. 82 30 W.	19 30 E.
E.N.E.	N. 83 30 W.	20 30 E.
E. by N.	N. 84 5 W.	21 5 E.
East	N. 83 20 W.	20 20 E.
E. by S.	N. 82 15 W.	19 15 E.
E.S.E.	N. 81 5 W.	18 5 E.
S.E. by E.	N. 79 30 W.	16 30 E.
S.E.	N. 77 40 W.	14 40 E.
S.E. by S.	N. 75 5 W.	12 5 E.
S.S.E.	N. 72 40 W.	9 40 E.
S. by E.	N. 69 0 W.	6 0 E.
South	N. 66 10 W.	3 10 E.
S. by W.	N. 63 5 W.	0 5 E.
S.S.W.	N. 60 0 W.	3 0 W.
S.W. by S.	N. 56 30 W.	6 30 W.
S.W.	N. 53 20 W.	9 40 W.
S.W. by W.	N. 50 0 W.	13 0 W.
W.S.W.	N. 46 50 W.	16 10 W.
W. by S.	N. 43 45 W.	19 15 W.
West	N. 41 50 W.	21 10 W.
W. by N.	N. 39 40 W.	23 20 W.
W.N.W.	N. 39 0 W.	24 0 W.
N.W. by W.	N. 39 25 W.	23 35 W.
N.W.	N. 41 0 W.	22 0 W.
N.W. by N.	N. 44 0 W.	19 0 W.
N.N.W.	N. 48 10 W.	14 50 W.
N. by W.	N. 53 45 W.	9 15 W.
North	N. 59 50 W.	3 10 W.

PART I. OBSERVATIONS for determining the Deviations of the Standard Compass
in H.M. Ship *Trident*, 23rd December 1856, at *Greenhithe*.

The Standard Compass and a compass marked No. 33 were compared on shore, by taking the bearing of a distant object with each of them ; but kept several feet apart, in order to prevent their having any influence on each other. The bearings, by the mean of several repetitions, were found to agree (see note to Art. 6, and Art. 13), and No. 33 being retained on shore, the Standard Compass was then replaced on board, and their mutual bearings from each other were simultaneously observed when the ship's head was successively on each of the 32 points as follows :—

TABLE II. *Form for registering the Process by Reciprocal Bearings.—*
See Art. 11.

Time.	Ship's Head by the Standard Compass.	Simultaneous Bearings.		Deviation of the Standard Compass.
		From Standard Compass on Board.	From the Shore Compass.	
h. m.		° ' "	° ' "	' "
9 10	North.	S. 37 50 E.	N. 41 0 W.	3 10 W.
9 14	N. by E.	S. 45 0 E.	N. 42 25 W.	2 35 E.
9 17	N.N.E.	S. 51 40 E.	N. 43 30 W.	8 10 E.
9 21	N.E. by N.	S. 57 20 E.	N. 44 10 W.	13 10 E.
And in like manner at all points of the Compass.				

TABLE III. *Form for tabulating the Results* (see Art. 17); or the

PART I.

DEVIATION TABLE FOR H.M.S. TRIDENT.

Greenhithe, 23rd December 1856.

I. Ship's Head, or Course by the Standard Compass.	II. Deviation of the Standard Compass.	III. Magnetic Course steered.
	° /	° /
North	3 10 W.	N. 3 10 W.
N. by E.	2 35 E.	N. 13 50 E.
N.N.E.	8 10 E.	N. 30 40 E.
N.E. by N.	13 10 E.	N. 46 55 E.
N.E.	16 50 E.	N. 61 50 E.
N.E. by E.	19 30 E.	N. 75 45 E.
E.N.E.	20 30 E.	N. 88 0 E.
E. by N.	21 5 E.	S. 80 10 E.
East	20 20 E.	S. 69 40 E.
E. by S.	19 15 E.	S. 59 30 E.
E.S.E.	18 5 E.	S. 49 25 E.
S.E. by E.	16 30 E.	S. 39 45 E.
S.E.	14 40 E.	S. 30 20 E.
S.E. by S.	12 5 E.	S. 21 40 E.
S.S.E.	9 40 E.	S. 12 50 E.
S. by E.	6 0 E.	S. 5 15 E.
South	3 10 E.	S. 3 10 W.
S. by W.	0 5 E.	S. 11 20 W.
S.S.W.	3 0 W.	S. 19 30 W.
S.W. by S.	6 30 W.	S. 27 15 W.
S.W.	9 40 W.	S. 35 20 W.
S.W. by W.	13 0 W.	S. 43 15 W.
W.S.W.	16 10 W.	S. 51 20 W.
W. by S.	19 15 W.	S. 59 30 W.
West	21 10 W.	S. 68 50 W.
W. by N.	23 20 W.	S. 77 55 W.
W.N.W.	24 0 W.	S. 88 30 W.
N.W. by W.	23 35 W.	N. 79 50 W.
N.W.	22 0 W.	N. 67 0 W.
N.W. by N.	19 0 W.	N. 52 45 W.
N.N.W.	14 50 W.	N. 37 20 W.
N. by W.	9 15 W.	N. 20 30 W.
North	3 10 W.	N. 3 10 W.

To obtain the correct magnetic course of the vessel from the course shown by the Standard Compass, look in the 1st column of the above Table for the latter; the 2d column gives the deviation when her head is on that point; and in the 3rd column (the deviation having been applied as directed in Art. 8), the seaman will find the correct magnetic course given there, by inspection.

In order to correct any Bearings taken by the Standard Compass, the Table is to be entered with the direction of the ship's head at the time, in the 1st column, and, corresponding thereto, in the 2d column will be found the amount of deviation to be applied. See Article 9.

PART I.

TABLE IV.

To prevent any mistake in shaping a course by the Standard Compass, it will be expedient to prepare a Table like the following, where the Corrections for Deviation have been applied conversely, as suggested in Art. 18.

Example. Steering Table, H.M.S. *Trident*. *English Channel*, December 1856.

Magnetic Course proposed to be made.	Course therefore to be steered by Standard Compass in order to make the proposed Correct Magnetic Course.	
	° /	
North	N. 2 0 E.	or nearly N. $\frac{1}{4}$ E.
N. by E.	N. 9 30 E.	N. $\frac{1}{2}$ E.
N.N.E.	N. 16 50 E.	N. by E. $\frac{1}{4}$ E.
N.E. by N.	N. 23 30 E.	N.N.E. $\frac{1}{4}$ E.
N.E.	N. 32 40 E.	N.N.E. $\frac{1}{2}$ E.
N.E. by E.	N. 41 0 E.	N.E. $\frac{1}{4}$ N.
E.N.E.	N. 49 30 E.	N.E. $\frac{1}{2}$ E.
E. by N.	N. 59 15 E.	N.E. by E. $\frac{1}{4}$ E.
East	N. 69 20 E.	E.N.E. $\frac{1}{2}$ E.
E. by S.	N. 80 30 E.	E. $\frac{1}{4}$ N.
E.S.E.	S. 87 30 E.	E. $\frac{1}{2}$ S.
S.E. by E.	S. 75 0 E.	E. by S. $\frac{1}{4}$ S.
S.E.	S. 62 10 E.	S.E. by E. $\frac{1}{4}$ E.
S.E. by S.	S. 49 0 E.	S.E. $\frac{1}{2}$ E.
S.S.E.	S. 34 45 E.	S.E. $\frac{3}{4}$ S.
S. by E.	S. 19 40 E.	S. by E. $\frac{1}{2}$ E.
South	S. 4 15 E.	S. $\frac{3}{4}$ E.
S. by W.	S. 11 15 W.	S. by W.
S.S.W.	S. 26 50 W.	S.S.W. $\frac{1}{4}$ W.
S.W. by S.	S. 43 0 W.	S.W. $\frac{1}{4}$ S.
S.W.	S. 58 30 W.	S.W. by W. $\frac{1}{4}$ W.
S.W. by W.	S. 74 0 W.	W. by S. $\frac{1}{4}$ S.
W.S.W.	S. 88 40 W.	W. $\frac{1}{4}$ S.
W. by S.	N. 78 15 W.	W. by N.
West	N. 66 15 W.	N.W. by W. $\frac{1}{4}$ W.
W. by N.	N. 55 20 W.	N.W. $\frac{1}{2}$ W.
W.N.W.	N. 45 40 W.	N.W.
N.W. by W.	N. 36 35 W.	N.W. $\frac{1}{4}$ N.
N.W.	N. 27 50 W.	N.N.W. $\frac{1}{4}$ W.
N.W. by N.	N. 20 5 W.	N. by W. $\frac{1}{4}$ W.
N.N.W.	N. 12 40 W.	N. by W. $\frac{1}{2}$ W.
N. by W.	N. 5 20 W.	N. $\frac{1}{4}$ W.
North	N. 2 0 E.	N. $\frac{1}{4}$ E.

Whenever, from the alteration of guns, iron fittings near the Compass, or cargo, or from any considerable change of latitude, the Deviation of the Standard Compass is found to vary, a new Table for the above purpose should be immediately constructed.

TABLE

PART I.

For converting points of the Compass, and their fractional parts, into Degrees, &c.

POINTS.	No.	DEGREES, &c.	No.	POINTS.
NORTH.	0	0 0 0	0	SOUTH.
	$\frac{1}{8}$	1 24 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	2 48 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	4 13 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	5 37 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	7 1 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	8 26 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	9 50 37	$-\frac{7}{8}$	
N. by E....N. by W.....	1	11 15 0	1	S. by W.....S. by E.
	$\frac{1}{8}$	12 39 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	14 3 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	15 28 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	16 52 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	18 16 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	19 41 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	21 5 37	$-\frac{7}{8}$	
N.N.E.....N.N.W.....	2	22 30 0	2	S.S.W.....S.S.E.
	$\frac{1}{8}$	23 54 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	25 18 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	26 43 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	28 7 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	29 31 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	30 56 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	32 20 37	$-\frac{7}{8}$	
N.E. by N....N.W. by N.	3	33 45 0	3	S.W. by S.....S.E. by S.
	$\frac{1}{8}$	35 9 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	36 33 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	37 58 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	39 22 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	40 46 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	42 11 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	43 35 37	$-\frac{7}{8}$	
N.E.....N.W.....	4	45 0 0	4	S.W.....S.E.
	$\frac{1}{8}$	46 24 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	47 48 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	49 13 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	50 37 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	52 1 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	53 26 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	54 50 37	$-\frac{7}{8}$	
N.E. by E....N.W. by W.	5	56 15 0	5	S.W. by W.....S.E. by E.
	$\frac{1}{8}$	57 39 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	59 3 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	60 28 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	61 52 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	63 16 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	64 41 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	66 5 37	$-\frac{7}{8}$	
E.N.E.....W.N.W.....	6	67 30 0	6	W.S.W. E.S.E.
	$\frac{1}{8}$	68 54 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	70 18 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	71 43 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	73 7 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	74 31 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	75 56 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	77 20 37	$-\frac{7}{8}$	
E. by N.....W. by N...	7	78 45 0	7	W. by S.....E. by S.
	$\frac{1}{8}$	80 9 22	$-\frac{1}{8}$	
	$\frac{1}{4}$	81 33 45	$-\frac{1}{4}$	
	$\frac{3}{8}$	82 58 7	$-\frac{3}{8}$	
	$\frac{1}{2}$	84 22 30	$-\frac{1}{2}$	
	$\frac{5}{8}$	85 46 52	$-\frac{5}{8}$	
	$\frac{3}{4}$	87 11 15	$-\frac{3}{4}$	
	$\frac{7}{8}$	88 35 37	$-\frac{7}{8}$	
EAST WEST...	8	90 0 0	8	WEST EAST.

PART II.

GRAPHIC METHOD OF CORRECTING THE DEVIATIONS OF A SHIP'S COMPASS.—NAPIER'S CURVE.

1. The deviations of the Standard Compass on a certain number of points having been observed in the manner described in the "Practical Rules, &c.," two problems remain to be solved in order to derive the greatest advantage from the observations:—1st. From observations made on a small number of points to find the most probable value of the deviations on the intermediate points. 2d. From observations made on a large number of points to find the most probable value of the deviations on each point.

The solution of one of these problems is the solution of the other. A mathematical solution depending on the method known to mathematicians as the method of least squares is given in the Admiralty Manual of the Deviations of the Compass. This solution is the best when applicable, but it requires that the points on which the observations have been made should be equidistant, and it requires a considerable amount of calculation.

2. A Graphic solution (which is due to J. R. Napier, Esq., F.R.S.,) has the advantage of being equally applicable, whether the points on which observations have been made are or are not precisely equidistant. It requires no calculation, and only a moderate degree of neat-handedness.

The method consists of two parts, the diagram and the curve. The diagram is the same for all vessels; it is therefore engraved, and furnished to Her Majesty's ships.*

Description of the Diagram.

3. The diagram consists of a vertical line of convenient length,—say 18 inches,—divided into 32 equal parts, the divisions representing the 32 points of the compass in the following order, beginning at the top: North, N. by E., N.N.E., &c. The line is also divided into 360 equal parts, representing degrees, and these divisions are numbered from 0° at

* Sold by J. D. Potter, Admiralty Chart Agent, 31, Poultry, London.

PART II. the top to 360° at the bottom. They are also numbered, according to the usual mode of dividing the circumference of the compass card, from 0° at North and South, up to 90° at East and West. The line may in fact be considered as the margin of a compass card cut at the North point and straightened. The vertical line is intersected at each of the 32 points by two straight lines, inclined to it at angles of 60°. One, a *plain* line, inclined to the right; the other, a *dotted* line, inclined to the left.

Requisite Observations to be made.

4. The least number of observed deviations available for obtaining a complete curve are the deviations on 4 points distributed equally, or nearly so, round the compass; but, if possible, the deviations should be observed on 8 or more points. If the deviations are observed on 4 points only, these should be at or near N.E., S.E., S.W., and N.W. The points next in importance are North, East, South, West.

Cases may also occur in which by the ship swinging round at her anchor in a tide-way, or to the wind, or by the aid of a steam-tug, the deviations may be observed on various directions of the ship's head, not being necessarily exact points of the compass; or similarly whilst under steam or sail at sea, a number of azimuths of the sun may be observed, and hence the deviation obtained. See Art. 16.

In these cases the Graphic method here described furnishes a ready and effectual mode of obtaining a result in which the errors of individual observation are as far as possible compensated, and any egregious errors eliminated. The following examples for constructing the curve of deviations will render intelligible this method.

Construction of the Curve of Deviations.

5. The observed deviations are to be laid down on the diagram in the following manner. Take, on the vertical line, one of the *compass** courses on which an observation has been made, and lay off the amount of the observed deviation on the dotted line which passes through it, or if no dotted line passes through it, then in a direction parallel to the dotted lines, to the right if the observed deviation be easterly, to the left if westerly, and mark the point so determined with a cross, or dot encircled in ink. Observe that the scale on which the deviations are to be so laid off is the same as the scale of the vertical line, viz., $\frac{1}{20}$ of an inch to the degree. Perform the same operation for each observed deviation. Then

* If in the table the deviations are given for the *correct magnetic* and not the *compass* courses, the same process is gone through, except that the deviations are in that case laid off on the *plain* lines.

with a pencil and a light hand, draw a flowing curve, passing as nearly as possible through all the crosses or dots encircled; and when satisfied with the curve in pencil, draw it in ink. This is the curve of deviations.

6. The curve of deviations having been completed, the diagram affords a ready and convenient method of applying the deviation to the ship's courses. This correction may be required as follows:—1st, from the compass course which has been steered, it may be required to find the magnetic course to be laid down on the chart; or, 2dly, from the correct magnetic course given by the chart, it may be required to find the compass course on which the ship's head ought to be kept. The corrections are given by the following rules.*

RULE I.—From a given Compass Course, to find the corresponding Magnetic Course.

On the vertical line take the given compass course. Move in a direction parallel to the *dotted* lines till you arrive at the curve, and then move in a direction parallel to the *plain* lines till you get back to the vertical line. The point in the vertical line at which you arrive is the magnetic course required.

Example.—If in H.M.S. *Trident* the Standard Compass course be N.N.E., follow the *dotted* line of that rhumb till it meets the curve, and then return to the vertical line in a direction parallel to the *plain* line; it will be found to intersect it at N. 30° 40' E., the required magnetic course.

RULE II.—From a given Magnetic Course, to find the corresponding Compass Course.

On the vertical line take the given magnetic course. Move in a direction parallel to the *plain* lines till you arrive at the curve, and then move in a direction parallel to the *dotted* lines till you get back to the vertical line. The point on the vertical line at which you arrive is the compass course required.

Example.—If in H.M.S. *Trident* the magnetic course required be N.N.E., follow the *plain* lines of that rhumb till it meets the curve, and then returning to the vertical line in a direction parallel to the *dotted* line, it will be found to intersect it at N. 16° 50' E. or N. by E. $\frac{1}{2}$ E., the required course by Standard Compass.

* A modification of the graphic method of correcting the ship's course for the Deviation of the Compass, by Rear Admiral A. P. Ryder, is also published by the Admiralty, and sold by J. D. Potter, Admiralty Chart Agent, 31, Poultry, London.

PART II. 7. The only difficulty in applying these rules is, to remember in each case whether we ought to depart from the central line by a *dotted* line, and return to it by a *plain* line; or whether we ought to depart from the central line by a *plain* line, and return to it by a *dotted* line.

The doubt will be removed if the following lines, in which the two rules are versified, are committed to memory:—

I.

“From compass course magnetic course to gain,
Depart by dotted and return by plain.”

II.

“But, if you seek to steer a course allotted,
Take plain from chart and keep her head on dotted.”

8. Illustrations of the diagrams are appended, in which **Example I.** represents the curve of Deviations of H.M. Steam-vessel *Trident*, when swung at Greenhithe in 1856. See Table I., page 9.

EXAMPLE II. represents the curve of Deviations of H.M. Steamship *Warrior*, obtained at Spithead between the 12th and 17th October 1861, while swinging to the wind and tide, and partly towed by a tug steam vessel. PART II.

Direction of Ship's Head by Standard Compass.	Observed Deviations.	Direction of Ship's Head by Standard Compass.	Observed Deviations.
N. 8 30 E.	7 50 W.	S. 12 0 W.	*18 45 E.
N. 16 40 E.	8 0 W.	S. 20 0 W.	*22 15 E.
N. 30 40 E.	10 10 W.	S. 29 0 W.	25 20 E.
N. 33 0 E.	10 40 W.	S. 31 0 W.	*25 45 E.
N. 41 30 E.	13 0 W.	S. 34 0 W.	27 40 E.
N. 52 30 E.	15 0 W.	S. 39 20 W.	*28 5 E.
N. 62 0 E.	15 50 W.	S. 42 0 W.	28 20 E.
N. 71 0 E.	18 40 W.	S. 47 30 W.	28 0 E.
N. 86 0 E.	22 20 W.	S. 50 30 W.	*28 0 E.
S. 81 0 E.	23 0 W.	S. 53 0 W.	28 0 E.
S. 76 30 E.	24 0 W.	S. 53 30 W.	*29 0 E.
S. 67 0 E.	25 20 W.	S. 57 30 W.	27 20 E.
S. 59 0 E.	26 10 W.	S. 62 0 W.	*26 10 E.
S. 52 30 E.	24 20 W.	S. 64 0 W.	26 35 E.
S. 46 0 E.	24 20 W.	S. 67 30 W.	*26 25 E.
S. 39 0 E.	20 40 W.	N. 89 0 W.	18 40 E.
S. 35 30 E.	19 35 W.	N. 78 30 W.	14 40 E.
S. 32 0 E.	18 0 W.	N. 67 0 W.	10 20 E.
S. 25 20 E.	*10 20 W.	N. 57 0 W.	6 40 E.
S. 17 30 E.	*7 50 W.	N. 45 0 W.	3 30 E.
S. 11 20 E.	*3 25 W.	N. 38 30 W.	1 0 E.
S. 8 0 W.	*7 30 E.	N. 27 30 W.	3 0 W.
S. 4 40 W.	*8 15 E.	N. 21 0 W.	2 40 W.
S. 6 30 W.	*9 45 E.	N. 5 0 W.	6 20 W.

Note.—The above deviations were chiefly observed by reciprocal bearings with a shore compass; those marked * were obtained by the sun's true bearing or Azimuth, as described in Art. 15.

PART II. From the curve of Deviations in EXAMPLE II. the following Tables are
— deduced by measurement from the Diagram.

H.M.S. *Warrior*, Spithead, October 1861. Deviation Table.

Ship's Head by Standard Compass.	Deviation.	Ship's Head by Standard Compass.	Deviation.
	° /		° /
NORTH	6 30 W.	SOUTH	5 30 E.
N. by E.	7 35 W.	S. by W.	14 20 E.
N.N.E.	9 0 W.	S.S.W.	22 30 E.
N.E. by N.	10 50 W.	S.W. by S.	27 10 E.
N.E.	13 0 W.	S.W.	28 35 E.
N.E. by E.	15 10 W.	S.W. by W.	28 0 E.
E.N.E.	17 40 W.	W.S.W.	25 40 E.
E. by N.	20 15 W.	W. by S.	22 30 E.
EAST	22 15 W.	WEST	19 15 E.
E. by S.	23 50 W.	W. by N.	15 0 E.
E.S.E.	25 0 W.	W.N.W.	10 30 E.
S.E. by E.	25 0 W.	N.W. by W.	6 30 E.
S.E.	23 30 W.	N.W.	3 0 E.
S.E. by S.	18 50 W.	N.W. by N.	0 0
S.S.E.	11 10 W.	N.N.W.	3 0 W.
S. by E.	3 0 W.	N. by W.	5 0 W.

Table of Deviations on *Magnetic* Points.

Ship's Head <i>Magnetic</i> .	Deviation.	Ship's Head <i>Magnetic</i> .	Deviation.
	° /		° /
NORTH	7 10 W.	SOUTH	3 10 E.
N. by E.	8 45 W.	S. by W.	7 50 E.
N.N.E.	10 50 W.	S.S.W.	12 50 E.
N.E. by N.	13 40 W.	S.W. by S.	17 50 E.
N.E.	16 30 W.	S.W.	22 10 E.
N.E. by E.	19 30 W.	S.W. by W.	26 15 E.
E.N.E.	22 10 W.	W.S.W.	23 15 E.
E. by N.	24 0 W.	W. by S.	28 25 E.
EAST	25 10 W.	WEST	26 40 E.
E. by S.	25 0 W.	W. by N.	22 45 E.
E.S.E.	24 20 W.	W.N.W.	16 50 E.
S.E. by E.	20 10 W.	N.W. by W.	10 15 E.
S.E.	16 0 W.	N.W.	4 35 E.
S.E. by S.	11 10 W.	N.W. by N.	0 0
S.S.E.	6 25 W.	N.N.W.	3 40 W.
S. by E.	1 40 W.	N. by W.	6 0 W.

PART III.

APPLICATION OF THEORY TO PRACTICE IN THE CORRECTION OF THE DEVIATIONS OF A SHIP'S COMPASS.

SECTION I.

Introductory Observations.

THE methods of correcting the deviations of the compass described in Parts I. and II. are purely practical.

A table of the deviations of the compass, formed as directed in Part I., or a Napier's curve constructed as directed in Part II., conveys sufficient information for the ordinary purposes of navigation at the time and place at which the ship has been swung. The table so formed, and the curve so constructed, are the results of observation alone, and are independent of any theory.

We find, however, when we apply theory to the observed deviations, that they follow simple laws, the knowledge of which enables us to construct a table of deviations from a smaller number of observed deviations than would otherwise be possible; and likewise to express the magnetic character of the ship by means of a few coefficients; and by means of these to anticipate, with some degree of confidence, the changes which the deviations will undergo on a change of magnetic latitude. Theory also enables us, from observations of deviation, combined with observations of the horizontal and vertical force, to infer the amount of the deviation arising from the heeling of the ship.

These results depend principally on this, that the deviation (on an even keel) may in all cases be represented by means of *five* coefficients, of which *two* are in ordinary cases so small that they may be left out of account; and the principal, and in general the only important part of the heeling deviation, by *two* additional coefficients. The principal object of this part of the Instructions is to give methods of determining these coefficients readily and with accuracy from either a large or a small number of observed deviations; methods of deducing the deviation on all points of the compass from the coefficients, and of anticipating the changes which take place in the deviation on a change of geographical position; and also methods of determining the heeling deviation, and we shall proceed in this section—

1. To describe generally the nature of the deviation of the compass in iron and wood built vessels.
2. To explain the causes of the several parts of the deviation and of their change on a change of geographical position.

PART III. 3. To explain the nature and cause of the additional deviation arising from the heeling of the ship.

We shall then give —

In Section II., formulæ and examples requiring observations of deviation only.

In Section III., formulæ and examples requiring observations of deviation and of horizontal force.

In Section IV., formulæ and examples requiring observations of deviation and horizontal and vertical force.

§ 1. *Description of the Deviation of the Compass in Iron and Wood-built Vessels.*

Since the publication of these Instructions, in the year 1855, the increased use of iron in the construction and armature of ships has materially altered this question.

The deviation of the compass in iron-built vessels differs from that in wood-built vessels, not only in amount, but also in the proportion and arrangement of its constituent parts; and in the law according to which those parts change with a change of geographical position and with the lapse of time.

In wood-built vessels the deviations are more regular in appearance than in iron-built vessels. But in all vessels the deviation, however irregular in appearance, consists of two principal parts, the "Semicircular" and the "Quadrantal," with the addition in some cases of a small "Constant" part. Each of these parts is in itself perfectly regular, any apparent irregularity arising simply from the mode of super-position of the parts.

Semicircular Deviation.—The "Semicircular" deviation is so called from being easterly in one semicircle and westerly in the other, as the ship's head moves round a complete circle in azimuth. The points of no semicircular deviation, sometimes called the neutral points, are opposite to each other, and the deviation, or more exactly, the sine of the deviation, in each semicircle is proportional to the sine of the azimuth of the ship's head measured from the neutral points, as shown by the disturbed compass. In wood-built ships the neutral points coincide nearly with the north and south points, *i.e.*, the semicircular deviation is zero when the ship's head is north or south by compass, and the deviation when the ship's head is east is, (in northern latitudes,) generally easterly, and when the ship's head is west, westerly.* In iron-built ships the neutral points or points of no

* This results from the north point of the compass being in such cases drawn towards the ship's head. The effect of the north point being so drawn is generally shown by the ship's true place when her course is nearly east or west being found to the southward of her place by dead reckoning. In the English channel therefore the effect of deviation unallowed for is generally to keep a vessel off the English shore; but in the cases mentioned at the end of this paragraph to throw her on the English shore.

semicircular deviation approximate to those points of the compass to which the ship's head and stern were directed in building, the deviation being easterly when *the part of the ship which was south in building* is east, westerly when it is west. It follows from this that in an iron ship built head north, the deviations will be *westerly* when the ship's head is *easterly*, and *easterly* when the ship's head is *westerly*. This being opposite to what is generally found in wood-built ships in northern latitudes is often a source of perplexity to the seaman.*

Quadrantal Deviation.—The “Quadrantal” deviation is so called from its being easterly and westerly alternately in the four quadrants as the ship's head moves round a complete circle in azimuth; its zero points coinciding very nearly with the cardinal points, and its amount, or more exactly, the sine of its amount being proportional to the sine of twice the azimuth of the ship's head, measured, however, neither from the correct magnetic north nor from the direction of the disturbed needle, but from a line half-way between these two directions. With very rare exceptions, the quadrantal deviation is easterly in the N.E. and S.W. quadrants, westerly in the N.W. and S.E.†

Constant Deviation.—The “Constant” part of the deviation is, (unless the compass be improperly placed,) in all cases in which the observations have been carefully made with good instruments, small in amount, and it

* In northern latitudes the part of the ship which was north in building is much less strongly magnetised than that which was south in building; and it is desirable, if possible, to have a standard compass near that end which was north in building, but of course not so near as to be powerfully influenced by a vertical iron sternpost or stem. This position has the additional advantage, that the vertical disturbing force will probably then act upwards, and so counteract instead of conspiring with the transverse induction in causing the heeling deviation (see p. 33, *post*). As the most convenient position of a standard compass is near the stern, it seems desirable on this ground that iron ships should, in northern latitudes, be built head south.

† When the observed deviations are laid down graphically by Napier's curve, in which easterly deviations are represented by positive ordinates, westerly deviations by negative, the semicircular deviation is represented (the ordinates being measured along the *dotted lines*) by a regular curve of sines cutting the axis at two points 180° distant from each other; the quadrantal deviation (the ordinates being measured *perpendicularly to the central line*) by a regular curve of sines cutting the axis at 0° , 90° , 180° , and 270° , being + between 0° and 90° , and between 180° and 270° , and – between 90° and 180° and between 270° and 360° . If the zero points of the semicircular and quadrantal curves coincide, the curves of easterly and westerly deviation, which arise from the superposition of the semicircular and quadrantal curves, are of equal extent, and are inversely similar, and are regular in appearance; but if a maximum of the semicircular happens to correspond with a maximum of the quadrantal, the curve on one side of the axis will be found to be narrowed and peaked, on the other side to be broadened and flattened, and the curves of easterly and westerly deviation may be entirely dissimilar. In this case, too, the table of deviations will present an appearance of irregularity.

PART III. probably more often arises from index or other instrumental error, or from error in the assumed direction of the magnetic north than from the action of the iron in the ship. This part of the deviation, so far as it arises from the action of the iron of the ship, does not change with a change of latitude, but, of course, if it arises from index or other error, it may have different apparent values at different places and times. When the mean of the several directions of the north point of the compass on the several courses on which observations have been made, is taken as the magnetic north, there is apparently no constant deviation.

The Quadrantal deviation, besides the remarkable uniformity in its zero points and in its direction in the several quadrants, has the further remarkable property of remaining unchanged in all magnetic latitudes, and of being little changed by the lapse of time. When, therefore, it has once been ascertained for a particular compass in a particular spot in a particular ship, its unchangingness may be relied on so long as the distribution of the iron in the ship is unchanged.

The Semicircular deviation differs from the quadrantal and the constant deviations in those respects. In all ships the semicircular deviation changes with a change of geographical position. In wood-built ships it varies nearly as the tangent of the magnetic dip at the place of the ship.* It consequently becomes zero at or near the magnetic equator, or in dip 0° , and changes its sign when the ship goes from northern to southern magnetic latitudes; the north point of the compass in such ships is generally drawn towards the bow in northern latitudes, and towards the stern in southern latitudes. In iron-built ships the semicircular deviation consists of two parts—one, the induced part, which is generally the smaller, resembling the semicircular deviation of wood-built ships, and varying (approximately) as the tangent of the dip; the other having its zero point when the ship's head or stern is on or near the point on which the ship's head was when building, and varying (approximately) inversely as the earth's horizontal force at the place. If the two parts varied exactly, and not merely approximately according to these laws, we might, by observations made in two different latitudes, discriminate the two parts, and might predict the changes the deviations would undergo in different latitudes. But although we cannot do this merely from such observations, we can in ships of certain classes, the magnetic peculiarities of which have been studied, and when the lapse of a year or two from launching has caused the magnetism to assume a

* The induced magnetism in the iron of wood-built ships seem to require a little time to acquire its full charge of induced magnetism, and the semicircular deviation is on this account more nearly proportional to the tangent of the dip at the place at which the ship was a few days previously.

nearly permanent state, predict these changes with a considerable degree of approximation. The changes which take place in the magnetism of iron ships on a change of latitude are, however, a matter on which extended and accurate observations are still much wanted. PART III.

The amount of the maximum of Semicircular deviation in England, in wood-built ships of war, does not generally exceed 10° , but in iron-built ships it frequently exceeds 30° even at the standard compass. The Quadrantal deviation in wood-built ships does not often exceed 1° or 2° ; in ordinary iron-built ships it ranges from 3° and 4° to 6° and 7° , and in some of the armour plated iron-built ships of war it has been found to amount to $8\frac{1}{2}^\circ$ at the standard compass, and to 15° in compasses less favourably placed.

§ 2. *Explanation of the Causes of the several parts of the Deviation, and of their Changes on a change of Latitude.*

In order to understand the effect of the ship's iron in producing the deviation of the compass, it is necessary to understand in some degree the general nature of the action of the Earth's magnetism on the needle.

Effect of Earth's magnetism.—The general effect of the Earth's magnetism on the needle is to draw the north end of the needle to the north, the south end to the south. The effect on the south end of the needle is simply to double the effect on the north end without altering its direction, and as we are only dealing with comparative effects, we may confine our attention to the effect on the north end of the needle.

Line of force.—The direction of this force, sometimes called the "line of force," is not, in general, towards the geographical or "true" north, but to the east or west of it, by an angle known to seamen as the "variation of the compass," sometimes called the "declination."

The line of force is not, in general, horizontal, but lies below or above the horizontal plane, by an angle which is known as the "dip" or "inclination" of the needle.

Magnetic equator.—The line of force is horizontal, or the dip is zero, along one particular line on the globe, not differing very much from the terrestrial equator, and called by analogy *the magnetic equator*. North of the magnetic equator the north end of the needle is drawn downwards, or in other words, in addition to the horizontal force there is a vertical force acting downwards. South of the magnetic equator the north end of the needle is drawn upwards or, in addition to the horizontal force, there is a vertical force acting upwards.

The dip increases from the magnetic equator where it is zero, to the magnetic poles, where it is 90° .

PART III. If we call the distance of a point on the earth's surface from the magnetic equator its magnetic latitude we have approximately, taking the horizontal force at the magnetic equator as unit,—

$$\text{Hor. force} = \cos \text{mag. lat.}$$

$$\text{Vert. force} = 2 \sin \text{mag. lat.}$$

$$\text{Tan dip.} = 2 \tan \text{mag. lat.}$$

$$\text{*Total force} = \sqrt{1 + 3 \sin^2 \text{mag. lat.}}$$

According to the formulæ the total magnetic forces on the earth's surface, would increase from being 1 at the magnetic equator, to being 2 at

* These expressions would be exact if the earth were a sphere of matter uniformly magnetised in parallel lines, or, which comes to the same thing as regards all forces external to the earth, if its magnetism were caused by a small magnet at its centre.

The mathematical investigation of the effect of such a magnet may be made so simple that it seems desirable to insert it here, particularly as it is useful in enabling us to estimate the action of any magnet on a needle at a considerable distance from it.

1. To find the effect of a small magnet on a magnetic particle situate in the prolongation of its axis—



Let A B be the magnet, P the particle.

The magnet may be supposed to consist of two magnetic particles, one of south magnetism at A, one of north magnetism at B.

Let A B = l , C P = r , then the force of the magnet will be proportional to—

$$\begin{aligned} & \frac{1}{AP^2} - \frac{1}{BP^2} \\ &= \frac{1}{\left(r - \frac{l}{2}\right)^2} - \frac{1}{\left(r + \frac{l}{2}\right)^2} \\ &= \frac{2rl}{\left\{r^2 - \left(\frac{l}{2}\right)^2\right\}^2} \\ &= \frac{2l}{r^3} \text{ if } l \text{ be so small that terms involving } l^3 \text{ may be neglected.} \end{aligned}$$

2. To find the effect of a small magnet on a magnetic particle situate on a line perpendicular to its axis and passing through its centre.



In this case we have a force $\frac{1}{AP^2}$ acting from P to A, and a like force acting from B to P. The resultant will act in a direction parallel to A B, and will be equal to—

$$\begin{aligned} & \frac{AB}{AP} \cdot \frac{1}{AP^2} \\ &= \frac{AB}{AP^3} \\ &= \frac{l}{r^3} \end{aligned}$$

PART III

Corollary.

PART III. there are, at least in the northern hemisphere, two maxima, one on each side of it. This description will, it is hoped be sufficient to enable the reader to follow generally the changes of amount and direction of the magnetic force in different parts of the globe. A more particular description of the changes will be given in Part IV.

Causes of the several parts of the deviation.—In investigating the causes of the several parts of the deviation, we must consider separately the effects produced by iron of the qualities designated (magnetically) “hard iron” and “soft iron.”

“Soft iron” is iron which becomes instantaneously magnetised by induction when exposed to any magnetic force, but which has no power of retaining magnetism, and it has therefore no independent magnetism. All its magnetism is transient induced magnetism.

“Hard iron” is iron less susceptible of being magnetised by induction, but which when once magnetised retains its magnetism permanently.

Hard iron.—The effect of the permanent magnetism of hard iron is extremely simple. It attracts the north end of the needle towards a particular direction in the ship. When the azimuth of the ship’s head is such that this direction is north or south of the compass no deviation is produced, but the directive force on the needle is increased or diminished as the case may be ; when the direction is east of the compass, an easterly deviation is produced ; when west, a westerly. The deviation in each case is proportional to the sine of the azimuth of the direction of the force measured from the direction of the disturbed needle. This deviation is therefore what we have called a *semicircular* deviation. The force which causes this deviation being constant, the deviation produced on a given azimuth will be inversely proportional to the antagonistic force which it has to overcome in producing deviation, *i.e.*, to the horizontal force of the earth.

Instead of considering a single force acting in a direction which does not generally coincide with either of the principal directions in a ship, it is more convenient to consider the hard iron as giving rise to two forces ; (1) a force acting in the fore and aft line of the ship (+ when drawing the north end of the needle forwards, — when drawing it aft), and (2) a force acting athwart-ships (+ when drawing the north end of the needle to starboard, — when drawing it to port).

Soft iron.—The deviation caused by the magnetism induced in the soft iron of the ship by the earth’s magnetic force is less simple in its law, as well as less easy of popular explanation ; but it may be understood without difficulty if we first familiarize ourselves with the action of the inductive force of the earth on masses of soft iron of a very simple shape, *viz.*, thin rods.

A rod of soft iron held in the direction of the line of force becomes instantly magnetic. Its south (in England the upper) end becomes a south pole, and attracts the north end of the compass needle; its north (in England the lower) end becomes a north pole, and repels the north end of the compass needle. PART III.

If the rod be inclined to the line of force its magnetism diminishes, being proportional to the cosine of the angle of inclination, and when at right angles to the line of force the bar ceases to be magnetic. If the inclination of the bar to the line of force be still further increased, the end which was a north pole, and which then repelled the north end of the needle, becomes a south pole and attracts it.

It follows from this that if a rod of soft iron be in a vertical position, its upper end will in north magnetic latitudes attract the north end of the compass needle with a force proportional to the earth's vertical force. At the magnetic equator the line of force of the earth is horizontal, and there a vertical rod of soft iron will not be magnetic. In south magnetic latitudes the line of force of the earth is inclined upwards, and in those latitudes the upper end of a vertical bar of soft iron will repel the north end of the compass needle and attract the south end.

If the rod of soft iron be horizontal and in an east and west position, it is at right angles to the line of force, and will produce no effect. If it be horizontal, and in any other position, its south end will attract, and its north end will repel the north end of the compass needle with a force proportional to the earth's horizontal force multiplied by the cosine of the azimuth of the bar measured from the magnetic north.

If one of the rods we have been considering is retained in position and exposed to mechanical violence by being hammered, bent, or twisted, the amount of magnetism it receives is increased, and part of the magnetism so received is retained on a change of position till dissipated by lapse of time, or renewed mechanical violence applied to it in a different position. The greater the hardness of the iron the greater is the amount of magnetism which it can thus receive and retain.*

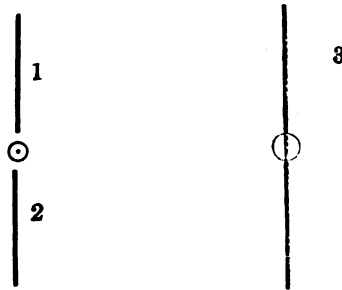
These considerations will enable us to understand the effect of the earth's magnetism on the irregular masses of iron of which a ship is composed. This will be most easily understood by considering separately the effects of the vertical and horizontal components of the earth's total magnetic force.

The vertical component of the earth's magnetic force (called the vertical force) induces in the soft iron of the ship a magnetic state proportional

* For an explanation of the method of completely representing the action of the soft iron of a ship by means of nine imaginary thin rods, *a, b, c, d, e, f, g, h, k*, see Appendix I.

PART III. to the vertical force, and which it will be evident does not alter as the ship turns in azimuth. It therefore produces a *semicircular* deviation, following precisely the same law as that caused by the permanent magnetism, with the exception that the effect produced being directly proportional to the vertical force, as well as inversely proportional to the horizontal force on board ship, it is on the whole directly proportional to the tangent of the dip.

It may be shown that when the soft iron of the ship is symmetrically distributed on each side of the fore-and-aft line, and when the ship is on an even keel, the whole of the action on the compass needle of the magnetism induced in the soft iron by the horizontal force of the earth may be represented by the action of two horizontal soft iron rods, the directions of which pass through the compass, one of such rods lying fore and aft, the other athwartship.



Let 1 or 2 and 3 represent such a rod lying fore and aft. The effect will be different, according as the rod lies in either position 1 or 2 entirely on one side of the compass, or in the position 3, with the ends extending on each side the compass.

When the rod 1, or the rod 2, is north or south of the compass, although it attracts the nearest end of the needle strongly and so increases its directive force, it produces no deviation, because it acts in the direction in which the needle is pointing. When the rod 1, or the rod 2, is east or west of the compass it ceases to be magnetic, and neither affects the direction nor the directive force of the needle. When 1, or 2, is N.E. or S.W. of the compass it produces an easterly deviation, when N.W. or S.E. a westerly deviation.

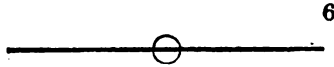
Consequently, a soft iron rod in the position 1 or 2 *increases* the mean directive force, and causes a *positive quadrantal* deviation.

So a soft iron rod in the position 3 *decreases* the mean directive force and causes a *negative quadrantal* deviation.

A soft iron rod placed athwartship in the position 4 or 5 *increases* the directive force, but causes a *negative quadrantal* deviation.

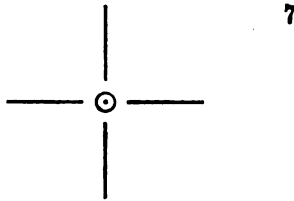


In the position 6 it *decreases* the directive force and causes a *positive* PART III.
quadrantal deviation.



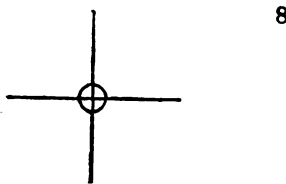
In iron ships in general, and particularly in those with iron decks, the mean horizontal force is found by observation to be *diminished*, and the quadrantal deviation is *positive*, we thence infer that the quadrantal deviation in such ships is principally caused by horizontal induction in transverse iron in the position 6.

Equal rods in the position 7



would *increase* the directive force without producing deviation.

Equal rods in the position 8



would *diminish* the directive force without producing deviation; but in each of the cases 7 and 8 the soft iron would have an indirect effect on the deviation produced by other causes by *increasing* or *diminishing* the directive force to be overcome in order to produce deviation, *i.e.*, the first arrangement would diminish the deviation produced by other causes, the second would increase it.

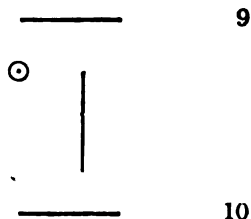
Soft iron magnetised by horizontal induction may, therefore, besides its effect in producing quadrantal deviation, have an indirect effect on the amount of the semicircular deviation.

Soft iron magnetized by vertical induction and hard iron have no such effect: if they increase the directive force in one azimuth they decrease it in another and do not alter its mean value.

When the soft iron is not symmetrically arranged on each side of the fore-and-aft midship section, or when the compass is not in the midship

PART III. line, a *constant* term may be caused in the expression for the deviation, as may be seen by the following simple cases.

Two soft iron rods in position 9 would produce a *constant* easterly deviation.



Two soft iron rods in position 10 would produce a *constant* westerly deviation.



One rod in any one of these positions would produce a deviation partly *quadrantal* partly *constant*.

Such arrangements are of course rare on board ship, and the *true* constant deviation is in general very small. But an *apparent* constant deviation is caused when there is any index error in the standard compass, or in the shore compass which is compared with it; or, if the magnetic bearing be derived from astronomical observation, when there is any error in the assumed amount of the variation of the compass at the place.

From these observations the principal peculiarities in the magnetism of wood-built and iron-built ships will be understood. In a wood-built ship not only is the amount of iron much smaller, but the iron which it does contain is not exposed to mechanical violence after it has been fixed in position in the ship. The permanent magnetism of such a ship has therefore no general character. The magnetism of one mass of iron may counteract that of another, and the result is that the magnetism of such a ship is chiefly the transient magnetism temporarily induced from the magnetism of the earth. In an iron-built ship the case is different. The plates in the process of riveting are hammered after being fixed in position, and a general magnetic character, due to the position of the ship in building, is developed in the ship as a whole, which is never entirely lost; and the effect of this permanent magnetism in producing deviation is generally much greater than that of the transient induced magnetism, though that is often of considerable amount.

§ 3. *Explanation of the Nature and Cause of the Heeling Deviation.*

In an iron ship built in England the north end of the needle is generally drawn downwards. When such a ship is on a north or south course, and heels over, this force becomes partly a lateral force and draws the north

end of the needle to windward: at the same time the horizontal deck beams and all other transverse horizontal iron become magnetized by induction, the upper or weather end attracting the north end of the needle to windward. These two causes therefore, so long as the ship is in the northern magnetic hemisphere, act in the same direction and often produce a large heeling error to windward. In the southern hemisphere the second and sometimes the first cause will alter the direction of its effect, and the heeling error may be to windward or to leeward as one or other preponderates.

The effect of different arrangements of soft iron in a ship, in producing heeling error, will be understood from studying the figures 11 and 12, one or other of which represents the effect of every usual arrangement of soft iron in producing the principal part of the heeling error.

Fig. 11.

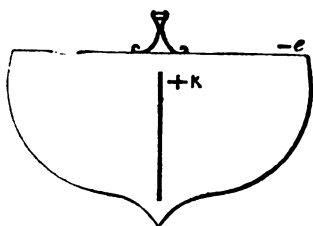
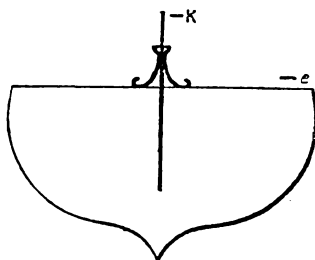


Fig. 12.



It will be seen that in the northern hemisphere in fig. 11, both $+k$ and $-e^*$ conspire to give a heeling error to windward, while in fig. 12, $-e$ will give a heeling error to windward, but $-k$ will give a heeling error to leeward; the direction of the heeling error will therefore depend on the predominance of the one or the other.

It will be obvious that in ships which have a large heeling deviation the rolling of the ship will call into play a force to the weather side which becomes alternately stronger and weaker as the ship's inclination is increased or diminished, or which may even act alternately to starboard or to port if the ship rolls on each side of the horizontal. This alternating force tends to produce an oscillation of the compass which sometimes becomes extremely inconvenient.

* $+e$ or $-e$ and $+k$ or $-k$ are two of the nine imaginary rods, by means of which the whole action of the soft iron on the compass may be represented, and of which a full explanation will be found in Appendix I., p. 112.

PART III.

SECTION II.

FORMULÆ AND EXAMPLES REQUIRING OBSERVATIONS OF DEVIATION ONLY.

§ I.

Approximate expression. . Coefficients A, B, C, D, E.

If the deviation is of moderate amount, not exceeding say 20° on any point, it is expressed with sufficient exactness, for practical purposes, by the formula—

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'.$$

In this expression δ is the deviation, reckoned + when the north end of the needle is drawn to the east, — when drawn to the west.

ζ' is the “compass course,” or the azimuth of the ship’s head from the direction of the disturbed needle, reckoned + to the eastward — to the westward.

$\zeta' + \delta = \zeta$ is therefore the “magnetic course,” or the azimuth of the ship’s head from the direction of the magnetic north or of the undisturbed needle, reckoned + to the eastward — to the westward.

In this expression—

A is the *constant* deviation,

$B \sin \zeta' + C \cos \zeta'$ is the *semicircular* deviation,

$D \sin 2\zeta' + E \cos 2\zeta'$ is the *quadrantal* deviation.

When the compass is in the middle fore and aft line of the ship the coefficients A and E are so small that they may be neglected. The expression for the deviation then becomes,—

$$\delta = B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta'.\dagger$$

* ζ' is considered as increasing from 0° at N. to 90° at E., 180° at S., 270° at W., and 360° or 0° again at N., and in using the formulæ it is necessary to be familiar with the practice of taking out from the tables the natural sines and cosines of arcs up to 360° , and giving them their proper signs. The rules for this will be found in the common treatises on trigonometry. For those who have not acquired complete familiarity with the process it may be convenient to be here reminded that,—

1. We may always subtract 180° or 360° from an angle, or subtract the angle from 180° or 360° , without altering the *numerical* value of its sine or cosine.
2. All lines measured to the E. or N. are +, to the W. or S. are negative; and, therefore, the sine of an angle in the East semicircle is +, in the West semicircle —; the cosine of an angle in the North semicircle is +, in the South semicircle is —.
3. The tangent of an angle in the N.E. and S.W. quadrants is +, in the S.E. and N.W. is —.
4. It follows, that the sine of a negative angle less than 90° is negative; the cosine positive.

† It will be easily seen from the expression that—

B is approximately the deviation at E.; or the deviation at W. with its sign changed.

C is approximately the deviation at N.; or the deviation at S. with its sign changed.

D is the mean of the deviations at N.E. and S.W.; or is the mean of the deviations at S.E. and N.W. with their signs changed.

In this case therefore there is no constant deviation ; the semicircular deviation is $B \sin \zeta' + C \cos \zeta'$; * the quadrantal deviation is $D \sin 2\zeta'$. PART III.

The coefficients A, B, C, D, E are best determined from observations of deviation made with the ship's head on a certain number of equidistant points, we shall therefore first give forms and examples for the computation of the coefficients A, B, C, D, E from observations of deviation on the 32, 16, and 8 principal points, and, so far as possible, from the four cardinal points and the four quadrantal points.

The computation of these coefficients from the deviations on the 32 points, or on the 16 or 8 principal points, is most readily performed by tabular forms, as in the examples which follow. These tabular forms contain directions for the performance of all the necessary operations, and no difficulty should be felt in any case if the examples here given are once mastered.

The examples are taken from the Deviations of H.M.S. *Trident*, swung at Greenhithe, December 23, 1856, the deviation table of which vessel is given in Part I., p. 11.

CASE I.

Deviations observed on 32 (Compass) Points.

The whole process of computing A, B, C, D, and E for H.M.S. *Trident* from deviations observed on 32 points is exhibited in the two tables which follow :—

Thus, from the table of deviations of the standard compass of H.M.S. *Trident* in p. 11, we get at once the following approximate values :—

$$B = + 20^{\circ} 20' \text{ or } + 21^{\circ} 10'$$

$$C = - 3^{\circ} 10'$$

$$D = \frac{1}{2} (16^{\circ} 50' - 9^{\circ} 40') = + 3^{\circ} 35'$$

$$\text{or } \frac{1}{2} (- 14^{\circ} 40' + 22^{\circ} 0') = + 3^{\circ} 40'.$$

We shall see immediately that the best values derived from the observations on 32 points are—

$$B = + 21^{\circ} 38\frac{1}{2}'$$

$$C = - 3^{\circ} 27'$$

$$D = + 3^{\circ} 42',$$

which do not appreciably differ from the preceding.

The student is recommended to familiarize himself with obtaining the approximate values of B, C, and D in this way, by inspection of tables of deviation or Napier's curves.

* $B \sin \zeta' + C \cos \zeta' = \sqrt{B^2 + C^2} \cdot \sin (\zeta' + \alpha)$, in which α is an auxiliary angle such that $\tan \alpha = \frac{C}{B}$.

$\sqrt{B^2 + C^2}$ is the maximum of semicircular deviation, called for shortness "the semicircular deviation." α is the "starboard angle" or the angle to the right of the ship's head of the direction in which the force causing the semicircular deviation acts.

FORM I.

PART III. TABLE A.—COMPUTATION OF COEFFICIENTS B and C, from Deviations observed on 32 Compass Points.

Name of Ship, Trident.
Place of Observation, Greenhithe.

Commander,
Date, 23rd December 1856.

Direction of Ship's Head by Standard Compass.	I.	Direction of Ship's Head by Standard Compass.	II.	III.	IV.	V.		VI.	
	Observed Deviation of Standard Compass.		Observed Deviation of Standard Compass.	Half-Sum of Quantities in Cols. I. and II. — <i>Constant and Quadrantal Deviation.</i>		Computation of B.		Computation of C.	
						Multipliers.	Products of Col. IV. by Multipliers.	Multipliers.	Products of Col. IV. by Multipliers.
NORTH.	° ' — 3 10	SOUTH.	° ' + 3 10	° ' 0 0	° ' — 3 10	0	° ' 0 0	1	° ' — 3 10
N. by E.	+ 2 35	S. by W.	+ 0 5	+ 1 20	+ 1 15	S ₁	+ 0 15	S ₇	+ 1 14
N.N.E.	+ 8 10	S.S.W.	— 3 0	+ 2 35	+ 5 35	S ₂	+ 2 8	S ₆	+ 5 10
N.E. by N.	+ 13 10	S.W. by S.	— 6 30	+ 3 20	+ 9 50	S ₃	+ 5 28	S ₅	+ 8 11
N.E.	+ 16 50	S.W.	— 9 40	+ 3 35	+ 13 15	S ₄	+ 9 22	S ₄	+ 9 22
N.E. by E.	+ 19 30	S.W. by W.	— 13 0	+ 3 15	+ 16 15	S ₅	+ 13 31	S ₃	+ 9 2
E.N.E.	+ 20 30	W.S.W.	— 16 10	+ 2 10	+ 18 20	S ₆	+ 16 56	S ₂	+ 7 1
E. by N.	+ 21 5	W. by S.	— 19 15	+ 0 55	+ 20 10	S ₇	+ 19 47	S ₁	+ 3 56
EAST.	+ 20 20	WEST.	— 21 10	— 0 25	+ 20 45	1	+ 20 45	0	0 0
E. by S.	+ 19 15	W. by N.	— 23 20	— 2 2	+ 21 18	S ₇	+ 20 53	— S ₁	— 4 10
E.S.E.	+ 18 5	W.N.W.	— 24 0	— 2 57	+ 21 3	S ₆	+ 19 25	— S ₂	— 8 3
S.E. by E.	+ 16 30	N.W. by W.	— 23 35	— 3 33	+ 20 3	S ₅	+ 16 40	— S ₃	— 11 9
S.E.	+ 14 40	N.W.	— 22 0	— 3 40	+ 18 20	S ₄	+ 12 58	— S ₄	— 12 58
S.E. by S.	+ 12 5	N.W. by N.	— 19 40	— 3 47	+ 15 53	S ₃	+ 8 49	— S ₅	— 13 12
S.S.E.	+ 9 40	N.N.W.	— 14 50	— 2 35	+ 12 15	S ₂	+ 4 41	— S ₆	— 11 19
S. by E.	+ 6 0	N. by W.	— 9 15	— 1 37	+ 7 37	S ₁	+ 1 30	— S ₇	— 7 29
Sum of + terms = + 173 8						+ 43 56		+ 43 56	
Sum of — terms = —						— 71 30		— 71 30	
Divisor 8						+ 173 8		8	
B =						+ 21 38½		C = — 3 27	

N.B.—Easterly Deviations are to be entered in this Table with the sign +; Westerly Deviations with the sign —.

FORM I.

PART III.

TABLE B.—COMPUTATION OF COEFFICIENTS A, D, E, from Deviations observed on 32 Compass Points.

I. Upper Half of Table A., Col. III.	II. Lower Half of Table A., Col. III.	III. Half Sum of Quantities in Cols. I. and II. — Constant Deviation.	IV. Half-Sum of Cols. I. and II. (changing Signs of Col. II.) ; — Quadrantal Deviation.	V. Computation of D.		VI. Computation of E.	
				Multipliers.	Products of Col. IV. by Multipliers.	Multipliers.	Products of Col. IV. by Multipliers.
° / + 0 0	° / - 0 25	° / - 0 12	° / + 0 12	0	° / 0 0	1	° / + 0 12
+ 1 20	- 2 2	- 0 21	+ 1 41	S ₂	+ 0 38	S ₆	+ 1 33
+ 2 35	- 2 57	- 0 11	+ 2 46	S ₄	+ 1 58	S ₄	+ 1 58
+ 3 20	- 3 33	- 0 7	+ 3 26	S ₆	+ 3 10	S ₂	+ 1 20
+ 3 35	- 3 40	- 0 2	+ 3 37	1	+ 3 37	0	0 0
+ 3 15	- 3 47	- 0 16	+ 3 31	S ₆	+ 3 15	-S ₂	- 1 20
+ 2 10	- 2 35	- 0 12	+ 2 22	S ₄	+ 1 40	-S ₄	- 1 40
+ 0 55	- 1 37	- 0 21	+ 1 16	S ₂	+ 0 30	-S ₆	- 1 10
Sum of + terms = +			Sum of + terms = + 14 48		+ 5 3		
Sum of - terms = - 1 42			Sum of - terms = -		- 4 10		
Divisor 8			Divisor 4		4		
A = - 0 13			D = + 3 42		E = + 0 13		

Note.—S₁ = .195, S₂ = .383, S₃ = .556, S₄ = .707, S₅ = .831, S₆ = .924, S₇ = .981.

PART III. The operations indicated in this example are to be performed in the following manner :—

First write down the observed deviations in the first and second columns of Table A, prefixing the sign + to the easterly deviations and the sign — to the westerly.

Next form column III. by taking half the sum (attending to the signs) of the quantities in column I. and column II.

Next form column IV. by changing (mentally) the signs of the quantities in column II., and then taking half the sum of the quantities in column I. and column II.

After columns III. and IV. have been computed in this manner they should be *proved*.

Both columns are proved at once and very rapidly by observing that the sum of the quantities on any line in columns III. and IV. ought to be equal to the quantity on the same line in column I.

It will be observed that the result of these operations, is the separation of the *semicircular* from the other parts of the deviation, the quantities in column IV. being the semicircular deviations on the points from N. to S. by E., and the same quantities with the signs changed being the semicircular deviations from S. to N. by W., while the quantities in column III. are the sums of the *constant* and the *quadrantal* deviations on the same points.

Next form column V. and column VI. by multiplying each of the semicircular deviations by the multiplier set opposite it in columns V. and VI. respectively. These multipliers are the sines of the rhumbs, the values of which are given above.

The multiplications are performed most easily by means of the table appended, and which gives the product of every fifth minute of arc from $0^{\circ} 5'$ to $34^{\circ} 55'$ by $S_1, S_2, S_3, S_4, S_5, S_6$, and S_7 .

In these multiplications the rule of signs must be attended to, viz., that + multiplied by + or — multiplied by — has the sign +, and that + multiplied by — or — multiplied by + has the sign —.

Next add all the + quantities in column V. and VI., and likewise all the — quantities, subtract the least from the greatest, prefixing the sign of the greatest, and divide by 8 ; this gives B and C.

To find A, D, E, we have another set of operations to perform by means of Table B.

First write down the upper half of column III. of Table A in column I. of Table B, and the lower half in column II.

Next from these form columns III. and IV. in the same way in which the corresponding columns of Table A were formed.

It will be seen that we have now separated the *constant* and the *quadrantal* parts of the deviation. Column III. containing the constant part, and the quantity in each of the 8 lines being, in fact, the value of A derived from one of the eight sets of 4 points distant 90° from each other. The sum of these values divided by 8 gives the best value of A. Column IV. contains the quadrantal deviation on the points from N. to E., and from S. to W., and, with the signs changed, the quadrantal deviation on the points from E. to S., and from W. to N.

From the quantities in column IV., D and E are obtained in the same way as B and C from column IV. of Table A.*

* It may be observed that the sum of the 1st and 9th lines in column V. is the value of B which would be obtained from deviation on the four cardinal points; the sum of the 2nd and 10th, the value of B from the deviation on the N. by E., E. by S., S. by W., and W. by N. points, and so on. We may thus get the *eight* values of B, which would be given by every set of *four* equidistant points. The mean of the 1st and 5th of these is the value from the eight principal points, and in the same way we get the values from each of the *four* sets of *eight* equidistant points. The mean of the 1st and 3rd of these gives the value derived from the sixteen principal points, the mean of the 2nd and 4th the value derived from the sixteen secondary points. Similar observations apply to the values of C, A, D, and E, except that D and E cannot generally be derived from observation on four points. There is, however, this exception, that E may be derived from observations on the four *cardinal* points, D from observations on the four *quadrantal* points.

These values may be exhibited in the following way :—

Values from					Values from				
	4 points.	8 points.	16 points.	32 points.		4 points.	8 points.	16 points.	32 points.
A	12	7	10	13	C	3 10	3 23	3 29	3 27
	21	18	16			2 56	3 33	3 25	
	11	12				2 53	3 35		
	7	14				2 53	3 16		
	12					3 36			
	16					4 10			
	12					4 18			
	21					3 33			
B	+ 20 45	+ 21 32	+ 21 33	+ 21 38	D	+ 3 37	+ 3 37	+ 3 37	+ 3 42
	+ 21 8	+ 21 44	+ 21 43				+ 3 53	+ 3 47	
	+ 21 33	+ 21 35					+ 3 38		
	+ 22 8	+ 21 42					+ 3 40		
	+ 22 20				E	+ 12	+ 12	+ 15	+ 13
	+ 22 20						+ 13	+ 11	
	+ 21 37						+ 18		
	+ 21 17						+ 10		

The very close agreement of the results of each set of eight observations is a test of the goodness of the observations.

In the case of the eight sets from four observations, the same agreement is not to be looked for, as will be seen from the expressions for these sets contained in Appendix I, p. 127.

PART III.

CASE II.

Deviations observed on the 16 principal compass Points.

The example which follows will sufficiently show the method without any further explanation.

FORM II.

TABLE A.—COMPUTATION OF COEFFICIENTS B AND C from Deviations observed on the 16 principal compass Points.

Name of Ship, Trident.

Commander,

Place of Observation, Greenhithe.

Date, 23rd December 1856.

Direction of Ship's Head by Standard Compass.	I.	Direction of Ship's Head by Standard Compass.	II.	III.	IV.	V.		VI.	
	Observed Deviation of Standard Compass.		Observed Deviation of Standard Compass.	Half-Sum of Quantities in Cols. I. and II. — <i>Constant and Quadrantal Deviation.</i>	Half-Sum of Cols. I. and II. (changing Signs of Col. II.) — <i>Semi-circular Deviation.</i>	Computation of B.		Computation of C.	
						Multipliers.	Products of Col. IV. by Multipliers.	Multipliers.	Products of Col. IV. by Multipliers.
NORTH.	° / — 3 10	SOUTH.	° / + 3 10	° / 0 0	° / — 3 10	0	° / 0 0	1	° / — 3 10
N.N.E.	+ 8 10	S.S.W.	— 3 0	+ 2 35	+ 5 35	S ₂	+ 2 8	S ₄	+ 5 10
N.E.	+ 16 50	S.W.	— 9 40	+ 3 35	+ 13 15	S ₄	+ 9 22	S ₄	+ 9 22
E.N.E.	+ 20 30	W.S.W.	— 16 10	+ 2 10	+ 18 20	S ₆	+ 16 56	S ₂	+ 7 1
EAST.	+ 20 20	WEST.	— 21 10	— 0 25	+ 20 45	1	+ 20 45	0	0 0
E.S.E.	+ 18 5	W.N.W.	— 24 0	— 2 57	+ 21 3	S ₆	+ 19 25	— S ₂	— 8 3
S.E.	+ 14 40	N.W.	— 22 0	— 3 40	+ 18 20	S ₄	+ 12 58	— S ₄	— 12 58
S.S.E.	+ 9 40	N.N.W.	— 14 50	— 2 35	+ 12 15	S ₂	+ 4 41	— S ₆	— 11 19
						Sum of + terms = + 86 15		+	21 33
						Sum of — terms = —		—	35 30
						Divisor 4	+ 86 15	4	— 13 57
						B = + 21 34		C = — 3 29	

N.B.—Easterly Deviations are to be entered in this Table with the sign + ; Westerly Deviations with the sign —.

FORM II.

PART III.

TABLE B.—COMPUTATION OF COEFFICIENTS A, D, and E, from Deviations observed on the 16 principal compass Points.

I. Upper Half of Table A., Col. III.	II. Lower Half of Table A., Col. III.	III. Half-Sum of Quantities in Cols. I. and II. — <i>Constant Deviation.</i>	IV. Half Sum of Cols. I. and II. (changing Signs of Col. II.) — <i>Quadrantal Deviation.</i>	V. Computation of D.		VI. Computation of E.	
				Multipliers.	Products of Col. IV. by Multipliers.	Multipliers.	Products of Col. IV. by Multipliers.
° / 0 0	° / -0 25	° / -0 12	° / +0 12	0	° / 0 0	1	° / +0 12
+ 2 35	-2 57	-0 11	+2 46	S ₁	+1 58	S ₁	+1 58
+ 3 35	-3 40	-0 2	+3 37	1	+3 37	0	0 0
+ 2 10	-2 35	-0 12	+2 22	S ₁	+1 40	-S ₁	-1 40
Sum of + terms = +			Sum of + terms = + 7 15			+2 10	
Sum of - terms = - 37			Sum of - terms = -			-1 40	
Divisor 4 - 37			Divisor 2 +7 15			2 + 0 30	
A = - 0 9			D = + 3 37			E = + 0 15	

Note.—S₁ = .383, S₂ = .707, S₃ = .924.

PART III.

CASE III.

Deviations observed on the 8 principal compass Points.

The example which follows will sufficiently show the method without further explanation.

FORM III.

TABLE A.—COMPUTATION of COEFFICIENTS B and C from Deviations observed on the 8 principal compass Points.

Name of Ship, Trident.

Commander,

Place of Observation, Greenhithe.

Date, 23rd December 1856.

Direction of Ship's Head by Standard Compass.	I.	Direction of Ship's Head by Standard Compass.	II.	III.	IV.	V.		VI.	
	Observed Deviation of Standard Compass.		Observed Deviation of Standard Compass.	Half-Sum of Quantities in Cols. I. and II.	Half-Sum of Cols. I. and II. (changing Signs of Col. II.)	Computation of B.		Computation of C.	
				<i>Constant and Quadrantal Deviation.</i>	<i>Semi-circular Deviation.</i>	Multipliers.	Products of Col. IV. by Multipliers.	Multipliers.	Products of Col. IV. by Multipliers.
NORTH.	° / - 3 10	SOUTH.	° / + 3 10	° / 0 0	° / - 3 10	0	° / 0 0	1	° / - 3 10
N.E.	+ 16 50	S.W.	- 9 40	+ 3 35	+ 13 15	S ₁	+ 9 22	S ₁	+ 9 22
EAST.	+ 20 20	WEST.	- 21 10	- 0 25	+ 20 45	1	+ 20 45	0	0 0
S.E.	+ 14 40	N.W.	- 22 0	- 3 40	+ 18 20	S ₁	+ 12 58	- S ₁	- 12 58
						° / Sum of + terms = + 43 5 Sum of - terms = -		° / + 9 22 - 16 8	
						Divisor 2	+ 43 5	2	- 6 46
						B = + 21 32		C = - 3 23	

Note. S₁ = ° 707.

N.B.—Easterly Deviations are to be entered in this Table with the sign + ; Westerly Deviations with the sign -.

TABLE B.—COMPUTATION of COEFFICIENTS A, D, E.

I. Upper Half of Table A., Col. III.	II. Lower Half of Table A., Col. III.	III. Half-Sum of Quantities in Cols. I. and II. <i>Constant Deviation.</i>	IV. Half-Sum of Cols. I. and II. (changing Signs of Col. II.) <i>Quadrantal Deviation.</i>
° / 0 0	° / - 0 25	° / - 0 12	° / + 0 12 = E
+ 3 35	- 3 40	- 0 2	+ 3 37 = D
		Divisor 2	- 0 14
		A = - 0 7	

CASE IV.

PART III.

Deviations observed on the Four Cardinal Compass Points.

In this case the deviations do not enable us to determine D, but if D be known from previous observation we may determine the other coefficients as follows :—

FORM IV.

COMPUTATION OF COEFFICIENTS A, B, C, and E, from Deviations observed on the Four Cardinal Compass Points.

Name of Ship, Trident.

Commander,

Place of Observation, Greenhithe.

Date, 23rd December 1856.

	I.		II.		III.		IV.
Direction of Ship's Head by Standard Compass.	Observed Deviation of Standard Compass.	Direction of Ship's Head by Standard Compass.	Observed Deviation of Standard Compass.		Half Sum of Quantities in Cols. I. and II.		Half Sum of Quantities in Cols. I. and II. (changing Signs of Col. II.)
N.	° ' - 3 10	S.	° ' + 3 10		° ' 0 0		° ' - 3 10 = C
E.	+ 20 20	W.	- 21 10		- 0 25		+ 20 45* = B
Half sum of terms in Col. III. = A = - 12							D is known to be 3° 42'
Do. (changing sign of 2nd term) = E = + 12							

CASE V.

Deviations observed on the Four Quadrantal Compass Points.

In this case the deviations do not enable us to determine E., but the other coefficients may be determined as follows :—

* In this case we generally get a more exact value of B by multiplying by $1 + \frac{1}{2} \sin D$, and of C by multiplying by $1 - \frac{1}{2} \sin D$.

$$\begin{aligned} \text{Example :} \quad D &= 3^\circ 42' & 1 + \frac{1}{2} \sin D &= 1.032 \\ \sin D &= .064 & 1 - \frac{1}{2} \sin D &= .968 \\ \frac{1}{2} \sin D &= .032 \end{aligned}$$

$$\text{Best value of B} = + 20^\circ 45' \times 1.032 = + 21^\circ 25'$$

$$\text{" " C} = - 3^\circ 10' \times .968 = - 3^\circ 4'$$

Without this correction the values of B and C are only exact to terms of the 1st order. With this correction they are exact as far as terms of the 2nd order inclusive.

In the particular case we have chosen the value of B is greatly improved by the multiplication; the value of C is, however, as will sometimes happen, made rather worse.

PART III.

FORM V.

COMPUTATION OF COEFFICIENTS A, B, C, and D, from Deviations observed on the N.E., S.E., S.W., and N.W. Points.

Name of Ship, Trident.

Commander,

Place of Observation, Greenhithe.

Date, 23rd December 1856.

Direction of Ship's Head by Standard Compass.	I.	Direction of Ship's Head by Standard Compass.	II.	III.	IV.	V.	
	Observed Deviation of Standard Compass.		Observed Deviation of Standard Compass.	Half Sum of Quantities in Cols. I. and II.	Half Sum of Cols. I. and II. (changing Signs of Col. II.)	Multipliers.	Products of Col. IV. by Multipliers.
N.E.	° / + 16 50	S.W.	° / - 9 40	° / + 3 35	° / + 13 15	S ₁	° / + 9 22
S.E.	+ 14 40	N.W.	- 22 0	- 3 40	+ 18 20	S ₁	+ 12 58
Half sum of terms in Col. III. = A =					Sum of terms in Col. V.* = B =	° / + 22 20	
Do. (changing sign of 2nd term) = D =					Do. (changing sign of 2nd term) = C =	- 3 36	

§ II.

Use of the Coefficients A, B, C, D, E.

Having obtained A, B, C, D, E,† we may then compute the deviations on the 16 principal compass points by the following form:—

* In this case we generally get a more exact value of B by multiplying by $1 - \frac{1}{2} \sin D$, and of C by multiplying by $1 + \frac{1}{2} \sin D$, thus—

$$\text{Best value of B} = + 22^{\circ} 20' \times .968 = 21^{\circ} 32'$$

$$\text{„ „ C} = + 3^{\circ} 36' \times 1.032 = - 3^{\circ} 43'$$

† In some cases the deviations are given, not on the compass courses, but on the magnetic courses. In those cases we may either obtain the compass courses by laying down a Napier's curve by distances measured along the plain lines and measuring off the compass courses along the dotted lines, and then proceed as above; or we may suppose the deviations expressed in terms of the magnetic azimuth by a similar formula, viz:—

$$\delta = A_1 + B_1 \sin \zeta + C_1 \cos \zeta + D_1 \sin 2\zeta + E_1 \cos 2\zeta,$$

in which A_1, B_1, C_1, D_1 , and E_1 are coefficients nearly, but not exactly, equal to A, B, C, D, E, and which may be determined by a similar method; and from A_1, B_1, C_1, D_1, E_1 we may determine A, B, C, D, E by the following formulæ:—

$$A = A_1$$

$$B = B_1 \left\{ 1 - \frac{1}{2} \sin D_1 - \frac{1}{4} \text{versin } B_1 + \frac{3}{4} \text{versin } C_1 + \frac{1}{2} \text{versin } D_1 \right\} - (A_1 + \frac{1}{2} E_1) \sin C_1.$$

$$C = C_1 \left\{ 1 + \frac{1}{2} \sin D_1 + \frac{3}{4} \text{versin } B_1 - \frac{1}{4} \text{versin } C_1 + \frac{1}{2} \text{versin } D_1 \right\} + (A_1 - \frac{1}{2} E_1) \sin B_1.$$

$$D = D_1 + \frac{1}{2} B_1 \sin B_1 - \frac{1}{2} C_1 \sin C_1.$$

$$E = E_1 + B_1 \sin C_1 + 2A_1 \sin D_1.$$

COMPUTATION OF DEVIATION.

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PART III.

FORM VI.

COMPUTATION of the Deviations of the Standard Compass on the sixteen Principal Points from the Coefficients A, B, C, D, E.
Name of Ship, Trident.
Place of Observation, Greenhithe.
Commander,
Date, 23rd December 1856.

I. A =	II. D = + 3° 43'		III. E = + 13'		IV. Sum of Cols. II. and III. Quadrantal Deviation.	V. Sum of Columns I. & IV. Constant and Quadrantal Deviation.	VI. B = + 21° 40'		VII. C = - 3° 27'		VIII. Sum of Columns VI. and VII. Semi- circular Deviation.	IX. Sum of Columns V. and VIII. Computed Deviation of Standard Compass.
	Products of D by Multi- pliers.	Multipliers.	Products of E by Multi- pliers.	Multipliers.	Products of B by Multi- pliers.	Multipliers.	Products of C by Multi- pliers.	Multipliers.	Products of B by Multi- pliers.	Multipliers.		Direction of Ship's Head by Standard Compass.
- 13	0 0 0	1	+ 0 13	S_4	0 0 0	0	0 0 0	1	- 3 27	S_6	- 3 27	NORTH.
"	+ 2 37	S_4	+ 0 9	S_4	+ 2 33	S_4	+ 8 18	S_6	- 3 10	S_4	+ 5 8	N.N.E.
"	+ 3 42	0 0	0 0	S_4	+ 3 29	S_4	+ 15 19	S_6	- 2 27	S_4	+ 12 52	N.E.
"	+ 2 37	S_4	- 0 9	S_4	+ 2 28	S_6	+ 20 1	S_6	- 1 20	S_4	+ 18 41	E.N.E.
"	0 0 0	- 1	- 0 13	S_4	- 0 13	1	+ 21 40	0	0 0	0	+ 21 40	EAST.
"	- 2 37	S_4	- 0 9	S_4	- 2 46	S_6	+ 20 1	- S_4	+ 1 20	- S_4	+ 22 21	E.S.E.
"	- 3 42	0 0	0 0	0	- 3 42	S_4	+ 15 19	- S_4	+ 2 27	- S_4	+ 17 46	S.E.
"	- 2 37	S_4	+ 0 9	S_4	- 2 28	- S_4	+ 8 18	- S_6	+ 3 10	- S_4	+ 11 28	S.S.E.
Repeat the upper part of Col. V. in the lower part.						0 0	Repeat the upper part of Col. VIII. in the lower part, changing the signs.					
						+ 2 33						
						+ 3 29						
						+ 2 15						
Repeat the upper part of Col. V. in the lower part.						- 0 26	Repeat the upper part of Col. VIII. in the lower part, changing the signs.					
						- 2 59						
						- 3 55						
						- 2 41						

NOTE.—The sign + prefixed to a quantity in Col. IX. signifies a deviation of the North Point of the Compass to the East; the sign - a deviation to the West.
 N.B. $S_1 = .195$, $S_2 = .383$, $S_3 = .556$, $S_4 = .707$, $S_5 = .831$, $S_6 = .924$, $S_7 = .981$.

PART III.

FORM VII.

ADJUSTMENT of Table of Deviations for Change of Magnetic Latitude.

Name of Ship, Trident,*Commander*,*Place of Orig. Obser.*, Greenhithe.*Date*, Dec. 23, 1856.*Dip*, 68° 22' N.*Nat. Tan. Dip*, 2.52.*Hor. Force*, 1.0.

$$\frac{1}{\text{Hor. Force.}} = 1.0.$$

New Place of Ship, Coast of Labrador.*Date*,*Dip*, 77° 50' N.*Nat. Tan. Dip*, 4.62.*Hor. Force*, 0.605.

$$\frac{1}{\text{Hor. Force.}} = 1.65.$$

Direction of Ship's Head by Standard Compass.	I. <i>Semicircular Deviation at place of Original Observation.</i> Same as Form VI., Col. VIII., or Form I., Tab. A., Col. IV.	II. <i>Constant and Quadrantal Deviation.</i> Same as Form VI., Col. V., or Form I., Table A., Column III.	III. <i>Semicircular Deviation at new place of Ship, being 175 Semicircular Deviation at place of original Observation.</i>	Direction of Ship's Head by Standard Compass.	IV. Sum of Cols. II. and III. <i>Adjusted Deviation of Standard Compass at new place of Ship.</i>
NORTH.	— 3 27	0 0	— 6 2	NORTH.	— 6 2
N.N.E.	+ 5 8	+ 2 33	+ 9 0	N.N.E.	+ 11 33
N.E.	+ 12 52	+ 3 29	+ 22 30	N.E.	+ 26 0
E.N.E.	+ 18 41	+ 2 15	+ 32 40	E.N.E.	+ 34 55
EAST.	+ 21 40	— 0 26	+ 37 55	EAST.	+ 37 30
E.S.E.	+ 22 21	— 2 59	+ 39 6	E.S.E.	+ 36 7
S.E.	+ 17 46	— 3 55	+ 31 5	S.E.	+ 27 10
S.S.E.	+ 11 28	— 2 41	+ 20 4	S.S.E.	+ 17 23
SOUTH.	+ 3 27	0 0	+ 6 2	SOUTH.	+ 6 2
S.S.W.	— 5 8	+ 2 33	— 9 0	S.S.W.	— 6 27
S.W.	— 12 52	+ 3 29	— 22 30	S.W.	— 19 0
W.S.W.	— 18 41	+ 2 15	— 32 40	W.S.W.	— 30 25
WEST.	— 21 40	— 0 26	— 37 55	WEST.	— 38 21
W.N.W.	— 22 21	— 2 59	— 39 6	W.N.W.	— 42 5
N.W.	— 17 46	— 3 55	— 31 5	N.W.	— 35 0
N.N.W.	— 11 28	— 2 41	— 20 4	N.N.W.	— 22 45

In this case, if the magnetism were that of hard iron, the increase of semicircular deviation would be in the inverse ratio of the horizontal force, or as 1.65:1. If the magnetism were that of soft iron, the increase of semicircular deviation would be in the ratio of the tangents of the dip, i.e. of 4.62:2.50 or of 1.85:1. An intermediate ratio 1.75 is adopted, and the example (which is assumed) shows the resulting changes in the Table of Deviations. Great caution is however necessary in the use of this method, as the change may be much greater, when the two parts of which the semicircular deviation is composed are each large and have opposite signs.

§ III.

Exact Expressions. Coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$.

When the deviations on any course much exceed 20° , and also when we can only observe on a small number of points equally distributed round the horizon, it is necessary to use the exact expression for the deviation. This is—

$$\begin{aligned}\sin \delta &= \mathcal{A} \cos \delta + \mathcal{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathcal{D} \sin (\zeta + \zeta') + \mathcal{E} \sin (\zeta + \zeta'). \\ &= \mathcal{A} \cos \delta + \mathcal{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathcal{D} \sin (2\zeta' + \delta) + \mathcal{E} \cos (2\zeta' + \delta).\end{aligned}$$

In this expression the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, are nearly but not exactly the natural sines of the coefficients A, B, C, D, E.

CASE I.

$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, derived from A, B, C, D, E.

When A, B, C, D, E are known the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ may be derived from them by the following expressions, which are sufficiently exact for deviations not exceeding 40° .

$$\mathcal{A} = \sin A.$$

$$\mathcal{B} = \sin B \left\{ 1 + \frac{1}{2} \sin D + \frac{1}{4} \text{versin } B - \frac{1}{4} \text{versin } C \right\} + \frac{1}{2} \sin C \sin E.$$

$$\mathcal{C} = \sin C \left\{ 1 - \frac{1}{2} \sin D - \frac{1}{4} \text{versin } B + \frac{1}{4} \text{versin } C \right\} + \frac{1}{2} \sin B \sin E.$$

$$\mathcal{D} = \sin D \left\{ 1 + \frac{1}{2} \text{versin } D \right\}.$$

$$\mathcal{E} = \sin E - \sin A \sin D.$$

Example.

As an example of this method of determining $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ from A, B, C, D, E we may take the case of the *Trident* given above, in which we have—

$$A = - 0^\circ 13'$$

$$B = + 21^\circ 40'$$

$$C = - 3^\circ 27'$$

$$E = + 0^\circ 13'$$

$$D = + 3^\circ 42'$$

PART III.

$$\begin{aligned}
 \mathcal{A} &= \sin A = -\cdot 004 & \sin E &= +\cdot 004 \\
 \sin B &= +\cdot 369 & \sin C &= -\cdot 060 & \sin D &= +\cdot 065 \\
 \text{versin } B &= +\cdot 071 & \text{versin } C &= +\cdot 002 & \text{versin } D &= +\cdot 002 \\
 \mathcal{B} &= +\cdot 369 [1 + \cdot 032 + \cdot 006 - \cdot 000] + \frac{1}{2} (-\cdot 060) \times \cdot 004 = +\cdot 383 \\
 \mathcal{C} &= -\cdot 060 [1 - \cdot 032 - \cdot 018 + \cdot 000] + \frac{1}{2} \cdot 369 \times \cdot 004 = -\cdot 056 \\
 \mathcal{D} &= \cdot 065 [1 + \cdot 001] = \cdot 065 \\
 \mathcal{E} &= \cdot 004 + \cdot 004 \times \cdot 065 = \cdot 004
 \end{aligned}$$

As another example of this method of determining $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, we may take the case of H.M.S. *Warrior*, swung at Spithead in October 1861, as given in Part II., p. 20. In this case—

$$\begin{aligned}
 A &= -1^\circ 0' & E &= +0^\circ 44' \\
 B &= -22^\circ 12' & C &= -5^\circ 52' & D &= +8^\circ 56' \\
 \mathcal{A} &= \sin A = -\cdot 017 & \sin E &= +\cdot 013 \\
 \sin B &= -\cdot 378 & \sin C &= -\cdot 102 & \sin D &= +\cdot 155 \\
 \text{versin } B &= +\cdot 074 & \text{versin } C &= +\cdot 005 & \text{versin } D &= +\cdot 012 \\
 \mathcal{B} &= -\cdot 378 [1 + \cdot 077 + \cdot 006 - \cdot 001] + \frac{1}{2} (-\cdot 102 \times \cdot 013) = -\cdot 410 \\
 \mathcal{C} &= -\cdot 102 [1 - \cdot 077 - \cdot 018 + \cdot 000] + \frac{1}{2} (-\cdot 378 \times \cdot 013) = -\cdot 095 \\
 \mathcal{D} &= +\cdot 155 [1 + \cdot 004] = +\cdot 156 \\
 \mathcal{E} &= +\cdot 013 + \cdot 017 \times \cdot 155 = +\cdot 015
 \end{aligned}$$

This method is in general the most convenient when the ship has been completely swung and the deviations do not exceed the limits mentioned.

CASE II.

$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ derived from Observations on the eight principal Compass Points.

When the observations are very large and we can obtain the deviations on the eight principal compass points the following are the best formulæ to use.

$$\begin{aligned}
 2 \mathcal{A} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_4 + \sin \delta_{20})}{\frac{1}{2} (\cos \delta_4 + \cos \delta_{20})} + \frac{\frac{1}{2} (\sin \delta_{12} + \sin \delta_{28})}{\frac{1}{2} (\cos \delta_{12} + \cos \delta_{28})} \right. \\
 &\quad \left. + \frac{\frac{1}{2} (\sin \delta_8 + \sin \delta_{16})}{\frac{1}{2} (\cos \delta_8 + \cos \delta_{16})} + \frac{\frac{1}{2} (\sin \delta_0 + \sin \delta_{24})}{\frac{1}{2} (\cos \delta_0 + \cos \delta_{24})} \right\} \\
 \mathcal{D} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_4 + \sin \delta_{20})}{\frac{1}{2} (\cos \delta_4 + \cos \delta_{20})} - \frac{\frac{1}{2} (\sin \delta_{12} + \sin \delta_{28})}{\frac{1}{2} (\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 \mathcal{E} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_8 + \sin \delta_{16})}{\frac{1}{2} (\cos \delta_8 + \cos \delta_{16})} - \frac{\frac{1}{2} (\sin \delta_0 + \sin \delta_{24})}{\frac{1}{2} (\cos \delta_0 + \cos \delta_{24})} \right\} - \mathcal{D} \mathcal{A}
 \end{aligned}$$

$$\begin{aligned}
2 \mathfrak{B} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
&\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
&\quad + \mathfrak{C} \mathfrak{E} \\
&\quad + (1 + \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) \\
2 \mathfrak{C} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
&\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
&\quad + \mathfrak{B} \mathfrak{E} \\
&\quad + (1 - \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{16})
\end{aligned}$$

Example.

As an example of these computations we may take the deviations of H. M. S. *Warrior* on the several points by disturbed compass, as determined at Spithead in October 1861.—See Part II., p. 20, on which we have,

Ship's Head by Standard Compass.	Deviation.	Ship's Head by Standard Compass.	Deviation.
NORTH.	$\delta_0 = - 6 \ 30$	SOUTH.	$\delta_{16} = + 5 \ 30$
N.E.	$\delta_4 = - 13 \ 0$	S.W.	$\delta_{20} = + 28 \ 35$
EAST.	$\delta_8 = - 22 \ 15$	WEST.	$\delta_{24} = + 19 \ 15$
S.E.	$\delta_{12} = - 23 \ 30$	N.W.	$\delta_{28} = + 3 \ 0$

The computations for determining $\mathfrak{H}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are then the following:—

$$\begin{aligned}
\delta_4 &= - 13^\circ & \sin \delta_4 &= - \cdot 225 & \cos \delta_4 &= + \cdot 974 \\
\delta_{20} &= + 28^\circ 35' & \sin \delta_{20} &= + \cdot 478 & \cos \delta_{20} &= + \cdot 878 \\
\frac{1}{2} (\sin \delta_4 + \sin \delta_{20}) &= + \cdot 126 & \frac{1}{2} (\cos \delta_4 + \cos \delta_{20}) &= + \cdot 926 \\
\frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) &= - \cdot 351 & \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) &= + \cdot 048 \\
\delta_{12} &= - 23^\circ 30' & \sin \delta_{12} &= - \cdot 399 & \cos \delta_{12} &= + \cdot 917 \\
\delta_{28} &= + 3^\circ 00' & \sin \delta_{28} &= + \cdot 052 & \cos \delta_{28} &= + \cdot 998 \\
\frac{1}{2} (\sin \delta_{12} + \sin \delta_{28}) &= - \cdot 173 & \frac{1}{2} (\cos \delta_{12} + \cos \delta_{28}) &= + \cdot 957 \\
\frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) &= - \cdot 225 & \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) &= - \cdot 040 \\
\delta_8 &= - 6^\circ 30' & \sin \delta_8 &= - \cdot 113 & \cos \delta_8 &= + \cdot 993 \\
\delta_{16} &= + 5^\circ 30' & \sin \delta_{16} &= + \cdot 096 & \cos \delta_{16} &= + \cdot 995 \\
\frac{1}{2} (\sin \delta_8 + \sin \delta_{16}) &= - \cdot 008 & \frac{1}{2} (\cos \delta_8 + \cos \delta_{16}) &= + \cdot 994 \\
\frac{1}{2} (\sin \delta_8 - \sin \delta_{16}) &= - \cdot 104 \\
\delta_0 &= - 22^\circ 15' & \sin \delta_0 &= - \cdot 379 & \cos \delta_0 &= + \cdot 926 \\
\delta_{24} &= + 19^\circ 15' & \sin \delta_{24} &= + \cdot 330 & \cos \delta_{24} &= + \cdot 944 \\
\frac{1}{2} (\sin \delta_0 + \sin \delta_{24}) &= - \cdot 025 & \frac{1}{2} (\cos \delta_0 + \cos \delta_{24}) &= + \cdot 935 \\
\frac{1}{2} (\sin \delta_0 - \sin \delta_{24}) &= - \cdot 354 \\
2 \mathfrak{H} &= \frac{1}{2} \left\{ \begin{array}{cccc} \cdot 126 & - \cdot 173 & \cdot 008 & \cdot 025 \\ \cdot 926 & \cdot 957 & \cdot 994 & \cdot 935 \end{array} \right\} \\
&= \frac{1}{2} \{ \cdot 136 - \cdot 180 - \cdot 008 - \cdot 027 \}
\end{aligned}$$

PART III.

$$\mathfrak{A} = -\cdot020$$

$$\begin{aligned} \mathfrak{D} &= \frac{1}{2} \{ \cdot136 + \cdot180 \} \\ &= \cdot158 \end{aligned} \quad \begin{aligned} \mathfrak{D} \mathfrak{S}_4 &= +\cdot112 \\ 1 + \mathfrak{D} &= 1\cdot158 \\ 1 - \mathfrak{D} &= \cdot842 \end{aligned}$$

$$\begin{aligned} \mathfrak{G} &= \frac{1}{2} \{ -\cdot008 + \cdot027 \} + \cdot020 \times \cdot158 \\ &= \cdot009 + \cdot003 \\ &= \cdot012 \end{aligned}$$

$$\begin{aligned} 2 \mathfrak{B} &= \mathfrak{S}_4 \{ -\cdot351 - \cdot225 \} \\ &\quad - \cdot112 \{ +\cdot048 + \cdot040 \} \\ &\quad + \cdot012 \mathfrak{C} \\ &\quad - 1\cdot158 \times \cdot354 \\ \mathfrak{B} &= -\cdot414 \end{aligned}$$

$$\begin{aligned} 2 \mathfrak{C} &= \mathfrak{S}_4 \{ -\cdot351 + \cdot225 \} \\ &\quad - \cdot112 \{ +\cdot048 - \cdot040 \} \\ &\quad + \cdot012 \mathfrak{B} \\ &\quad - \cdot842 \times \cdot104 \\ \mathfrak{C} &= -\cdot090 \end{aligned}$$

CASE III.

$\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{G}$ derived from Observations on the Four Cardinal Compass Points.

When we observe on the four cardinal compass points only the following formulæ may be used. $\mathfrak{D}$ being supposed either known from previous observation or estimated.

$$\begin{aligned} \mathfrak{A} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_0 + \sin \delta_{16})}{\frac{1}{2} (\cos \delta_0 + \cos \delta_{16})} + \frac{\frac{1}{2} (\sin \delta_8 + \sin \delta_{24})}{\frac{1}{2} (\cos \delta_8 + \cos \delta_{24})} \right\} - \mathfrak{D} \mathfrak{G} \\ \mathfrak{G} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_0 + \sin \delta_{16})}{\frac{1}{2} (\cos \delta_0 + \cos \delta_{16})} - \frac{\frac{1}{2} (\sin \delta_8 + \sin \delta_{24})}{\frac{1}{2} (\cos \delta_8 + \cos \delta_{24})} \right\} - \mathfrak{D} \mathfrak{A} \\ \mathfrak{B} &= (1 + \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) \\ \mathfrak{C} &= (1 - \mathfrak{D}) \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) \end{aligned}$$

Example.

H.M.S. *Warrior*, Spithead, October 1861.

$$\begin{aligned} \delta_0 &= -6^\circ 30' & \sin \delta_0 &= -\cdot113 & \cos \delta_0 &= +\cdot993 \\ \delta_{16} &= +5^\circ 30' & \sin \delta_{16} &= +\cdot096 & \cos \delta_{16} &= +\cdot995 \\ \frac{1}{2} (\sin \delta_0 + \sin \delta_{16}) &= -\cdot008 & \frac{1}{2} (\cos \delta_0 + \cos \delta_{16}) &= +\cdot994 \\ \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) &= -\cdot104 \\ \delta_8 &= -22^\circ 15' & \sin \delta_8 &= -\cdot379 & \cos \delta_8 &= +\cdot926 \\ \delta_{24} &= +19^\circ 15' & \sin \delta_{24} &= +\cdot330 & \cos \delta_{24} &= +\cdot944 \\ \frac{1}{2} (\sin \delta_8 + \sin \delta_{24}) &= -\cdot025 & \frac{1}{2} (\cos \delta_8 + \cos \delta_{24}) &= +\cdot935 \\ \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) &= -\cdot354 \end{aligned}$$

Suppose $\mathfrak{D} = \cdot 158$, then $(1 + \mathfrak{D}) = 1\cdot 158$ and $(1 - \mathfrak{D}) = \cdot 842$

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ -\frac{\cdot 008}{\cdot 994} - \frac{\cdot 025}{\cdot 935} \right\} - \cdot 158 \mathfrak{E} \\ &= \frac{1}{2} \{ -\cdot 008 - \cdot 027 \} - \cdot 158 \times \cdot 012 \\ &= -\cdot 017 - \cdot 002 \\ &= -\cdot 019\end{aligned}$$

$$\begin{aligned}\mathfrak{E} &= \frac{1}{2} \{ -\cdot 008 + \cdot 027 \} + \cdot 158 \times \cdot 019 \\ &= \cdot 009 + \cdot 003 \\ &= \cdot 012\end{aligned}$$

$$\begin{aligned}\mathfrak{B} &= -1\cdot 158 \times \cdot 354 \\ &= -\cdot 410\end{aligned}$$

$$\begin{aligned}\mathfrak{C} &= -\cdot 842 \times \cdot 104 \\ &= -\cdot 088\end{aligned}$$

CASE IV.

$\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, derived from Observations on the Four Quadrantal Compass Points.

When we observe on the four quadrantal compass points only the following formulæ may be used. $\mathfrak{E}$ being known or supposed = 0.

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_4 + \sin \delta_{20})}{\frac{1}{2} (\cos \delta_4 + \cos \delta_{20})} + \frac{\frac{1}{2} (\sin \delta_{12} + \sin \delta_{28})}{\frac{1}{2} (\cos \delta_{12} + \cos \delta_{28})} \right\} + \mathfrak{D} \mathfrak{E} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{\frac{1}{2} (\sin \delta_4 + \sin \delta_{20})}{\frac{1}{2} (\cos \delta_4 + \cos \delta_{20})} - \frac{\frac{1}{2} (\sin \delta_{12} + \sin \delta_{28})}{\frac{1}{2} (\cos \delta_{12} + \cos \delta_{28})} \right\} \\ \mathfrak{B} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathfrak{C} \mathfrak{E} \\ &\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\ \mathfrak{C} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathfrak{B} \mathfrak{E} \\ &\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\}\end{aligned}$$

Example.

H.M.S. *Warrior*, Spithead, October 1861.

$$\begin{aligned}\delta_4 &= -13^\circ 00' & \sin \delta_4 &= -\cdot 225 & \cos \delta_4 &= +\cdot 974 \\ \delta_{20} &= +28^\circ 35' & \sin \delta_{20} &= +\cdot 478 & \cos \delta_{20} &= +\cdot 878 \\ \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) &= +\cdot 126 & \frac{1}{2}(\cos \delta_4 + \cos \delta_{20}) &= +\cdot 926 \\ \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) &= -\cdot 351 & \frac{1}{2}(\cos \delta_4 - \cos \delta_{20}) &= +\cdot 048 \\ \delta_{12} &= -23^\circ 30' & \sin \delta_{12} &= -\cdot 399 & \cos \delta_{12} &= +\cdot 917 \\ \delta_{28} &= +3^\circ 00' & \sin \delta_{28} &= +\cdot 052 & \cos \delta_{28} &= +\cdot 998 \\ \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) &= -\cdot 173 & \frac{1}{2}(\cos \delta_{12} + \cos \delta_{28}) &= +\cdot 957 \\ \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) &= -\cdot 225 & \frac{1}{2}(\cos \delta_{12} - \cos \delta_{28}) &= -\cdot 040\end{aligned}$$

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{\cdot 126}{\cdot 926} - \frac{\cdot 173}{\cdot 957} \right\} \\ &= \frac{1}{2} \{ \cdot 136 - \cdot 180 \} \\ &= -\cdot 022\end{aligned}$$

PART III.

$$\begin{aligned}
 \mathfrak{D} &= \frac{1}{2}\{\cdot 136 + \cdot 180\} \\
 &= 158 \qquad \mathfrak{S}_4 = \cdot 112 \\
 \mathfrak{B} &= \mathfrak{S}_4\{-\cdot 351 - \cdot 225\} - \cdot 112\{\cdot 048 + \cdot 040\} \\
 &= -\cdot 249 - \cdot 160 - \cdot 005 - \cdot 004 \\
 &= -\cdot 418 \\
 \mathfrak{C} &= -\cdot 249 + \cdot 160 - \cdot 005 + \cdot 004 \\
 &= -\cdot 090
 \end{aligned}$$

CASE V.

$\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ derived from Observations on two adjacent Cardinal points and the intermediate Quadrantal Compass Point.

When we observe on the three principal points in one quadrant we must assume $\mathfrak{H}$ and $\mathfrak{E}$ to be zero; we then have the following formulæ:—

1.

North-east Quadrant.

$$\begin{aligned}
 \mathfrak{D} &= \frac{\mathfrak{S}_4 \sin \delta_1 - \frac{1}{2}(\sin \delta_0 + \sin \delta_2)}{\mathfrak{S}_4 \cos \delta_1 - \frac{1}{2}(\sin \delta_0 - \sin \delta_2)} \\
 \mathfrak{B} &= (1 + \mathfrak{D}) \sin \delta_1 \\
 \mathfrak{C} &= (1 - \mathfrak{D}) \sin \delta_0
 \end{aligned}$$

Example.

$$\begin{aligned}
 \delta_1 &= -13^\circ 0' & \mathfrak{S}_4 &= \cdot 707 \\
 \sin \delta_1 &= -\cdot 225 & \mathfrak{S}_4 \sin \delta_1 &= -\cdot 160 \\
 \cos \delta_1 &= +\cdot 974 & \mathfrak{S}_4 \cos \delta_1 &= +\cdot 689 \\
 \delta_0 &= -6^\circ 30' & \sin \delta_0 &= -\cdot 113 \\
 \delta_2 &= -22^\circ 15' & \sin \delta_2 &= -\cdot 379 \\
 & & \frac{1}{2}(\sin \delta_0 + \sin \delta_2) &= -\cdot 246 \\
 & & \frac{1}{2}(\sin \delta_0 - \sin \delta_2) &= -\cdot 133 \\
 \mathfrak{D} &= \frac{-\cdot 160 + \cdot 246}{+\cdot 689 - \cdot 133} = \frac{\cdot 086}{\cdot 556} \\
 &= +\cdot 155 \\
 1 + \mathfrak{D} &= 1\cdot 155 & 1 - \mathfrak{D} &= \cdot 845 \\
 \mathfrak{B} &= -1\cdot 155 \times \cdot 379 = -\cdot 438 \\
 \mathfrak{C} &= -\cdot 845 \times \cdot 113 = -\cdot 095
 \end{aligned}$$

2.

South-east Quadrant.

$$\begin{aligned}
 \mathfrak{D} &= \frac{-\mathfrak{S}_4 \sin \delta_{12} + \frac{1}{2}(\sin \delta_9 + \sin \delta_{16})}{\mathfrak{S}_4 \cos \delta_{12} - \frac{1}{2}(\sin \delta_9 - \sin \delta_{16})} \\
 \mathfrak{B} &= (1 + \mathfrak{D}) \sin \delta_9 \\
 \mathfrak{C} &= -(1 - \mathfrak{D}) \sin \delta_{16}
 \end{aligned}$$

Example.

$$\begin{aligned}
 \delta_{12} &= -23^\circ 30' & S_4 &= .707 \\
 \sin \delta_{12} &= -.399 & S_4 \sin \delta_{12} &= -.282 \\
 \cos \delta_{12} &= +.917 & S_4 \cos \delta_{12} &= +.648 \\
 \delta_8 &= -22^\circ 15' & \sin \delta_8 &= -.379 \\
 \delta_{16} &= +5^\circ 30' & \sin \delta_{16} &= +.096 \\
 \frac{1}{2}(\sin \delta_8 + \sin \delta_{16}) &= -.141 \\
 \frac{1}{2}(\sin \delta_8 - \sin \delta_{16}) &= -.238 \\
 \mathcal{D} &= \frac{+.282 - .141}{+.648 + .238} = \frac{.141}{.886} \\
 &= +.159 \\
 1 + \mathcal{D} &= 1.159 & 1 - \mathcal{D} &= .841 \\
 \mathcal{B} &= -1.159 \times .379 = -.439 \\
 \mathcal{C} &= -.841 \times .096 = -.081
 \end{aligned}$$

3.

South-west Quadrant.

$$\begin{aligned}
 \mathcal{D} &= \frac{S_4 \sin \delta_{20} - \frac{1}{2}(\sin \delta_{16} + \sin \delta_{24})}{S_4 \cos \delta_{20} - \frac{1}{2}(\sin \delta_{16} - \sin \delta_{24})} \\
 \mathcal{B} &= -(1 + \mathcal{D}) \sin \delta_{24} \\
 \mathcal{C} &= -(1 - \mathcal{D}) \sin \delta_{16}
 \end{aligned}$$

Example.

$$\begin{aligned}
 \delta_{20} &= +28^\circ 35' & S_4 &= .707 \\
 \sin \delta_{20} &= +.478 & S_4 \sin \delta_{20} &= +.337 \\
 \cos \delta_{20} &= +.878 & S_4 \cos \delta_{20} &= +.620 \\
 \delta_{16} &= +5^\circ 30' & \sin \delta_{16} &= +.096 \\
 \delta_{24} &= +19^\circ 15' & \sin \delta_{24} &= +.330 \\
 \frac{1}{2}(\sin \delta_{16} + \sin \delta_{24}) &= +.213 \\
 \frac{1}{2}(\sin \delta_{16} - \sin \delta_{24}) &= -.117 \\
 \mathcal{D} &= \frac{+.337 - .213}{+.620 + .117} = \frac{.124}{.737} \\
 &= .168 \\
 1 + \mathcal{D} &= 1.168 & 1 - \mathcal{D} &= .832 \\
 \mathcal{B} &= -1.168 \times .330 = -.385 \\
 \mathcal{C} &= -.832 \times .096 = -.080
 \end{aligned}$$

4.

North-west Quadrant.

$$\begin{aligned}
 \mathcal{D} &= \frac{-S_4 \sin \delta_{24} + \frac{1}{2}(\sin \delta_{24} + \sin \delta_8)}{S_4 \cos \delta_{24} - \frac{1}{2}(\sin \delta_{24} - \sin \delta_8)} \\
 \mathcal{B} &= -(1 + \mathcal{D}) \sin \delta_{24} \\
 \mathcal{C} &= (1 - \mathcal{D}) \sin \delta_8
 \end{aligned}$$

PART III.

Example.

$$\begin{aligned}
\delta_{23} &= + 3^\circ 0' & S_4 &= \cdot 707 \\
\sin \delta_{23} &= + \cdot 052 & S_4 \sin \delta_{23} &= + \cdot 036 \\
\cos \delta_{23} &= + \cdot 999 & S_4 \cos \delta_{23} &= + \cdot 706 \\
\delta_{24} &= + 19^\circ 15' & \sin \delta_{24} &= + \cdot 330 \\
\delta_0 &= - 6^\circ 30' & \sin \delta_0 &= - \cdot 113 \\
\frac{1}{2}(\sin \delta_{24} + \sin \delta_0) &= + \cdot 108 \\
\frac{1}{2}(\sin \delta_{24} - \sin \delta_0) &= + \cdot 222 \\
\mathcal{D} &= \frac{-\cdot 036 + \cdot 108}{+ \cdot 706 - \cdot 222} = \frac{\cdot 072}{\cdot 484} \\
&= + \cdot 149 \\
1 + \mathcal{D} &= 1 \cdot 149 & 1 - \mathcal{D} &= \cdot 851 \\
\mathcal{B} &= - 1 \cdot 149 \times \cdot 330 = - \cdot 379 \\
\mathcal{C} &= - \cdot 851 \times \cdot 113 = - \cdot 096
\end{aligned}$$

CASE VI.

$\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ derived from Observations on the principal Compass Points in one Semicircle.

When we observe on the principal compass points in one semicircle we have the following formulæ:—

1.

Eastern Semicircle.

$$\begin{aligned}
\mathcal{D} &= \frac{\frac{1}{2}(\sin \delta_4 - \sin \delta_{12}) - S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})}{\frac{1}{2}(\cos \delta_4 + \cos \delta_{12}) - S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})} \\
\mathcal{B} &= (1 + \mathcal{D}) \sin \delta_8 \\
\mathcal{C} &= (1 - \mathcal{D}) \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})
\end{aligned}$$

Example.

$$\begin{aligned}
\delta_4 &= - 13^\circ 0' & \sin \delta_4 &= - \cdot 225 & \cos \delta_4 &= + \cdot 974 \\
\delta_{12} &= - 23^\circ 33' & \sin \delta_{12} &= - \cdot 399 & \cos \delta_{12} &= + \cdot 917 \\
\frac{1}{2}(\sin \delta_4 - \sin \delta_{12}) &= + \cdot 087 & \frac{1}{2}(\cos \delta_4 + \cos \delta_{12}) &= + \cdot 945 \\
\delta_8 &= - 22^\circ 15' & \sin \delta_8 &= - \cdot 379 \\
\delta_0 &= - 6^\circ 30' & \sin \delta_0 &= - \cdot 113 & \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) &= - \cdot 105 \\
\delta_{16} &= + 5^\circ 30' & \sin \delta_{16} &= + \cdot 096 & S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) &= - \cdot 074 \\
\mathcal{D} &= \frac{\cdot 087 + \cdot 074}{\cdot 945 + \cdot 074} = \frac{\cdot 161}{1 \cdot 019} = \cdot 158 \\
\mathcal{B} &= 1 \cdot 158 \times (- \cdot 379) = - \cdot 438 \\
\mathcal{C} &= \cdot 842 \times (- \cdot 105) = - \cdot 088
\end{aligned}$$

2.

Southern Semicircle.

$$\mathcal{D} = \frac{\frac{1}{2}(\sin \delta_{20} - \sin \delta_{12}) + S_4 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})}{\frac{1}{2}(\cos \delta_{20} + \cos \delta_{12}) - S_4 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})}$$

$$\mathcal{B} = (1 + \mathcal{D}) \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})$$

$$\mathcal{C} = -(1 - \mathcal{D}) \sin \delta_{16}$$

Example.

$$\delta_{20} = + 28^\circ 35' \quad \sin \delta_{20} = + \cdot 478 \quad \cos \delta_{20} = + \cdot 878$$

$$\delta_{12} = - 23^\circ 30' \quad \sin \delta_{12} = - \cdot 399 \quad \cos \delta_{12} = + \cdot 917$$

$$\frac{1}{2}(\sin \delta_{20} - \sin \delta_{12}) = + \cdot 439 \quad \frac{1}{2}(\cos \delta_{20} + \cos \delta_{12}) = + \cdot 897$$

$$\delta_8 = - 22^\circ 15' \quad \sin \delta_8 = - \cdot 379 \quad \frac{1}{2}(\sin \delta_8 - \sin \delta_{24}) = - \cdot 354$$

$$\delta_{24} = + 19^\circ 15' \quad \sin \delta_{24} = + \cdot 330 \quad S_4 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24}) = - \cdot 250$$

$$\delta_{16} = + 5^\circ 30' \quad \sin \delta_{16} = + \cdot 096$$

$$\mathcal{D} = \frac{+ \cdot 439 - \cdot 250}{+ \cdot 897 + \cdot 250} = \frac{\cdot 189}{1 \cdot 147} = \cdot 165$$

$$\mathcal{B} = 1 \cdot 165 \times (- \cdot 354) = - \cdot 412$$

$$\mathcal{C} = - \cdot 835 \times \cdot 096 = - \cdot 080$$

3.

Western Semicircle.

$$\mathcal{D} = \frac{\frac{1}{2}(\sin \delta_{20} - \sin \delta_{28}) + S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})}{\frac{1}{2}(\cos \delta_{20} + \cos \delta_{28}) + S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})}$$

$$\mathcal{B} = -(1 + \mathcal{D}) \sin \delta_{24}$$

$$\mathcal{C} = +(1 - \mathcal{D}) \frac{1}{2}(\sin \delta_0 - \sin \delta_{16})$$

Example.

$$\delta_{20} = + 28^\circ 35' \quad \sin \delta_{20} = + \cdot 478 \quad \cos \delta_{20} = + \cdot 878$$

$$\delta_{28} = + 3^\circ 00' \quad \sin \delta_{28} = + \cdot 052 \quad \cos \delta_{28} = + \cdot 998$$

$$\frac{1}{2}(\sin \delta_{20} - \sin \delta_{28}) = + \cdot 213 \quad \frac{1}{2}(\cos \delta_{20} + \cos \delta_{28}) = + \cdot 938$$

$$\delta_{24} = + 19^\circ 15' \quad \sin \delta_{24} = + \cdot 330$$

$$\delta_0 = - 6^\circ 30' \quad \sin \delta_0 = - \cdot 113 \quad \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) = - \cdot 104$$

$$\delta_{16} = + 5^\circ 30' \quad \sin \delta_{16} = + \cdot 096 \quad S_4 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) = - \cdot 074$$

$$\mathcal{D} = \frac{+ \cdot 213 - \cdot 074}{+ \cdot 938 - \cdot 074} = \frac{+ \cdot 139}{+ \cdot 914} = \cdot 161$$

$$\mathcal{B} = - 1 \cdot 161 \times \cdot 330 = - \cdot 383$$

$$\mathcal{C} = \cdot 839 \times (- \cdot 074) = - \cdot 062$$

4.

Northern Semicircle.

$$\mathcal{D} = \frac{\frac{1}{2}(\sin \delta_8 - \sin \delta_{20}) - S_4 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})}{\frac{1}{2}(\cos \delta_8 + \cos \delta_{20}) + S_4 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})}$$

$$\mathcal{B} = (1 + \mathcal{D}) \frac{1}{2}(\sin \delta_8 - \sin \delta_{24})$$

$$\mathcal{C} = (1 - \mathcal{D}) \sin \delta_0$$

PART III.

Example.

$$\delta_4 = -13^\circ 0' \quad \sin \delta_4 = -\cdot 225 \quad \cos \delta_4 = +\cdot 974$$

$$\delta_{28} = +3^\circ 0' \quad \sin \delta_{28} = +\cdot 052 \quad \cos \delta_{28} = +\cdot 998$$

$$\frac{1}{2} (\sin \delta_4 - \sin \delta_{28}) = -\cdot 138 \quad \frac{1}{2} (\cos \delta_4 + \cos \delta_{28}) = +\cdot 986$$

$$\sin \delta_8 = -22^\circ 15' \quad \sin \delta_8 = -\cdot 379 \quad \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) = -\cdot 355$$

$$\sin \delta_{24} = +19^\circ 15' \quad \sin \delta_{24} = +\cdot 330 \quad S_4 \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) = -251$$

$$\delta_6 = -6^\circ 30' \quad \sin \delta_6 = -\cdot 113$$

$$\mathcal{D} = \frac{-\cdot 138 + \cdot 251}{\cdot 986 - \cdot 251} = \frac{+ \cdot 113}{\cdot 735} = +\cdot 154$$

$$\mathcal{B} = 1\cdot 154 \times (-\cdot 355) = -\cdot 410$$

$$\mathcal{C} = \cdot 846 \times (-\cdot 113) = -\cdot 096^*$$

* If we have the deviations on a semicircle terminated by two quadrantal compass points, then in addition to the expressions for $\mathcal{D}$ $\mathcal{B}$ and $\mathcal{C}$ from the complete quadrant which is included in it we have the following expressions from each set of three quadrantal points ($\mathcal{A}$ and $\mathcal{G}$ being considered as zero):

N.E. Semicircle.

$$\mathcal{D} = - \frac{\sin \delta_{12} + \sin \delta_{28}}{\cos \delta_{12} + \cos \delta_{28}}$$

$$\mathcal{B} = S_1 (\sin \delta_4 + \sin \delta_{12}) - \mathcal{D} S_1 (\cos \delta_4 - \cos \delta_{12})$$

$$\mathcal{C} = S_1 (\sin \delta_4 + \sin \delta_{28}) - \mathcal{D} S_1 (\cos \delta_4 - \cos \delta_{28})$$

S.E. Semicircle.

$$\mathcal{D} = \frac{\sin \delta_4 + \sin \delta_{28}}{\cos \delta_4 + \cos \delta_{28}}$$

$$\mathcal{B} = S_1 (\sin \delta_4 + \sin \delta_{12}) - \mathcal{D} S_1 (\cos \delta_4 - \cos \delta_{12})$$

$$\mathcal{C} = - S_1 (\sin \delta_{12} + \sin \delta_{28}) - \mathcal{D} S_1 (\cos \delta_{12} - \cos \delta_{28})$$

S.W. Semicircle.

$$\mathcal{D} = - \frac{\sin \delta_{12} + \sin \delta_{28}}{\cos \delta_{12} + \cos \delta_{28}}$$

$$\mathcal{B} = - S_1 (\sin \delta_{28} + \sin \delta_{28}) + \mathcal{D} S_1 (\cos \delta_{28} - \cos \delta_{28})$$

$$\mathcal{C} = - S_1 (\sin \delta_{12} + \sin \delta_{28}) - \mathcal{D} S_1 (\cos \delta_{12} - \cos \delta_{28})$$

N.W. Semicircle.

$$\mathcal{D} = \frac{\sin \delta_4 + \sin \delta_{28}}{\cos \delta_4 + \cos \delta_{28}}$$

$$\mathcal{B} = - S_1 (\sin \delta_{28} + \sin \delta_{28}) + \mathcal{D} S_1 (\cos \delta_{28} - \cos \delta_{28})$$

$$\mathcal{C} = S_1 (\sin \delta_4 + \sin \delta_{28}) - \mathcal{D} S_1 (\cos \delta_4 - \cos \delta_{28})$$

The mean of the values of $\mathcal{B}$ $\mathcal{C}$ and $\mathcal{D}$ given by these equations and the values given by the equations in pp. 52, 53, is to be used.

CASE VII.

PART III.

$\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ derived from Observations on any three Compass Courses.

If we observe the deviations δ_1 δ_2 δ_3 on any three compass courses ζ'_1 ζ'_2 and ζ'_3 , then assuming $\mathfrak{A}$ and $\mathfrak{E}$ to be zero—

$$\mathfrak{D} = \frac{\sin(\zeta'_3 - \zeta'_2) \sin \delta_1 + \sin(360 + \zeta'_1 - \zeta'_3) \sin \delta_2 + \sin(\zeta'_2 - \zeta'_1) \sin \delta_3}{\sin(\zeta'_3 - \zeta'_2) \sin(2\zeta'_1 + \delta_1) + \sin(360 + \zeta'_1 - \zeta'_3) \sin(2\zeta'_2 + \delta_2) + \sin(\zeta'_3 - \zeta'_1) \sin(2\zeta'_3 + \delta_3)}$$

$$\mathfrak{B} = -\frac{\{\sin \delta_1 - \mathfrak{D} \sin(2\zeta'_1 + \delta_1)\} \cos \zeta'_2 + \{\sin \delta_2 - \mathfrak{D} \sin(2\zeta'_2 + \delta_2)\} \cos \zeta'_1}{\sin(\zeta'_2 - \zeta'_1)}$$

$$\mathfrak{C} = \frac{\{\sin \delta_1 - \mathfrak{D} \sin(2\zeta'_1 + \delta_1)\} \sin \zeta'_2 - \{\sin \delta_2 - \mathfrak{D} \sin(2\zeta'_2 + \delta_2)\} \sin \zeta'_1}{\sin(\zeta'_2 - \zeta'_1)}$$

Example.

H.M.S. *Warrior*, Spithead, October 1861.

$\zeta'_1 = 8^\circ 30'$	$\delta_1 = - 7^\circ 50'$	$\sin \delta_1 = - \cdot 136$
$2\zeta'_1 + \delta_1 = + 9^\circ 10'$	$\sin 2\zeta'_1 + \delta_1 = + \cdot 159$	
$\zeta'_2 = 113^\circ 0'$	$\delta_2 = - 25^\circ 20'$	$\sin \delta_2 = - \cdot 428$
$2\zeta'_2 + \delta_2 = 200^\circ 40'$	$\sin 2\zeta'_2 + \delta_2 = - \cdot 352$	
$\zeta'_3 = 200^\circ 0'$	$\delta_3 = + 22^\circ 15'$	$\sin \delta_3 = + \cdot 379$
$2\zeta'_3 + \delta_3 = 62^\circ 15'$	$\sin 2\zeta'_3 + \delta_3 = + \cdot 885$	
$\zeta'_3 - \zeta'_2 = 87^\circ 0'$	$\sin(\zeta'_3 - \zeta'_2) = + \cdot 999$	
$360 + \zeta'_1 - \zeta'_3 = 168^\circ 30'$	$\sin(360 + \zeta'_1 - \zeta'_3) = + \cdot 199$	
$\zeta'_2 - \zeta'_1 = 104^\circ 30'$	$\sin(\zeta'_2 - \zeta'_1) = + \cdot 968$	
$\sin \zeta'_1 = \cdot 148$	$\sin \zeta'_2 = + \cdot 920$	
$\cos \zeta'_1 = \cdot 989$	$\cos \zeta'_2 = - \cdot 391$	

$$\mathfrak{D} = \frac{+ \cdot 999 \times (- \cdot 136) + \cdot 199 \times (- \cdot 428) + \cdot 968 \times \cdot 379}{+ \cdot 999 \times \cdot 159 + \cdot 199 \times (- \cdot 352) + \cdot 968 \times \cdot 885}$$

$$\begin{aligned} &= \frac{- \cdot 136 - \cdot 085 + \cdot 367}{+ \cdot 159 - \cdot 070 + \cdot 857} \\ &= \frac{\cdot 146}{\cdot 946} \\ &= + \cdot 154 \end{aligned}$$

$$\mathfrak{B} = \frac{+ \{ - \cdot 136 - \cdot 154 \times \cdot 159 \} \cdot 391 + \{ - \cdot 428 + \cdot 154 \times \cdot 352 \} \cdot 989}{\cdot 968}$$

$$\begin{aligned} &= \frac{- \cdot 160 \times \cdot 391 - \cdot 374 \times \cdot 989}{\cdot 963} \\ &= \frac{- \cdot 063 - \cdot 370}{\cdot 968} \end{aligned}$$

$$\begin{aligned} &= \frac{- \cdot 438}{+ \cdot 968} \\ &= - \cdot 447 \end{aligned}$$

$$\mathfrak{C} = \frac{- \cdot 160 \times \cdot 920 + \cdot 374 \times \cdot 148}{\cdot 968}$$

$$\begin{aligned} &= \frac{- \cdot 147 + \cdot 055}{\cdot 968} \\ &= \frac{- \cdot 092}{\cdot 968} \\ &= - \cdot 095 \end{aligned}$$

PART III. It is interesting to compare the values of the coefficients obtained by the different methods which have been given. The near accordance of these values is a test of the accuracy of the observations. For this purpose the following Table is formed.

Comparative Table of the Value of B, C, and D obtained by the several preceding Methods.				B.	C.	D.
From 32 points	-	-	-	- '410	- '095	+ '156
From the 8 principal points	-	-	-	- '414	- '090	+ '158
From the 4 cardinal points	-	-	-	- '410	- '088	
From the 4 quadrantal points	-	-	-	- '418	- '090	+ '158
N.E. quadrant	-	-	-	- '438	- '095	+ '155
S.E. „	-	-	-	- '439	- '081	+ '159
S.W. „	-	-	-	- '385	- '080	+ '168
N.W. „	-	-	-	- '379	- '096	+ '149
East semicircle	-	-	-	- '438	- '088	+ '158
South „	-	-	-	- '412	- '080	+ '165
West „	-	-	-	- '383	- '062	+ '161
North „	-	-	-	- '410	- '096	+ '154
Three points	-	-	-	- '447	- '095	+ '154

§ IV.

Determination of $\mathfrak{B}$ and $\mathfrak{C}$, $\mathfrak{D}$ being supposed known, and $\mathfrak{A}$ and $\mathfrak{E}$ being supposed zero.

As the coefficient $\mathfrak{D}$ does not change with a change of geographical position and only very slightly with the lapse of time we may when its value for a compass in any particular place in a ship has been once determined make use of that value in any subsequent observations on a compass in the same place in the ship. This enables us to determine $\mathfrak{B}$ and $\mathfrak{C}$ from two observations, provided they are not too near or too nearly opposite each other, and it also enables us when observations of deviation have been made on several points situate at irregular intervals to apply the method of least squares to determine $\mathfrak{B}$ and $\mathfrak{C}$, a method which would be far too laborious if we had to determine $\mathfrak{D}$ as well.

If the deviations are given on the 32, 16, or 8 principal compass points we may use the tabular forms, using only Table A. of each. These will give $\mathfrak{B}$ and $\mathfrak{C}$, and $\mathfrak{B}$ and $\mathfrak{C}$ may be determined from them as shown above.

Column III. might seem unnecessary. It is, however, desirable to retain it and find the mean of its values so as to obtain $\mathfrak{A}$, in order to detect

and allow for the index and other errors which frequently give A an apparent value. PART III.

CASE I.

The four Cardinal Compass Points.

If we observe on the four cardinal points, then—

$$B = (1 + D) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24})$$

$$C = (1 - D) \frac{1}{2} (\sin \delta_8 - \sin \delta_{16})$$

Example.

H.M.S. *Warrior*.

$$D = .158$$

$$1 + D = 1.158$$

$$1 - D = .842$$

$$\delta_8 = -22^\circ 15' \quad \sin \delta_8 = -.379$$

$$\delta_{24} = +19^\circ 15' \quad \sin \delta_{24} = +.330$$

$$\frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) = -.354$$

$$\delta_0 = -6^\circ 30' \quad \sin \delta_0 = -.113$$

$$\delta_{16} = +5^\circ 30' \quad \sin \delta_{16} = +.096$$

$$\frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) = -.104$$

$$B = -1.158 \times .354$$

$$= -.410$$

$$C = -.842 \times .104$$

$$= -.088$$

CASE II.

The four Quadrantal Compass Points.

If the observations are made on the four quadrantal points—

$$B + C = S_4 (\sin \delta_4 - \sin \delta_{20}) - D S_4 (\cos \delta_4 - \cos \delta_{20})$$

$$B - C = S_4 (\sin \delta_{12} - \sin \delta_{28}) + D S_4 (\cos \delta_{12} - \cos \delta_{28})$$

Half the sum of these expressions gives B, half the difference gives C.

Example.

H.M.S. *Warrior*.

$$D = .158 \quad D S_4 = .112$$

$$\delta_4 = -13^\circ \quad \sin \delta_4 = -.225 \quad \cos \delta_4 = +.974$$

$$\delta_{20} = +28^\circ 35' \quad \sin \delta_{20} = +.478 \quad \cos \delta_{20} = +.878$$

$$\sin \delta_4 - \sin \delta_{20} = -.703 \quad \cos \delta_4 - \cos \delta_{20} = +.096$$

$$B + C = -S_4 .703 - .112 \times .096$$

$$= -.496 - .011 = -.507$$

$$\delta_{12} = -23^\circ 30' \quad \sin \delta_{12} = -.399 \quad \cos \delta_{12} = +.917$$

$$\delta_{28} = +3^\circ 00' \quad \sin \delta_{28} = +.052 \quad \cos \delta_{28} = +.998$$

$$\sin \delta_{12} - \sin \delta_{28} = -.451 \quad \cos \delta_{12} - \cos \delta_{28} = -.081$$

PART III.

$$\begin{aligned}\mathfrak{B} - \mathfrak{C} &= -S_1 \times \cdot 451 + \cdot 112 \times (-\cdot 081) \\ &= -\cdot 319 - \cdot 009 = -\cdot 328\end{aligned}$$

Whence—

$$\begin{aligned}\mathfrak{B} &= \frac{1}{2} (-\cdot 507 - \cdot 328) = -\cdot 417 \\ \mathfrak{C} &= \frac{1}{2} (-\cdot 507 + \cdot 328) = -\cdot 089\end{aligned}$$

CASE III.*One Cardinal Compass Point.*

$\mathfrak{D}$ being known, an observation of deviation on any one of the cardinal points will give the value of one of the semicircular coefficients. Thus an observation of deviation at E. or W. will give $\mathfrak{B}$, at N. or S. will give $\mathfrak{C}$. The requisite formulæ are—

$$\begin{aligned}\mathfrak{B} &= + (1 + \mathfrak{D}) \sin \delta_e \\ \mathfrak{B} &= - (1 + \mathfrak{D}) \sin \delta_{24} \\ \mathfrak{C} &= + (1 - \mathfrak{D}) \sin \delta_s \\ \mathfrak{C} &= - (1 - \mathfrak{D}) \sin \delta_{16}\end{aligned}$$

Examples.

H.M.S. *Warrior*. $\mathfrak{D} = \cdot 158$.

$$1 + \mathfrak{D} = 1\cdot 158 \qquad 1 - \mathfrak{D} = \cdot 842$$

1. East.

$$\begin{aligned}\delta_e &= -22^\circ 15' & \sin \delta_e &= -\cdot 379 \\ \mathfrak{B} &= -1\cdot 158 \times \cdot 379 \\ &= -\cdot 439\end{aligned}$$

2. West.

$$\begin{aligned}\delta_{24} &= +19^\circ 15' & \sin \delta_{24} &= +\cdot 330 \\ \mathfrak{B} &= -1\cdot 158 \times \cdot 330 \\ &= -\cdot 382\end{aligned}$$

The mean of the two values is—

$$\mathfrak{B} = -\cdot 410$$

3. North.

$$\begin{aligned}\delta_s &= -6^\circ 30' & \sin \delta_s &= -\cdot 113 \\ \mathfrak{C} &= -\cdot 842 \times \cdot 113 \\ &= -\cdot 095\end{aligned}$$

4. South.

$$\begin{aligned}\delta_{16} &= +5^\circ 30' & \sin \delta_{16} &= +\cdot 096 \\ \mathfrak{C} &= -\cdot 842 \times \cdot 096 \\ &= -\cdot 081\end{aligned}$$

The mean of the two values is $-\cdot 088$.

These formulæ are extremely simple, and if a steam vessel for which $\mathfrak{D}$ is known, were put successively on the two cardinal points nearest her course, and the deviation on these courses ascertained, $\mathfrak{B}$ and $\mathfrak{C}$ might be easily determined.

CASE IV.

PART III.

Any two Compass Points.

If the deviations are observed on any two compass azimuths ζ'_1 and ζ'_2 , we have—

$$B = \frac{-\{\sin \delta_1 - D \sin (2\zeta'_1 + \delta_1)\} \cos \zeta'_2 + \{\sin \delta_2 - D \sin (2\zeta'_2 + \delta_2)\} \cos \zeta'_1}{\sin (\zeta'_2 - \zeta'_1)}$$

$$C = \frac{\{\sin \delta_1 - D \sin (2\zeta'_1 + \delta_1)\} \sin \zeta'_2 - \{\sin \delta_2 - D \sin (2\zeta'_2 + \delta_2)\} \sin \zeta'_1}{\sin (\zeta'_2 - \zeta'_1)}$$

*Example.**H.M.S. Warrior.*

$$D = .156$$

$$\begin{array}{lll} \zeta'_1 = 8^\circ 30' & \delta_1 = -7^\circ 50' & \sin \delta_1 = -.136 \\ \sin \zeta'_1 = .148 & 2\zeta'_1 + \delta_1 = +9^\circ 10' & D \sin (2\zeta'_1 + \delta_1) = +.025 \\ \cos \zeta'_1 = .989 & \sin (2\zeta'_1 + \delta_1) = +.160 & \sin \delta_1 - D \sin (2\zeta'_1 + \delta_1) = -.161 \\ \zeta'_2 = 86^\circ & \delta_2 = -22^\circ 20' & \sin \delta_2 = -.380 \\ \sin \zeta'_2 = .998 & 2\zeta'_2 + \delta_2 = +149^\circ 40' & D \sin (2\zeta'_2 + \delta_2) = +.079 \\ \cos \zeta'_2 = .070 & \sin (2\zeta'_2 + \delta_2) = +.505 & \sin \delta_2 - D \sin (2\zeta'_2 + \delta_2) = -.459 \end{array}$$

$$\zeta'_2 - \zeta'_1 = 77^\circ 30'$$

$$\sin (\zeta'_2 - \zeta'_1) = .976$$

$$B = \frac{.161 \times .070 - .459 \times .989}{.976}$$

$$= \frac{.011 - .454}{.976}$$

$$= -.453$$

$$C = \frac{-.161 \times .998 + .459 \times .148}{.976}$$

$$= \frac{-.161 + .068}{.976}$$

$$= -.095$$

CASE V.

Any Number of Compass Courses.

If the deviations are observed on the compass courses, $\zeta_1 \zeta_2 \dots \dots \zeta_n$ and if $\Delta = \sin \delta - D \cos (2\zeta + \delta)$

$$B = 2 \frac{\frac{1}{n} \Sigma \Delta \sin \zeta' + \frac{1}{n} \Sigma \Delta \sin \zeta' \times \frac{1}{n} \Sigma \cos 2\zeta' - \frac{1}{n} \Sigma \Delta \cos \zeta' \times \frac{1}{n} \Sigma \sin 2\zeta'}{1 - (\frac{1}{n} \Sigma \sin 2\zeta')^2 - (\frac{1}{n} \Sigma \cos 2\zeta')^2}$$

$$C = 2 \frac{\frac{1}{n} \Sigma \Delta \cos \zeta' - \frac{1}{n} \Sigma \Delta \cos \zeta' \times \frac{1}{n} \Sigma \cos 2\zeta' - \frac{1}{n} \Sigma \Delta \sin \zeta' \times \frac{1}{n} \Sigma \sin 2\zeta'}{1 - (\frac{1}{n} \Sigma \sin 2\zeta')^2 - (\frac{1}{n} \Sigma \cos 2\zeta')^2}$$

N.B.—The expression $\Sigma \Delta \sin \zeta'$ is a contraction for $\Delta_1 \sin \zeta'_1 + \Delta_2 \sin \zeta'_2 + \Delta_3 \sin \zeta'_3$ &c., &c., and so of the other expressions preceded by a Σ .

These calculations may be most easily performed by a tabular arrangement as in the following :—

Example. See *Warrior*, p. 62.

PART III.

FORM VII.

COMPUTATION of $\mathfrak{B}$ and $\mathfrak{C}$ from Deviations of the Standard Compass of H.M.S. *Warrior*, observed on six Compass Courses, $\mathfrak{D}$ being known ($= + \cdot 156$), and $\mathfrak{A}$ and $\mathfrak{E}$ being Zero.

ζ'	δ	$2\zeta'$	$2\zeta' + \delta$	$\sin(2\zeta' + \delta)$	$-\mathfrak{D} = -\cdot 156$ $\sin(2\zeta' + \delta)$	$\sin \delta$	$\frac{\sin \delta}{\sin(2\zeta' + \delta)}$ $= \Delta$	$\sin \zeta'$	$\cos \zeta'$	$\Delta \sin \zeta'$	$\Delta \cos \zeta'$	$\sin 2\zeta'$	$\cos 2\zeta'$
8 30	7 50	17	+ 9 10	+ .159	— .025	— .136	— .161	+ .148	+ .989	— .024	— .159	+ .292	+ .956
41 30	— 13 0	83	+ 940	+ .940	— .146	— .235	— .371	+ .663	+ .749	— .246	— .278	+ .992	+ .122
99 0	— 23 0	198	+ .087	+ .087	— .014	— .391	— .405	+ .988	— .156	— .400	+ .063	— .309	— .951
183 0	+ 7 30	6	+ .233	+ .233	— .036	+ .131	+ .095	— .052	— .999	— .005	— .095	+ .104	+ .994
244 0	+ 26 35	128	+ .429	+ .429	— .067	+ .447	+ .380	— .899	— .438	— .341	— .166	+ .788	— .616
321 50	+ 1 0	283	— .970	— .970	+ .151	+ .017	+ .168	— .623	+ .783	— .105	+ .132	— .974	+ .225
$\mathfrak{B} = 2 \quad \frac{\text{I.} + \text{I.} \times \text{IV.} - \text{II.} \times \text{III.}}{\text{V.}}$													
$= 2 \quad \frac{-\cdot 187 - \cdot 187 \times \cdot 122 - \cdot 084 \times \cdot 149}{\cdot 963}$													
$= -\cdot 412$													
$\mathfrak{C} = 2 \quad \frac{\text{II.} - \text{I.} \times \text{III.} - \text{II.} \times \text{IV.}}{\text{V.}}$													
$= 2 \quad \frac{-\cdot 084 + \cdot 187 \times \cdot 149 + \cdot 084 \times \cdot 122}{\cdot 963}$													
$= -\cdot 096$													
Sum													
$\frac{1}{2}$ sum													
Square													
Sum of squares													
1 — sum of squares													
V.													

CASE VI.

PART III.

One compass Course (the starboard angle not changing).

In those ships in which $\mathfrak{B}$ and $\mathfrak{C}$ have been ascertained to change in the same proportion on a change of latitude or in which the starboard angle does not change with a change of latitude, we can obtain the value of $\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$ and therefore of $\mathfrak{B}$ and $\mathfrak{C}$ from a single observation of deviation.

Let $\frac{\mathfrak{C}}{\mathfrak{B}} = \tan \alpha$. α being therefore what we have called the starboard angle.

Then—

$$\begin{aligned}\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} &= \frac{\sin \delta - \mathfrak{D} \sin (2\zeta' + \delta)}{\sin (\zeta' + \alpha)} \\ \mathfrak{B} &= \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} \cos \alpha \\ \mathfrak{C} &= \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} \sin \alpha.\end{aligned}$$

*Example.*H.M.S. *Warrior*. At Spithead.

We have—

$$\begin{aligned}\mathfrak{B} &= -\cdot 408 & \tan \alpha &= \frac{-\cdot 090}{-\cdot 408} \\ \mathfrak{C} &= -\cdot 090 & &= +\cdot 221 \\ \mathfrak{D} &= +\cdot 156 & \alpha &= 192^\circ \cdot 26' \\ & & \cos \alpha &= -\cdot 976 \\ & & \sin \alpha &= -\cdot 215\end{aligned}$$

Suppose that at Lisbon the value of α has been ascertained not to change, and that on a N.N.E. course (compass) the deviation is 6° W. ; then we have—

$$\begin{aligned}\zeta' &= + 56^\circ \cdot 15' & \sin \delta &= -\cdot 104 \\ \delta &= - 6^\circ & \sin (2\zeta' + \delta) &= +\cdot 959 \\ 2\zeta' + \delta &= + 106^\circ \cdot 30'\end{aligned}$$

whence—

$$\begin{aligned}\mathfrak{D} \sin (2\zeta' + \delta) &= \cdot 156 \times \cdot 959 = +\cdot 150 \\ \zeta' + \alpha &= 248^\circ 41' & \sin (\zeta' + \alpha) &= -\cdot 932\end{aligned}$$

and by the formula—

$$\begin{aligned}\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} &= \frac{\sin \delta - \mathfrak{D} \sin (2\zeta' + \delta)}{\sin (\zeta' + \alpha)} \\ &= \frac{-\cdot 104 - \cdot 150}{-\cdot 932} \\ &= +\cdot 273 \\ \mathfrak{B} &= -\cdot 273 \times \cdot 976 = -\cdot 266 \\ \mathfrak{C} &= -\cdot 273 \times \cdot 215 = -\cdot 059\end{aligned}$$

§ V.

Computation of the Deviation from $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$.

When the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are determined, or $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, if $\mathfrak{A}$ and $\mathfrak{E}$ be zero, the deviations on each *magnetic* point may be obtained *exactly* by the following formula :—

$$\tan \delta = \frac{\mathfrak{A} + \mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathfrak{E} \cos 2\zeta}{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta}$$

The computations may be made by the following tabular form :—

TABULAR FORM for obtaining the Deviations on the *Magnetic* Courses from α , β , γ , δ , and ϵ .
FORM VIII.

COMPUTATION of the Deviations of the Standard Compass of H.M.S. *Warrior* on the 8 principal *Magnetic* Points from the coefficients α , β , γ , δ , and ϵ , as determined at Spithead, October 1861.

I. $\alpha = - \cdot 017$. This Column contains the Value of α and $1 \cdot 000$ alternately.	II. $\delta = \cdot 156$.		III. $\epsilon = \cdot 015$.		IV. $\beta = - \cdot 408$.		V. $\gamma = - \cdot 090$.		VI. Sum of Cols. I, II, III, IV, and V.	VII. Logarithms of Nos. in Col. VI.	VIII. Difference of Logarithms of the Lines forming each Pair. in Col. VII.	IX. Angles of which the Nos. in Col. VIII. are the log. tan., being the Deviations on the Magnetic Courses.*	Magnetic Courses.
	Multipliers.	Products of δ by Multipliers.	Multipliers.	Products of ϵ by Multipliers.	Multipliers.	Products of β by Multipliers.	Multipliers.	Products of γ by Multipliers.					
$\alpha = - \cdot 017$	0	0	1	$\cdot 015$	0	0	1	$- \cdot 090$	$- \cdot 092$	8·9637	9·0898	$\begin{smallmatrix} 0 \\ - \end{smallmatrix} 7 \ 11$	N.
$1 \cdot 000$	1	$+ \cdot 156$	0	0	1	$- \cdot 408$	0	0	$\cdot 748$	9·8739			
$\alpha = - \cdot 017$	1	$+ \cdot 156$	0	0	S_1	$- \cdot 289$	S_1	$- \cdot 063$	$- \cdot 213$	9·3284	9·4482	$- 15 \ 41$	N.E.
$1 \cdot 000$	0	0	-1	$- \cdot 015$	S_1	$- \cdot 289$	$- S_1$	$+ \cdot 063$	$\cdot 759$	9·8802			
$\alpha = - \cdot 017$	0	0	-1	$- \cdot 015$	1	$- \cdot 408$	0	0	$- \cdot 440$	9·6424	9·6731	$- 25 \ 14$	E.
$1 \cdot 000$	-1	$- \cdot 156$	0	0	0	0	-1	$+ \cdot 090$	$\cdot 934$	9·9703			
$\alpha = - \cdot 017$	-1	$- \cdot 156$	0	0	S_1	$- \cdot 289$	$- S_1$	$+ \cdot 063$	$- \cdot 399$	9·6010	9·4652	$- 16 \ 16$	S.E.
$1 \cdot 000$	0	0	1	$+ \cdot 015$	$- S_1$	$+ \cdot 289$	$- S_1$	$+ \cdot 063$	$1 \cdot 367$	10·1358			
$\alpha = - \cdot 017$	0	0	1	$+ \cdot 015$	0	0	-1	$+ \cdot 090$	$\cdot 088$	8·9444	8·7502	$+ 3 \ 13$	S.
$1 \cdot 000$	1	$+ \cdot 156$	0	0	-1	$+ \cdot 408$	0	0	$1 \cdot 564$	10·1942			
$\alpha = - \cdot 017$	1	$+ \cdot 156$	0	0	$- S_1$	$+ \cdot 289$	$- S_1$	$+ \cdot 063$	$\cdot 491$	9·6911	9·6080	$+ 22 \ 4$	S.W.
$1 \cdot 000$	0	0	-1	$- \cdot 015$	$- S_1$	$+ \cdot 289$	S_1	$- \cdot 063$	$1 \cdot 211$	10·0831			
$\alpha = - \cdot 017$	0	0	-1	$- \cdot 015$	-1	$+ \cdot 408$	0	0	$\cdot 376$	9·5751	9·6977	$+ 26 \ 30$	W.
$1 \cdot 000$	-1	$- \cdot 156$	0	0	0	0	1	$- \cdot 090$	$\cdot 754$	9·8774			
$\alpha = - \cdot 017$	-1	$- \cdot 156$	0	0	$- S_1$	$+ \cdot 289$	$- S_1$	$- \cdot 063$	$\cdot 053$	8·7243	8·9028	$+ 4 \ 34$	N.W.
$1 \cdot 000$	0	0	1	$+ \cdot 015$	S_1	$- \cdot 289$	S_1	$- \cdot 063$	$\cdot 663$	9·8215			

* The signs of the angles of deviation will be found from the signs of the quantities in Col. VI. considered as fractions, and being the natural tangents of the angles of deviation.

As the deviations given by this form are for the *magnetic* courses it is PART III.
 necessary to construct a Napier's curve in order to determine the deviations on the *compass* courses; and as the curve must be constructed, and as it can be constructed, from the deviations on *eight* points it is unnecessary to increase the labour of the process by computing the deviations for more than eight points.

§ VI.

Graphic Methods.

There are several graphic methods which may be used with great advantage, principally to obtain the deviations on the magnetic or compass points from the coefficients, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$, or $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, but also to obtain the coefficients from the observations.

I.

Construction of Napier's curve from the coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$.

For this purpose we use the expression

$$\begin{aligned}\delta &= \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin (2\zeta' + \delta) \\ &= \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin 2 \left(\frac{\zeta + \zeta'}{2} \right)\end{aligned}$$

which is nearly exact when $\mathfrak{A}$ and $\mathfrak{E}$ are zero, and $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ not very large.

In this expression $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ are supposed expressed in degrees obtained either by taking the arcs, of which $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ are the natural sines, or by multiplying $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ by 57.3, the number of degrees in an arc = radius.

Then, take an engraved form of the diagram for Napier's curve, and suppose it placed vertically, the upper point being N., the next N. by E., and so on.

First, construct the curve of the quadrantal deviation, or $\mathfrak{D} \sin 2 \left(\frac{\zeta + \zeta'}{2} \right)$.

To do this take a pair of compasses and open them to $\mathfrak{D} \sin 60^\circ = .866 \mathfrak{D}$, or (which comes to the same thing) place one point of the compasses at the end of an arc $\mathfrak{D}$ measured from N., and open the compasses till the other point reaches to either the plain or dotted line passing through N.

This distance is then to be laid off *perpendicularly to the central line* at N.E., S.E., S.W., N.W.; to the right at N.E. and S.W.; to the left at S.E. and N.W.

Next, open the compasses to $S_4 \mathfrak{D} \sin 60^\circ = .612 \mathfrak{D}$, and lay off that distance perpendicularly to the central line at the eight semi-quadrantal points; to the right in the first and third quadrants to the left in the second and fourth.

Through the 12 points thus determined and the five N., E., S., W., N. points on the central line, the quadrantal curve may be drawn without difficulty.

PART III. As this curve does not change when once laid down for the standard compass of any particular ship, it may be constructed when the ship is first swung, and afterwards used.

Secondly, construct the curve of semicircular deviation,

$$\mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta'$$

by laying down the following distances along *the dotted lines*, i.e., $\mathfrak{C}$ at north, $\mathfrak{B}$ at east, $-\mathfrak{C}$ at south, and $-\mathfrak{B}$ at west,

$$\begin{array}{ll} +S, \mathfrak{B} + S, \mathfrak{C} \text{ at N.E.} & +S, \mathfrak{B} - S, \mathfrak{C} \text{ at S.E.} \\ -S, \mathfrak{B} - S, \mathfrak{C} \text{ at S.W.} & -S, \mathfrak{B} + S, \mathfrak{C} \text{ at N.W.} \end{array}$$

and through the eight points thus determined draw the semicircular curve.

Thirdly, superpose the quadrantal curve on the semicircular proceeding as follows:—

From any point Q in the quadrantal curve draw a line parallel to the dotted lines intersecting the central line in q . From q draw a line perpendicular to the central line intersecting the semicircular curve in p , then complete the parallelogram $QqpP$. P is a point in the Napier's curve.

When we have laid down a sufficient number of points we may draw the Napier's curve through them.

In practice we may, using a pair of compasses, dispense with actually drawing any line except a line through Q perpendicular to the central line, in which line the point P will of course lie.

Having drawn the Napier's curve, we may obtain the deviations in the usual way.

From the Napier's curve or from two observations to find the semicircular curve, the quadrantal curve being supposed laid down.—Take a point P on the Napier's curve or from observation. Through P draw a line perpendicular to the central line intersecting the quadrantal curve in Q ; from Q draw Qq parallel to the dotted lines intersecting the central line in q , and complete the parallelogram $PQqp$; p is a point in the semicircular curve.

If we have the Napier's curve any number of points p may be determined in the same way and the semicircular curve drawn.

If P is given by observation a second point may be obtained from the same observation by adding 180° to or subtracting it from the compass course and laying down the semicircular deviation in the opposite direction. We may thus from two observations, if not near or nearly opposite to each other, obtain four points from which the semicircular curve may be drawn, and from this the complete Napier's curve as in the last case.

II.

PART III.

*Dygogram.**

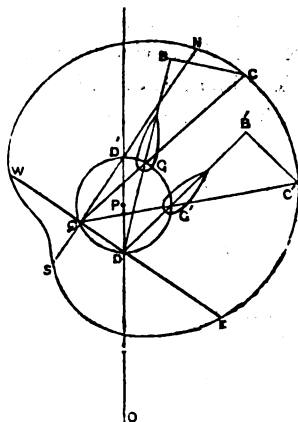
There is a graphic method of obtaining the deviations on the different magnetic points which is frequently very useful. It is adapted for the most general cases, viz., those in which $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ have sensible values, but its use is so much greater in the cases in which $\mathcal{A}$ and $\mathcal{E}$ may be neglected, that these cases alone will be considered in the text, the modifications which must be introduced when $\mathcal{A}$ and $\mathcal{E}$ have sensible values being fully considered in Appendix II.

$\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ being given, $\mathcal{A}$ and $\mathcal{E}$ being zero, to find the deviation on any given magnetic course.—From a point O draw a line upwards $OP=1\cdot000$. OP being any length conveniently divisible into 100 parts, as 5 inches, 100 millimetres, &c.

With P as centre and radius $= \mathcal{D}$ describe a circle cutting OP in D and OP produced in D' .

From D draw a line DG , cutting the circle in G such that PDG is the magnetic azimuth of the ship's head, if the ship's head is in the northern semicircle, or of the ship's stern if the ship's head is in the southern semicircle. DG being therefore the direction of the ship's keel, if the paper is so placed that P is to the magnetic north of O .

It adds greatly to clearness of conception if we draw a small outline or skeleton vessel, having its head in the direction of the ship's head, and so that G represents the position of its compass. Having drawn the outline ship, then from G lay off $GB=\mathcal{B}$ to head if +, to stern if —, and from B lay off BC to starboard if +, to port if —. Join CO †, POC is the deviation.



* This is the same figure as the dygogram represented in Plate II. of the first and second editions of this work, but with the difference that, in the present edition, the circle drawn round P as centre represents the *quadrantal* deviation on which the *semicircular* deviation is then superposed, while in the first and second editions the circle represented the *semicircular* deviation on which the *quadrantal* deviation was then superposed. The method of this edition (which was proposed by Lient. Colongue of the Imperial Russian Navy) has considerable advantages, partly depending on the permanence of the quadrantal coefficients, partly depending on the property of the circle, by reason of which all the lines CG pass through a fixed point Q , and the angle NQC is the magnetic azimuth of the ship's head.

† To avoid confusion the lines CO , $C'O$ are not actually drawn in the figure.

PART III. If we wish to find the Deviation for a second magnetic azimuth differing from the first by a known angle we may lay off $DG'B'$, making GDG' equal to the difference of azimuths and then lay off $G'B'=GB$, and $B'C'=BC$. POC' is the deviation required.

This process is much facilitated by observing that if we join CG , and $C'G'$, these lines, produced if necessary, will meet in a point Q in the circumference of the circle, which point remains the same whatever the azimuth of the ship's head may be and which point may be called the Pole of the Dygogram.* In order, therefore, to find the point C' , having found Q , we have only to take $GQG'=$ the given difference of azimuth and produce QG' to C' , so that $G'C'=GC$.

To find the deviation on magnetic N and S.—It will easily be seen that we may either measure $D'B,=\mathfrak{B}$, upwards from D' if +, downwards if —, and then $BN=C$, to right if +, to left if —, or if Q be given that we may draw $QD'N$, making $D'N = GC = \sqrt{\mathfrak{B}^2 + C^2}$. The deviation at magnetic south or the point S in the dygogram is found by laying off $D'S = D'N$, but in the opposite direction. No confusion can arise between N and S if we observe the signs of the quantities $\mathfrak{B}$ and C . PON , POS , are then the deviations at N . and S . (magnetic).

The line $ND'S$, will of course cut the circle in Q .

To find the deviation on magnetic E and W.—Through Q , draw EW , at right angles to NS , i.e., passing through D , and make $DE=DW=D'N$. E and W being placed so, N.E., S.W., succeed each other as on a compass card. POE , POW , are then the deviations at E . and W .

To find the deviations on all the magnetic courses.—We might find the deviation on every other magnetic course separately by the same method, but the best course is to proceed as follows:—

Find the point Q as above.

Then take a straight edge of paper, of length $=2GC$, and move it so that its centre may always lie on the circle, the edge, produced if necessary, always passing through Q . The two extremities will then trace out the dygogram and by marking a sufficient number of positions with pencil a curve may be drawn through the points, which curve is called the *dygogram*.†

To find the point of the dygogram corresponding to magnetic north or south draw through Q and D' , the line NS , cutting the dygogram in N and S . We can have no difficulty in distinguishing between N and S , if we observe that N will coincide with C , if the magnetic azimuth of the ship's head is zero.

* This permanence of position depends on the property of a circle, Euclid, Book III, prop. 21, by which the angles in the same segment of a circle are equal to each other.

† A contraction of *dynamo-gonio-gram*, i.e., a diagram giving the force and angle.

Draw another line through Q at right angles to QD', and therefore passing through D, and such that $DE=DW=GC$, and place E. and W., so that N. E. S. W., succeed each other as in the compass card. They are the points of the dygogram corresponding to the four cardinal magnetic courses. The deviations on any other magnetic course may be obtained by drawing a line from Q cutting the dygogram at C, so that DQC, is the magnetic azimuth of the ship's head. If we join CO, POC is then the deviation required.

The application of this method to the deviations of H.M.S. *Warrior* will be seen in Plate 2.

The reader should familiarise himself with the form of the dygogram, and practise himself in drawing the line DG, laying down the point Q, and sketching in the outline of the vessel for different courses and measuring off and determining the values of B and C.

The dygogram when drawn gives at once some things not easily obtained otherwise. Thus if OP cuts the dygogram in the two points P', P'', then NQP', NQP'', are the two magnetic courses on which there is no deviation. So if we draw OT, OT' tangents to the dygogram at T and T', NQT, NQT', give the courses of greatest deviation, the greatest deviation being TOP, T'OP.

The dygogram gives, as we have seen, very readily and exactly the deviations for each magnetic course. It does not give in the same way the deviations for the compass courses, but these may be obtained by laying down a Napier's curve on the plain lines and then measuring off the distances along the dotted lines.

SECTION III.

FORMULÆ AND EXAMPLES REQUIRING OBSERVATIONS OF DEVIATION AND HORIZONTAL FORCE.

There are some cases in which observations of deviation alone do not furnish data sufficient for determining the coefficients, but in which the requisite data may be supplied by the addition of observations of the directive force of the needle. The principal case is when we desire to determine the coefficients B and C in a ship still on the stocks or in dock, and therefore when we can only observe on one course.

§ I.

Determination of the Directive Force.

The directive force may be determined in different ways, that which has been found the most convenient in ordinary cases is the following:—

A small flat lenticular needle $2\frac{3}{4}$ inches (70^{mm}) long and one-third of an inch (8·5^{mm}) broad, fitted with a ruby or sapphire cap, having a pivot of

PART III. its own made to screw into the socket of the point of the standard compass or into a socket fixed in a small circular box with a glass top.

The observations are made as follows:—The observer holding a watch in his hand causes the needle to deviate through an arc of 45° , he then marks down the time, counts ten vibrations and marks down the time; and so on marking every tenth vibration till the vibrations become too small to be observed. A well constructed needle on shore in England should vibrate about 120 times. We thus obtain the times of 12 sets of 10 vibrations each, the mean of which is taken as the time of 10 vibrations. It will easily be seen from inspection whether the observer has in any of the sets made a mistake by taking 8 or 12 vibrations instead of 10.

If T be the time of 10 vibrations on shore and T' the time of 10 vibrations on board, and if H represents the directive force of the needle on shore H' the directive force on board then $H' = H \frac{T^2}{T'^2}$.

This observation is very simple and is completed in a few minutes. If the needle is of good quality the quantity T , or the time of 10 vibrations observed on shore, will continue sensibly the same for months, and the shore observations will therefore only require to be made occasionally for verification unless there is a change of geographical position.

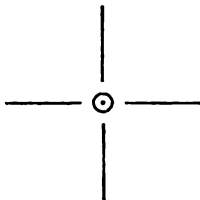
§ II.

Determination of λ .

We must now introduce the reader to the quantity λ (*lambda*) which plays an important part in the theory of ships' magnetism.

It is the proportion which the mean directive force of the needle, or to speak more precisely, the mean value of the component of that force, which is directed to the magnetic north bears to the directive force on shore.

To understand this we may observe that a compass placed in the centre of a uniform circular vessel of homogeneous soft iron would have no deviation but its directive force would be diminished. On the other hand if rods of perfectly soft iron are arranged round a compass as in the figure no deviation is caused,



but the directive force is increased; and if we represent the directive force

in these cases by $H' = \lambda H$, λ will be less than unity in the first case, and **PART III.** greater in the second.

In an iron ship the effect produced is, besides the deviation, a (generally) diminution of the mean directive force, which is represented by the coefficient λ .

The value of λ is given from observations of deviation and horizontal force on any azimuth (the coefficients $\mathfrak{B}$ $\mathfrak{C}$ $\mathfrak{D}$ being known) by the formula,—

$$\begin{aligned}\lambda &= \frac{H'}{H} \times \frac{\cos \delta}{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta} \\ &= \frac{H'}{H} \times \frac{1}{\text{radius of dygogram}}\end{aligned}$$

The value of the factor of $\frac{H'}{H}$ may therefore either be obtained by computation or measurement if the dygogram be constructed.

If observations are made on several magnetic courses we may use the following tabular form :—

Computation of λ and $\frac{1}{\lambda}$

T = time of ten vibrations of horizontal needle on shore.

=

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Magnetic Azimuth of Ship's Head. ζ	Deviation. δ	Time of Ten Vibrations. T'	T'^2	T^2	$\frac{T^2}{T'^2}$	Radius Vector of Dygogram or, $\{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta\} \frac{1}{\cos \delta}$.	Col. VI. Col. VII. = λ
Divide sum of Col. VIII. by number of observations =							

$$\lambda =$$

$$\frac{1}{\lambda} =$$

If the observations of horizontal force have been made at any number not less than four of equidistant magnetic points as $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ we may obtain λ more simply, as it is then the mean of the values of $\frac{H'}{H} \cos \delta$.

PART III. Thus for example,—

Accrington, 3rd Report Liverpool Compass Committee, p. 11.

ζ	δ	$\frac{H'}{H}$	$\cos \delta$	$\frac{H'}{H} \cos \delta$
° /	° /			
11 15	— 15 5	·897	·966	·866
101 15	— 8 55	1·174	·988	1·160
191 15	+ 13 5	1·244	·974	1·211
281 15	+ 10 50	·662	·982	·650
4				3·887
λ				= ·972

The values of $\mathfrak{D}$ and λ are so nearly the same for ships of the same class and remain so unchangeable that they may be estimated with great accuracy.

To assist us in so doing a table is annexed giving the values of $\mathfrak{D}$ and λ for a number of ships of different classes.

TABLE of VALUES of Coefficients $\mathfrak{D}$ λ a e and g for various classes of Iron Ships.

				$\mathfrak{D}$ +	$\mathfrak{D}$ +	λ	a	e	g
				° /					
Iron-built, and armour-plated ships of war. (Iron decks.)	<i>Minotaur</i> 6,621 tons.	- at standard compass	-	5 43	·100	·892	-·019	-·197	-·025
		steering "	-	5 56	·103	·811	-·106	-·272	
		main deck "	-	5 22	·093	·850	-·024	-·216	
	<i>Warrior</i> - 6,109 tons.	- at standard compass	-	5 49	·101	·867	-·045	-·221	+·069
		forward.							
		standard compass aft near iron tower.		8 27	·148	·873	+·002	-·256	
	<i>Black Prince</i> - 6,109 tons.	- at standard compass	-	11 56	·208	·833	+·006	-·340	
		steering "	-	7 38	·134	·783	-·112	-·322	+·118
		main deck "	-	10 32	·184	·760	-·100	-·380	
	<i>Defence</i> - 3,720 tons.	- at standard compass	-	13 16	·231	·757	-·068	-·418	
		steering "	-	7 0	·122	·822	-·078	-·278	+·157
		main deck "	-	10 16	·179	·794	-·064	-·348	
Wood-built, iron armour-plated ships of war. (Iron upper deck.)	<i>Royal Oak</i> - 4,056 tons.	- at standard compass	-	14 35	·254	·759	-·048	-·434	
		steering "	-	3 9	·055	·907	-·043	-·143	+·127
		main deck "	-	1 47	·031	·906	-·066	-·122	
	<i>Lord Warden</i> - 4,045 tons.	- at standard compass	-	1 28	·026	·862	-·116	-·160	
		steering "	-	3 0	·052	·934	-·022	-·110	+·060
			-	4 0	·069	·910	-·027	-·153	
Iron-built, screw ships, for troop and store service, H. M. Navy.	<i>Euphrates</i> - 4,173 tons.	- at standard compass	-	5 39	·098	·851	-·066	-·232	
		steering "	-	6 52	·120	·844	-·055	-·257	
	<i>Jumna</i> - 4,173 tons.	- at standard compass	-	6 08	·107	·865	-·044	-·226	+·070
		steering "	-	6 39	·116	·850	-·051	-·249	
	<i>Serapis</i> - 4,173 tons.	- at standard compass	-	4 55	·086	·905	-·017	-·173	
		steering "	-	4 10	·073	·853	-·085	-·209	
	<i>Orontes</i> - 2,812 tons.	- at standard compass	-	5 30	·096	·875	-·041	-·209	+·056
		steering "	-	7 16	·126	·862	-·029	-·247	
	<i>Industry</i> - 638 tons.	- at standard compass	-	2 58	·052	·937	-·014	-·112	
Iron paddle H.M.S.	<i>Caradoc</i> - 676 tons.	" "		2 3	·036	·945	-·021	-·089	
Iron-built sailing merchant ships.	<i>Clyde</i> - 1,100 tons.	" "		4 43	·082	·870	-·059	-·201	
	<i>City of Sydney</i> 1,181 tons.	" "		4 32	·079	·816	-·120	-·248	
Iron-built, screw and paddle, largest size. (Iron decks.)	<i>Great Eastern</i> 22,000 tons.	compass aft near stern		4 21	·076	·841	-·095	-·223	
		compass on fore bridge		4 31	·079	·892	-·038	-·178	

PART III. In the same table values are also given of two coefficients, a and e connected with $\mathfrak{D}$ and λ by the equations,—

$$1 + a = \lambda(1 + \mathfrak{D})$$

$$1 + e = \lambda(1 - \mathfrak{D})$$

or

$$\lambda = 1 + \frac{a + e}{2}$$

$$\mathfrak{D} = \frac{a - e}{2\lambda}$$

As a arises wholly from fore and aft iron, e wholly from transverse iron, it is sometimes more easy to allow for the effect of peculiarities of construction on the a and e than directly on the λ and $\mathfrak{D}$.*

The formulæ too are generally simpler when expressed in a and e than when expressed in λ and $\mathfrak{D}$; it is therefore desirable to accustom ourselves to both sets of formulæ, and to the estimate of both pairs of coefficients.

§ III.

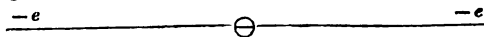
Determination of $\mathfrak{B}$ and $\mathfrak{C}$ from Observations of Deviation and Horizontal Force on one course, $\mathfrak{D}$ and λ being known, $\mathfrak{A}$ and $\mathfrak{E}$ neglected.

1.

By Computation.

When observations of the deviation and horizontal force have been made in a ship on any single course the horizontal force being also observed on shore the values of $\mathfrak{B}$ and $\mathfrak{C}$ are given by the following formulæ in which λ and $\mathfrak{D}$ or a and e are supposed to be known or estimated, $\mathfrak{A}$ and $\mathfrak{E}$ are neglected.

* e is always negative. It may be represented by an imaginary soft iron rod—



placed athwart ship and passing through the compass, such that when the ship's head is E. or W. its effect is to diminish the directive force on the needle by a quantity eH .

a is sometimes positive, in which case it is represented by an imaginary soft iron rod (Fig. 1.), increasing the directive force when the head is N. or S. by aH ; or negative, in which case it is represented by a soft iron rod (Fig. 2.), which would diminish the directive force on N. and S. courses by $-aH$.

Fig. 1.



Fig. 2.



Large masses of iron as boilers, &c. before or abaft the compass tend to give a $+a$, or if immediately below the compass to give a $-a$. It is for this reason that it is more easy to estimate the effect of any particular arrangement of iron in a ship by means of a and e than of λ and $\mathfrak{D}$.

$$\mathfrak{B} = + \frac{1}{\lambda} \cdot \frac{H'}{H} \cos \zeta' - (1 + \mathfrak{D}) \cos \zeta$$

$$\mathfrak{C} = - \frac{1}{\lambda} \cdot \frac{H'}{H} \sin \zeta' + (1 - \mathfrak{D}) \sin \zeta$$

or otherwise

$$\lambda \mathfrak{B} = + \frac{H'}{H} \cos \zeta' - (1 + a) \cos \zeta$$

$$\lambda \mathfrak{C} = - \frac{H'}{H} \sin \zeta' + (1 + e) \sin \zeta$$

Example.

H.M.S. *Orontes*, in dock, Portsmouth, June 8th 1863.

$$\zeta' = S 62^{\circ} 20' E = + 117^{\circ} 40' \quad \cos \zeta' = -.464 \quad \sin \zeta' = +.886$$

$$\zeta = S 68^{\circ} 55' E = + 111^{\circ} 05' \quad \cos \zeta = -.360 \quad \sin \zeta = +.933$$

$$\frac{H'}{H} = \frac{(25.0)^2}{(24.5)^2} = 1.042 \quad \text{assume } \begin{cases} \lambda = .875 \\ \mathfrak{D} = .096 \end{cases}$$

$$\frac{1}{\lambda} \cdot \frac{H'}{H} = \frac{1.042}{.875} = 1.191 \quad 1 + \mathfrak{D} = 1.096 \quad 1 - \mathfrak{D} = +.904$$

$$\mathfrak{B} = -1.191 \times .464 + 1.096 \times .360$$

$$= -.553 + .395$$

$$= -.158$$

$$\mathfrak{C} = -1.191 \times .886 + .904 \times .933$$

$$= -1.055 + .843$$

$$= -.212$$

2.

Graphically.

In order that this important graphic method may be the more easily acquired, it seems desirable to describe it from the beginning, although at the expense of some repetition.

To determine graphically $\mathfrak{B}$ and $\mathfrak{C}$, and the deviations on all magnetic courses from observations of deviation and horizontal force made on one course, $\mathfrak{D}$ and λ being known, and $\mathfrak{A}$ and $\mathfrak{E}$ neglected.

Observations to be made.—Suppose the ship to be on the stocks or in dock.

First, place a compass at the position intended for the standard compass and observe the compass azimuth of the ship's head (ζ'). Next obtain the magnetic azimuth of the ship's head ζ and the deviation ($\delta = \zeta - \zeta'$) either by observing with an azimuth compass on shore in the line of the ship's masts, or by observing on board the bearing of a distant object whose magnetic bearing is known, or by other known methods.

Secondly, with the horizontal needle described above observe the time of ten vibrations on shore (T), and also the time of ten vibrations at the position of the compass on board (T') and compute $\frac{T^2}{T'^2}$ and $\frac{1}{\lambda} \frac{T^2}{T'^2}$.

These are all the observations and computations which are necessary.

PART III. *Construction for finding B and C.*—Take paper, pencil, compass, and scale conveniently divisible into 100 parts, as 5 inches or 100^{mm}. Then from a point O draw a line OP upwards of length equal to 100 divisions of the scale. This length is to be taken as 1, each division being therefore .01. With centre P and radius D describe a circle cutting OP in D and OP produced in D'.

From D draw a line DG parallel to the ship's keel (OP being supposed to lie North and South magnetic), so that PDG is the magnetic azimuth of the ship's head if the ship's head is in the northern semicircle, or the magnetic azimuth of the ship's stern if the ship's head is in the southern semicircle.

Great clearness is given to the diagram by drawing around the point G an outline ship having its head in the direction of the ship's head, G being so placed as to represent the position of the standard compass.

So far the diagram may be drawn before going on board the ship.

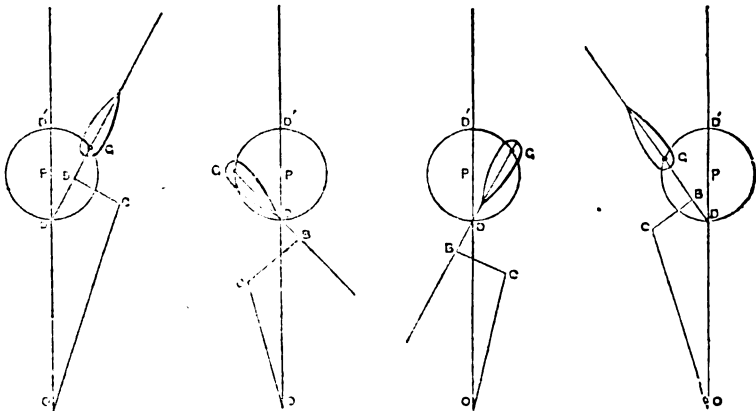
The following construction can only be added after the observations are made.

From O draw OC in the direction of the compass north, or such that POC is the deviation observed, and take $OC = \frac{1}{\lambda} \frac{T^2}{T'^2}$.

From C let fall a perpendicular CB on DG (produced if necessary) and measure GB and BC, then we have

$$B = GB$$

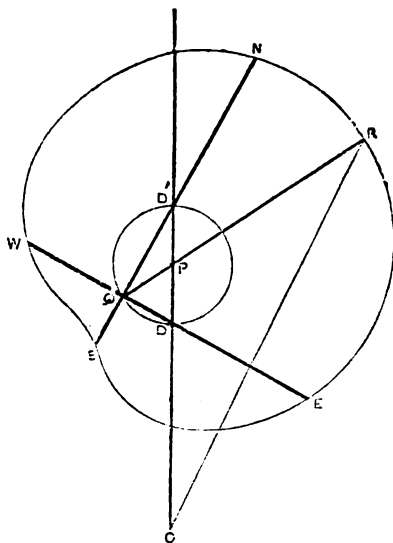
$$C = BC$$



B is + or - according as B is ahead or astern of G, C is + or - according as C is to starboard or to port of G.

The diagrams in this page represent imaginary cases of observations made in four different iron ships on the stocks with their heads in different positions.

PART III.



Proceed as in the last figure, but suppose the lines DG, OC, and CB, and also the line CGQ drawn in pencil and rubbed out as soon as the dygogram is drawn.

Join CG and let it (produced if necessary) cut the circle a second time in Q.

Take a straight edge of paper of length = $2GC$, and move it so that its centre may always lie on the circumference of the circle, the edge always passing through Q . The extremities of the edge will then trace out the dygogram, and by marking a sufficient number of positions with a pencil a curve may be drawn through all the positions, which curve is called the *dygogram*.

Next draw a line through Q and D' cutting the dygogram in N. and S. and another line, which will be at right angles to the first, through Q and D cutting the dygogram in E. and W. No difficulty will be found in placing N, E, S, W if we observe that they ought to come in the same order as on the compass card, and that C will coincide with N. if the magnetic azimuth of the ship's head be zero. Then if we suppose lines ON, OE, OS, OW drawn, the angles PON, POE, POS, POW are the deviations on the four cardinal magnetic courses.

To find the deviation on any other given magnetic azimuth.—Draw QR, cutting the dygogram in R such that NQR is the given azimuth, and join OR, then POR is the deviation required.

PART III.

§ IV.

Determination of B, C, D, and λ from Observations of Deviation and Horizontal Force on two Courses.

In some cases, particularly when ships lie in a river as the Tagus having rapid tides so that observations can only be conveniently made on or near two opposite courses, while on those courses very careful observations both of deviation and horizontal force can be made, it is desirable to have special formulæ for the purpose.

There are two principal cases to be considered,—

1. That in which we know H and T, the force and number of vibrations of the horizontal needle on shore, and,
2. That in which we do not know H and T.

CASE I.

To obtain B, C, D, λ from Observations of Deviation and Horizontal Force on two Courses, the Horizontal Force on shore being known.

The following are the most convenient formulæ :—

$$\begin{aligned}
 1+a &= \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta_1 - \frac{H'_2}{H} \cos \zeta_2 \right\}}{\frac{1}{2} (\cos \zeta_1 - \cos \zeta_2)} \\
 1+e &= \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta_1 - \frac{H'_2}{H} \sin \zeta_2 \right\}}{\frac{1}{2} (\sin \zeta_1 - \sin \zeta_2)} \\
 \lambda &= \frac{1}{2} \{ (1+a) + (1+e) \} \\
 B &= + \frac{1}{\lambda} \left\{ \frac{1}{2} \left(\frac{H'_1}{H} \cos \zeta_1 + \frac{H'_2}{H} \cos \zeta_2 \right) - (1+a) \frac{1}{2} (\cos \zeta_1 + \cos \zeta_2) \right\} \\
 C &= - \frac{1}{\lambda} \left\{ \frac{1}{2} \left(\frac{H'_1}{H} \sin \zeta_1 + \frac{H'_2}{H} \sin \zeta_2 \right) - (1+e) \frac{1}{2} (\sin \zeta_1 + \sin \zeta_2) \right\} \\
 D &= \frac{1}{\lambda} \frac{1}{2} \{ (1+a) - (1+e) \}
 \end{aligned}$$

It will be observed that we cannot obtain $1+a$ if $\cos \zeta_1 - \cos \zeta_2$ is zero or very small, i.e., if the mean point is at or near N. or S., and we cannot obtain $1+e$ if $\sin \zeta_1 - \sin \zeta_2$ is zero or very small, i.e., if the mean point is at or near East or West, and we cannot therefore in either of these cases obtain λ or D .

The formulæ are therefore insufficient if the observations be made on or near two adjoining quadrantal points or two opposite cardinal points, and in that case we must make the most probable estimate we can of the quantity a or e , as the case may be, which we cannot directly determine.

Example.

PART III.

Accrington. Third Report of Liverpool Compass Committee, p. 12.
 $T = 29^{\circ} 65$.

ζ	ζ'	T'	$\frac{H'}{H} = \frac{T^2}{T'^2}$	$\cos \zeta'$	$\sin \zeta'$	$\frac{H'}{H} \cos \zeta'$	$\frac{H'}{H} \sin \zeta'$	$\cos \zeta$	$\sin \zeta$
43 25	56 08	30°	·977	·558	·830	·545	·810	·726	·687
224 00	203 02	29° 44	1·015	−·920	−·391	−·933	−·397	−·719	−·695
						$\frac{1}{2}$ sum	(1) −·194 (2) +·207	(3) +·004 (4) −·004	
						$\frac{1}{2}$ diff.	(5) +·739 (6) +·603	(7) +·722 (8) +·691	

$$1 + a = \frac{(5)}{(7)} = \frac{·739}{·722} = 1·023$$

$$1 + e = \frac{(6)}{(8)} = \frac{·603}{·691} = ·873$$

$$\lambda = \frac{1}{2} \{1·023 + ·873\}$$

$$= ·948$$

$$B = \frac{1}{·948} \{ (1) - (1 + a) (3) \}$$

$$= \frac{1}{·948} \{ -·194 - 1·023 \times ·004 \}$$

$$= -\frac{·198}{·948}$$

$$= -·208$$

$$C = \frac{1}{·948} \{ -(2) + (1 + e) (4) \}$$

$$= \frac{1}{·948} \{ -·207 + ·873 \times (-·004) \}$$

$$= -\frac{·210}{·948}$$

$$= -·222$$

$$D = \frac{\frac{1}{2} \{1·023 - ·873\}}{·948}$$

$$= \frac{·075}{·948}$$

$$= ·079$$

In the example we have taken, the middle point between the two courses is $\frac{1}{2}\{43^{\circ} 25' + 224^{\circ} 00'\} = 133^{\circ} 42'$, or very near a quadrantal point, and

PART III. we can therefore obtain good values of $1+a$ and $1+e$, and therefore of λ or $\mathfrak{D}$.

When the two magnetic courses are diametrically opposite

$$\cos \zeta_1 + \cos \zeta_2 = 0, \sin \zeta_1 + \sin \zeta_2 = 0, \text{ and we have—}$$

$$\lambda \mathfrak{B} = \frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 + \frac{H'_2}{H} \cos \zeta'_2 \right\}$$

$$\lambda \mathfrak{C} = -\frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 + \frac{H'_2}{H} \sin \zeta'_2 \right\}$$

we have also—

$$1+a = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 - \frac{H'_2}{H} \cos \zeta'_2 \right\}}{\cos \zeta_1}$$

$$1+e = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 - \frac{H'_2}{H} \sin \zeta'_2 \right\}}{\sin \zeta_1}$$

In this case therefore we may always determine $\lambda \mathfrak{B}$ and $\lambda \mathfrak{C}$ with great ease, and we may also determine $1+a$ and $1+e$ unless the observations are made at or near two opposite cardinal points. When these points are N. and S. $\sin \zeta_1 = 0$ and the expression for $1+e$ takes the form $\frac{0}{0}$ and gives no result. When the points are E. and W. $\cos \zeta_1 = 0$, and we cannot determine $1+a$. If, however, λ is known or can be estimated then observations of deviation and horizontal force on any two opposite points will give very good values of $\mathfrak{B}$ and $\mathfrak{C}$.

$\cos \zeta_1 - \cos \zeta_2$ is also $= 0$, and the value of $1+a$ cannot be determined whenever the two points are at equal distances from N. and S., so $\sin \zeta_1 - \sin \zeta_2 = 0$, and $1+e$ cannot be determined when the two points are at equal distances from E. and W.

In most cases λ and $\mathfrak{D}$ are known with accuracy, and we have—

To obtain the best values of $\mathfrak{B}$ and $\mathfrak{C}$ from Observations of Deviation and Horizontal Force on two Courses, λ and $\mathfrak{D}$, and also the Horizontal Force on shore being known.

In this case we have more than sufficient data, but with the advantage that we are not limited in the selection of courses, and we may confine ourselves to the expressions—

$$\mathfrak{B} = \frac{1}{\lambda} \frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 + \frac{H'_2}{H} \cos \zeta'_2 \right\} - (1+\mathfrak{D}) \frac{1}{2} (\cos \zeta_1 + \cos \zeta_2)$$

$$\mathfrak{C} = -\frac{1}{\lambda} \frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 + \frac{H'_2}{H} \sin \zeta'_2 \right\} + (1-\mathfrak{D}) \frac{1}{2} (\sin \zeta_1 + \sin \zeta_2)$$

Example.

Let $\lambda = .950$ and $\mathfrak{D} = .075$, and taking the last case we have—

$$\begin{aligned} \mathfrak{B} &= \frac{1}{.950} \{ -.194 \} - 1.075 \times .004 \\ &= -.204 - .004 \\ &= -.208 \end{aligned}$$

$$\begin{aligned} C &= \frac{1}{.950} \{-.207\} - .925 \times .004 \\ &= -.217 - .004 \\ &= -.221 \end{aligned}$$

CASE II.

To obtain B , C , D from Observations of Deviation and Horizontal Force on two Courses, the Horizontal Force on shore not being known.

In this case the value of B , C , D being independent of H or T^2 , we may assume any convenient value for T^2 , as 100 or 1000, and using F and G as auxiliary quantities—

$$F = \frac{\frac{1}{2} \left\{ \frac{100}{T_1^2} \cos \zeta_1 - \frac{100}{T_2^2} \cos \zeta_2 \right\}}{\frac{1}{2} (\cos \zeta_1 - \cos \zeta_2)}$$

$$G = \frac{\frac{1}{2} \left\{ \frac{100}{T_1^2} \sin \zeta_1 - \frac{100}{T_2^2} \sin \zeta_2 \right\}}{\frac{1}{2} (\sin \zeta_1 - \sin \zeta_2)}$$

$$B = \frac{1}{\frac{1}{2}(F+G)} \left\{ \frac{1}{2} \left(\frac{100}{T_1^2} \cos \zeta_1 + \frac{100}{T_2^2} \cos \zeta_2 \right) - F \frac{1}{2} (\cos \zeta_1 + \cos \zeta_2) \right\}$$

$$C = -\frac{1}{\frac{1}{2}(F+G)} \left\{ \frac{1}{2} \left(\frac{100}{T_1^2} \sin \zeta_1 + \frac{100}{T_2^2} \sin \zeta_2 \right) - G \frac{1}{2} (\sin \zeta_1 + \sin \zeta_2) \right\}$$

$$D = \frac{\frac{1}{2}(F-G)}{\frac{1}{2}(F+G)}$$

It will be observed that we cannot make use of these formulæ if the two points are equidistant from a cardinal point, or, in other words, if the middle point between them is on or near a cardinal point; and that they are most correct when the middle point is on or near a quadrantal point.

Example.

Accrington.

ζ	ζ'	T	$\frac{1000}{T^2}$	$\cos \zeta$	$\sin \zeta$	$\frac{1000}{T^2} \cos \zeta$	$\frac{1000}{T^2} \sin \zeta$	$\cos \zeta$	$\sin \zeta$
43 25	56 03	30	1.111	.558	.830	+ .620	+ .922	.726	.687
224 00	203 02	29.44	1.153	-.920	-.391	-1.061	-.451	-.719	-.695
						(1)	(2)	(3)	(4)
						$\frac{1}{2}$ sum	-.220	+ .235	+ .003
						(5)	(6)	(7)	(8)
						$\frac{1}{2}$ diff.	+ .840	+ .686	+ .722

19423.

F

PART III.

$$F = \frac{(5)}{(7)} = \frac{.840}{.722} = 1.163$$

$$G = \frac{(6)}{(8)} = \frac{.686}{.691} = .993$$

$$\frac{1}{2}(F + G) = 1.078 \quad \frac{1}{2}(F - G) = .085$$

$$D = \frac{1}{1.078} \{(1) - F(3)\}$$

$$= \frac{1}{1.078} \{-.220 - 1.163 \times .003\}$$

$$= -\frac{.224}{1.078} = -.208$$

$$C = \frac{1}{1.078} \{-(2) + G(4)\}$$

$$= \frac{1}{1.078} \{-.235 - .993 \times .004\}$$

$$= -\frac{.239}{1.078} = -.222$$

$$D = \frac{.085}{1.078} = .079$$

SECTION IV.

FORMULÆ AND EXAMPLES REQUIRING OBSERVATIONS OF DEVIATION AND HORIZONTAL AND VERTICAL FORCE.

In the cases we have hitherto considered the ship has been supposed to be on an even keel. When an iron ship heels over her deviations are in general greatly altered. Forces which before acted vertically, and did not disturb the horizontal needle, now act to one side and produce deviation. New forces are developed by iron which was previously horizontal becoming inclined, and receiving magnetism by induction from the earth's vertical magnetism.

The new deviations might be determined by observation, by causing the ship to heel over, and swinging her in that position. But as the process of heeling a large ship is always difficult and sometimes impossible, it is of importance to be able to ascertain the change which will be caused in the table of deviations without actually heeling the ship. This is the object of the present section.

Heeling Error.

PART III.

The deviation on an even keel, when the soft iron is symmetrically distributed being—

$$\delta = B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta'$$

it will be shown in Appendix I. that when the vessel heels over i° ($+ i^\circ$ representing a heel to starboard, $- i^\circ$ a heel to port) the deviation will be, very nearly,

$$\delta_i = B \sin \zeta' + \left\{ C - \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta i^\circ \right\} \cos \zeta' + D \sin 2\zeta'$$

The effect of heeling is therefore to add to the deviation the term $-\left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta i^\circ \cos \zeta'$ which is the *heeling error*.

In this expression $\mathfrak{D}$, λ , are quantities already explained; θ is the dip; μ (μu) is the ratio of the mean value of the vertical force on board at the standard compass to the vertical force on shore.

The heeling error is therefore zero when $\zeta' = 90^\circ$ or 270° , *i.e.* when the ship's head is E. or W. by compass, and is a maximum when $\zeta' = 0^\circ$ or 180° , *i.e.* when the ship's head is N. or S. by compass. In the northern hemisphere $\tan \theta$ being +, and $\mathfrak{D} + \frac{\mu}{\lambda} - 1$ being generally +, the heeling error will generally have the opposite sign to i° , *i.e.* it will cause a deviation of the north point of the needle to *windward*; $\left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$ will therefore be the "coefficient of the heeling error to windward," or the amount in degrees of the deviation to windward caused by the ship heeling over one degree, and when her head is N. or S. by the disturbed compass.*

Determination of μ .—As μ changes with a change of geographical position, it is desirable that its value should be determined afresh after any

* The expression $\left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$ though the most convenient for deducing the heeling error from observations is not adapted for the estimation of the probable change of heeling error on a change of geographical position. For this purpose we may use the expression—

$$\frac{R}{\lambda H} + \frac{k - e}{\lambda} \tan \theta$$

in which R is the vertical force downwards from the hard iron, and is generally + in ships built in the northern hemisphere, but may change in the southern hemisphere. $\frac{k - e}{\lambda}$ is also generally +, and does not change.

A ship built in the northern hemisphere will therefore probably carry its heeling error to windward for some distance into southern magnetic latitudes, and until $\tan \theta$ has so large a negative value that—

$$\frac{R}{H} + (k - e) \tan \theta = 0.$$

PART III. considerable change in magnetic latitude. There are several cases for which it is convenient to have formulæ; these are—

CASE I.

Determination of μ from Observation of Vertical Force on One Course.

μ is found as follows: When the soft iron is symmetrically arranged (see App. I.).

$$\mu = \frac{Z'}{Z} - \frac{g}{\tan \theta} \cos \zeta$$

in which Z' is the vertical magnetic force at the position of the compass when the ship's magnetic course is ζ .

Z is the vertical magnetic force on shore, θ is the dip, and g is a small coefficient which does not change. The value of $\frac{Z'}{Z}$ may be determined in various ways. The method at present usually employed in the Royal Navy is to observe the vibration of a dipping needle placed with its face north or face south (magnetic) so that the needle when at rest stands vertically. The needle is then disturbed so as to begin vibrating through an angle of about 20° on each side of the vertical, and the time of 10 vibrations is observed as in the observations of the horizontal needle.

If T be the time of 10 vibrations of the vertical needle on shore, T' the time of 10 vibrations on board, then—

$$\frac{Z'}{Z} = \frac{T^2}{T'^2}$$

If we know g then μ may be found from the above formula. If we do not know g then μ can only be determined from one observation, if that has been made with the ship's head on the magnetic east or west, in which case we have $\mu = \frac{T^2}{T'^2}$

CASE II.

Determination of μ from Observations of Vertical Force on any Two Magnetic Courses.

If we do not know g we may determine μ from observations of vertical force on any two magnetic courses, ζ_1 and ζ_2 , by the formula—

$$\mu = \frac{\frac{1}{2} \left\{ \frac{Z'_2}{Z} \cos \zeta_2 - \frac{Z'_1}{Z} \cos \zeta_1 \right\}}{\frac{1}{2} \{ \cos \zeta_2 - \cos \zeta_1 \}}$$

Example.

PART III.

H.M.S. *Orontes*. Portsmouth, June 1863.

Time of 10 vibrations of vertical needle on shore $T = 20.88$ seconds.

$$T^2 = 436.0$$

Magnetic course N. 3° E. $\zeta_1 = +3^\circ$ $T_1 = 19.29$

" " S. 26° W. $\zeta_2 = +206^\circ$ $T_2 = 19.55$

$$\cos \zeta_1 = .999 \quad T_1^2 = 372.0$$

$$\cos \zeta_2 = -.899 \quad T_2^2 = 382.1$$

$$\mu = \frac{\frac{1}{2} \left\{ -\frac{436.0}{372.0} \times .899 - \frac{436.0}{382.1} \times .999 \right\}}{\frac{1}{2} \{ -.899 - .999 \}}$$

$$= \frac{1.097}{.949}$$

$$= 1.156$$

CASE III.

Determination of μ from Observations of Vertical Force on Three or more Magnetic Courses not equidistant.

If observations of vertical force have been made on a larger number of magnetic courses, $\zeta_1 \zeta_2 \zeta_3$ &c., then the method of least squares gives the following as the most exact expression for μ .

$$\mu = \frac{\left(\frac{Z_1}{Z} + \frac{Z_2}{Z} + \&c. \right) (\cos^2 \zeta_1 + \cos^2 \zeta_2 + \&c.) - \left(\frac{Z_1}{Z} \cos \zeta_1 + \frac{Z_2}{Z} \cos \zeta_2 + \&c. \right) (\cos \zeta_1 + \cos \zeta_2 + \&c.)}{n (\cos^2 \zeta_1 + \cos^2 \zeta_2 + \&c.) - (\cos \zeta_1 + \cos \zeta_2 + \&c.)^2}$$

Example.

H.M.S. *Orontes*. Portsmouth, June 1863.

Time of 10 vibrations of vertical needle on shore $T = 20.88$ seconds.

$$T^2 = 436.0.$$

Correct Magnetic Course.	ζ	Time of 10 Vibrations. T'	T'^2	$\frac{T^2}{T'^2} = \frac{Z'}{Z}$	$\cos \zeta$	$\cos^2 \zeta$	$\frac{Z'}{Z} \cos \zeta$
N. 3° E.	3°	19.29	372.0	1.172	+ .999	+ .998	+ 1.171
S. 26° W.	206°	19.55	382.1	1.141	- .899	+ .808	- 1.025
N. 54° W.	306°	19.09	364.4	1.196	+ .588	+ .346	+ .703
Sum				3.509	+ 1.587	+ 2.152	+ 1.874
					- .899		- 1.025
					+ .688		+ .850

$$\mu = \frac{3.509 \times 2.152 - .688 \times .850}{3 \times 2.152 - .688^2} = \frac{6.967}{5.983} = 1.164$$

PART III.

CASE IV.

Determination of μ from Observations of Vertical Force on Two or more equidistant Magnetic Courses.

If the observations have been made on equidistant courses $\cos \zeta_1 + \cos \zeta_2 + \&c. = 0$, and we have—

$$\mu = \frac{1}{n} \left\{ \frac{Z'_1}{Z} + \frac{Z'_2}{Z} + \frac{Z'_3}{Z} + \&c. \right\}$$

or in other words μ is the mean of the values of $\frac{Z'}{Z}$. This case is so simple that it does not require an example.

It most frequently occurs when the observations have been made on two exactly opposite courses, as when a ship is at anchor in a tideway. In this case by making one observation during flood tide, the other during ebb, bringing the ship's head to exactly opposite courses by the use of the rudder, the value of μ may be determined with great accuracy.

It may be added that if g is *very exactly* known from observations made with great care and under favourable circumstances, it will be better, when we have a number of observations of vertical force made under less favourable circumstances, to make use of this value of g , and to take for μ the mean of the values of $\frac{Z'}{Z} - \frac{g}{\tan \theta} \cos \zeta$, or—

$$\mu = \frac{1}{n} \left\{ \frac{Z'_1}{Z} + \frac{Z'_2}{Z} + \&c. \right\} + \frac{g}{\tan \theta} \frac{1}{n} \left\{ \cos \zeta_1 + \cos \zeta_2 + \&c. \right\}$$

Determination of g .—Although g is not directly required for the determination of the heeling error, yet it is of great use indirectly in enabling us to determine μ either from one observation or from several; and as it does not change with a change of geographical position, it is of importance to determine it with great care in some one geographical position. There are several cases for which it is convenient to have formulæ; these are—

CASE I.

PART III.

Determination of g from Observations of Vertical Force on any Two Magnetic Courses.

$$\frac{g}{\tan \theta} = \frac{\frac{1}{2} \left\{ \frac{Z_1'}{Z} - \frac{Z_2'}{Z} \right\}}{\frac{1}{2} \{ \cos \zeta_1 - \cos \zeta_2 \}}$$

Example.

H.M.S. *Orontes*. Portsmouth, June 1863.

Magnetic course N. 3° E. $\zeta_1 = + 3^\circ$ $\cos \zeta_1 = .999$ $\frac{Z_1'}{Z} = 1.172$

" S. 26 W. $\zeta_2 = + 206^\circ$ $\cos \zeta_2 = -.899$ $\frac{Z_2'}{Z} = 1.141$

$$\frac{g}{\tan \theta} = \frac{\frac{1}{2} \{ 1.172 - 1.141 \}}{\frac{1}{2} \{ .999 + .899 \}}$$

$$g = \frac{.015}{.949} \times 2.5$$

$$= .041$$

These observations should be made as nearly on a north and a south course as possible. The formula fails when the observations are near the east and west courses.

CASE II.

Determination of g from Observations of Vertical Force on Three or more Magnetic Courses.

$$\frac{g}{\tan \theta} = \frac{n \sum \frac{Z'}{Z} \cos \zeta - \sum \frac{Z'}{Z} \sum \cos \zeta}{n \sum \cos^2 \zeta - (\sum \cos \zeta)^2}$$

Example.

H.M.S. *Orontes*. Portsmouth, June 1863.

Taking the values from the example for the determination of μ , we have—

$$\frac{g}{\tan \theta} = \frac{3 \times .850 - 3.509 \times .688}{.5983}$$

$$g = .023 \tan \theta$$

$$= .057$$

PART III.

CASE III.

Determination of g from Observations of Vertical Force made on Three or more equidistant Magnetic Courses.

$$\frac{g}{\tan \theta} = \frac{2}{n} \sum \frac{Z'}{Z} \cos \zeta$$

This does not require an example.

Determination of the Coefficient of Heeling Error from μ , $\mathfrak{D}$, λ and θ .

The coefficient of heeling error to windward = $(\mathfrak{D} + \frac{\mu}{\lambda} - 1) \tan \theta$.

Example.

H.M.S. *Orontes*. Portsmouth, June 1863.

$$D = 5^\circ 30' \quad \lambda = .875 \quad \text{Dip } (\theta) = 68^\circ 12'$$

$$\mathfrak{D} = .096 \quad \frac{1}{\lambda} = 1.143 \quad \tan \theta = 2.500$$

$$\mu \text{ (determined as above)} = 1.164$$

$$\frac{\mu}{\lambda} = \frac{1.164}{.875} = 1.330$$

$$\mathfrak{D} + \frac{\mu}{\lambda} - 1 = .096 + .330$$

$$= .426$$

$$\left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta = .426 \times 2.5$$

$$= 1.065$$

The heeling error is therefore $(1.065) \times 1^\circ = 1^\circ 4'$ deviation to windward for every degree of heel when the ship's head is N. or S. by the disturbed compass.

In other words, we should have the following results if we swung the *Orontes* upright and heeled 10° to starboard and 10° to port.

<i>Coefficients.</i>	Heeling 10° to Port.	Upright.	Heeling 10° to Starboard.
	$^\circ \quad '$	$^\circ \quad '$	$^\circ \quad '$
B	- 8 40	- 8 40	- 8 40
C	- 2 10	- 12 50	- 23 30
D	+ 5 30	+ 5 30	+ 5 30

The mode of correcting the heeling error mechanically by a magnet will be found in Part IV., post, page 95.

Smaller Heeling Errors.—Besides the heeling error we have described, PART III. there will be two others, generally very much smaller, expressed by

$$\frac{c}{\lambda} \sin^2 \zeta' i^{\circ} - \frac{g}{\lambda} \cos^2 \zeta' i^{\circ}$$

$\frac{c}{\lambda} \sin^2 \zeta' i^{\circ}$ gives a small heeling error on E. and W. courses, $-\frac{g}{\lambda} \cos^2 \zeta' i^{\circ}$ causes the principal heeling error to be a little greater on northern than on southern courses, or *vice versâ*, the first if g be +, the last if g be —. These terms may have a sensible effect in compasses placed near the extremities of an iron-built vessel, but in a well-placed compass will have little effect, and may be disregarded by the navigator.

PART IV.

MECHANICAL CORRECTION OF THE DEVIATION.

The question of the mechanical correction of the deviation of the compass has materially changed its aspect of late years. Before that time the deviation of a properly placed standard compass in an iron-built ship was of moderate amount, its maximum seldom exceeding 20° , and the directive force which acted on it being generally comprised within the limits of two-thirds and four-thirds of the mean force. There was then no difficulty and some advantage in dispensing altogether with mechanical correction, or if mechanical correction was employed, it was possible, at least in vessels which did not change their magnetic latitude, to make the correction so complete that tabular correction might be dispensed with. But in the present day it is frequently impossible to find a position for the standard compass at which the deviation and the variations of directive force do not greatly exceed these limits. In such cases the application of magnets for the purpose of equalizing the directive force on different azimuths becomes a matter of necessity, while at the same time the danger of trusting to mechanical correction alone without ascertaining and applying the residual errors is increased.

This change of the condition of the question has produced a corresponding change in the practice in the Royal Navy.

The same care as before is still used in the selection of a place for the standard compass, but a magnet is frequently or generally introduced for the purpose of equalizing the directive force on different azimuths, and at the same time diminishing the semicircular deviation. The quadrantal deviation is not often corrected mechanically, but is generally left for tabular correction.

The heeling deviation is always ascertained, and is sometimes corrected mechanically.

We shall now describe the process of mechanical correction of the several parts of the deviation, viz. :—(1), the semicircular deviation ; (2), the quadrantal deviation ; (3), the heeling deviation.

CORRECTION OF THE SEMICIRCULAR DEVIATION.

PART IV.

First Method (One Magnet).

Find or estimate λ , and find $\mathcal{B}$ and $\mathcal{C}$ by any of the methods described in Part III.*

Find the angle of which the natural tangent is $\lambda\sqrt{\mathcal{B}^2 + \mathcal{C}^2}$, and call this M . Find also the angle of which the natural tangent is $\frac{\mathcal{C}}{\mathcal{B}}$, and call this N .

Take a bar magnet of convenient size and force, and placing it in an east and west direction with its centre due north or south (magnetic) of a compass in a convenient position on shore, free from the influence of iron; move the magnet to or from the compass till it produces a deviation M , and call its distance from the centre of the compass R .

Then take the magnet on board and place it with its axis horizontal and its centre at a distance R immediately below the centre of the compass to be corrected, the axis of the magnet making an angle N with the fore and aft line, and fix it there, taking care to place the north or marked end of the magnet in the direction of a point of which $+\mathcal{B}$ and $+\mathcal{C}$ are the co-ordinates.

In the Royal Navy the pillar which supports the standard compass is generally a cylindrical pillar of mahogany or teak, and in this pillar a magnet from 10 to 12 inches long is easily inserted in any required azimuth and at any required distance below the card by cutting a proper receptacle for it through the pillar. If a single magnet is not of sufficient power, two or more, placed at distances R, R^1 , from the magnet on shore and inserted in the pillar at like distances below the compass on board, may be used.

If the distance R is greater than the height of the card from the deck, as may happen in a steering compass, the magnet may be placed on or under the deck, and moved along the deck in a direction at right angles to the length of the magnet till its distance from the centre of the compass is R , and it may then be fixed.

* As the object aimed at is the reduction of the semicircular deviation to a small and manageable amount, not its entire removal, extreme accuracy in the determination of $\mathcal{B}$ and $\mathcal{C}$ is not required. This preliminary determination is therefore seldom made by the complete process of "swinging." When from the size of the ship or otherwise it is difficult to obtain observations on more than one azimuth, or when time presses, the method of determining $\mathcal{B}$ and $\mathcal{C}$ from observations of deviation and magnetic force on one azimuth is used and is found to be quite sufficient. In other cases, observations of deviation on a few points or in one semicircle while the ship swings at anchor, or shifts her berth in dock, furnish sufficient data for the determination of $\mathcal{B}$ and $\mathcal{C}$.

PART IV. The magnet placed as above directed will correct the whole semicircular deviation, but when a ship is new the semicircular deviation is generally in process of diminution, and it is best to leave a few degrees of it uncorrected.

Second Method (Two Magnets).

Find or estimate λ , and find $\mathfrak{B}$ and $\mathfrak{C}$ as before.

Find the distance S from a compass on shore at which a magnet B , placed as in the last case, will produce a deviation of which the natural tangent is $\lambda \mathfrak{B}$, and the distance T at which a magnet C will produce a deviation of which the natural tangent is $\lambda \mathfrak{C}$.

Take the magnets on board and place the magnet B fore and aft on or below the deck, with its centre to starboard or port of the foot of the compass pillar, and so that its distance from the centre of the compass = S , the marked end of B being forward if B be +, aft if —.

Place the magnet C athwartship on or below the deck, with its centre before or abaft the foot of the pillar, and so that its distance from the centre of the compass is T , the marked end of C being to starboard if C be +, to port if —.

These two magnets will then correct the whole of the semicircular deviation, but, as before, it may be better in a new iron ship to leave a few degrees uncorrected.

Third Method (without Calculation).

Provide two magnets of sufficient power B and C .

Place the ship with its head exactly N. or S. (magnetic).

Place the magnet C athwartship on or below the deck, with its centre before or abaft the foot of the pillar, and move it to or from the pillar keeping its centre in the fore and aft line, which passes through the centre of the pillar till the compass points correctly.*

Then place the ship with its head E. or W. (magnetic).

Place the magnet B fore and aft on or below the deck, with its centre to starboard or port of the foot of the pillar, and move it to or from the pillar keeping its centre in the transverse line which passes through the centre of the pillar till the compass points correctly.*

It will be prudent to turn the ship's head round, i.e., place her head on the other two cardinal points, and if the deviation exceeds one or two degrees to repeat the operation till the quantities left at each cardinal point are within the limits of error which may be neglected.

* It has been found advisable in the process to leave about $1^{\circ} 30'$ of the semicircular deviations uncorrected at the north and south points, and about $30'$ at the east and west; the deviation so left to be westerly, if the ship is swinging from left to right; easterly if from right to left. See p. 120.

It is to be observed that for these corrections it is of importance that the card should, as in the Admiralty compasses, have two needles with their ends distant 30° , or a little less, from the N. and S. diameter of the card, or should have four needles with their ends 15° and 45° , or a little less, from the N. and S. diameter of the card. With single needle cards, unless the correcting magnets are at a considerable distance, a sextantal* error will be produced by their use. PART IV.

These methods are in substance those proposed by Mr. Airy in 1839, and described by him in the Philosophical Transactions for that year, p. 167. The first is that generally used in the Royal Navy, the last is that generally used in the mercantile marine.

CORRECTION OF THE QUADRANTAL DEVIATION.

Two processes have been proposed for the mechanical correction of the quadrantal deviation.

First Process.—Mr. Airy's Soft Iron Correctors.—Mr. Airy, in the Philosophical Transactions for 1839, p. 190, proposed to correct the quadrantal deviation by a mass of soft iron at the same level as the compass, placed on the starboard or port side of the compass if D is positive, on the fore or aft side if D is negative. The mass which Mr. Airy preferred is a box filled with iron chain.

This mode of correction is considered in the 3rd Report of the Liverpool Compass Committee (1861), p. 34, and the preference there given to the use of cylinders of cast iron with hemispherical ends as correctors. Cylinders of two sizes were employed by the Committee, one nine inches long by three inches in diameter, the other twelve inches long by $3\frac{1}{4}$ inches in diameter.

The following are nearly the amounts of the quadrantal deviation corrected by the use of a pair of such cylinders.

Dimension of cylinder.	Distance of Ends of Cylinders from Centre of Compass in Inches.					
	7	8	9	10	11	12
Nine inches long - -	$\begin{smallmatrix} 0 & / \\ 10 & 10 \end{smallmatrix}$	$\begin{smallmatrix} 0 & / \\ 6 & 17 \end{smallmatrix}$	$\begin{smallmatrix} 0 & / \\ 4 & 8 \end{smallmatrix}$	$\begin{smallmatrix} 0 & / \\ 2 & 50 \end{smallmatrix}$	$\begin{smallmatrix} 0 & / \\ 2 & 0 \end{smallmatrix}$	$\begin{smallmatrix} 0 & / \\ 1 & 24 \end{smallmatrix}$
Twelve inches long -	$\begin{smallmatrix} 12 & 30 \end{smallmatrix}$	$\begin{smallmatrix} 8 & 30 \end{smallmatrix}$	$\begin{smallmatrix} 6 & 0 \end{smallmatrix}$	$\begin{smallmatrix} 4 & 0 \end{smallmatrix}$	$\begin{smallmatrix} 3 & 0 \end{smallmatrix}$	$\begin{smallmatrix} 2 & 0 \end{smallmatrix}$

It is shown in the Philosophical Transactions for 1861, p. 161, that compasses which are to be so corrected should have two or four needles, as

* i.e., a deviation having six maxima, three easterly and three westerly, as the ship is turned 360° . See Ph. Tr., 1861, p. 170, and Adm. Man., 2nd Edit., p. 141.

PART IV. in the Admiralty compass, and that with single needle compasses a large octantal* error is caused by the correctors.

Extended observations have shown that when $\mathcal{D}$ has an appreciable value it is always positive, and that therefore the correctors are always to be placed on the starboard or port side of the compass.

First Method.

Find $\mathcal{D}$ by swinging the ship.

Ascertain by trial the distance from a compass on shore at which the two correctors when placed at the level of the compass will produce a quadrantal deviation of the same amount.

This may be most simply done by placing the correctors in a line directed to the centre of the compass, the line making an angle $45^\circ + \frac{1}{2}\mathcal{D}$ with the N. and S. line, and then moving the correctors to and from the compass till they produce the deviation $\mathcal{D}$.†

* i.e., a deviation having eight maxima, four easterly and four westerly, as the ship is turned through 360° .

† As the correctors when on board ship are magnetised not merely by the earth's horizontal force H , but by that force as modified by the ship's force, which may be considered as acting on the correctors with the same force with which it acts on the compass, i.e., by a force $(1+a)X$ to head, and $(1+e)Y$ to starboard, it might seem at first sight that there would be some difference in amount between the quadrantal deviation which the correctors will produce on shore and that which they will correct on board, but this is not so. The deviation produced and corrected are the same, as may be shown as follows:—

Let $a \ e \ \lambda \ \mathcal{D}$ represent, as in App. I., the effect of the soft iron of the ship acting on the compass without the correctors.

Let $a' \ e' \ \lambda' \ \mathcal{D}'$ be the corresponding quantities for the correctors acting without the ship.

Let $a'' \ e'' \ \lambda'' \ \mathcal{D}''$ be the corresponding quantities for the ship and correctors acting together on the compass. Then it will be easily seen that—

$$a'' = a + a' (1 + a)$$

$$e'' = e + e' (1 + e)$$

or

$$1 + a'' = (1 + a) (1 + a')$$

$$1 + e'' = (1 + e) (1 + e')$$

or

$$\lambda'' (1 + \mathcal{D}'') = \lambda \lambda' (1 + \mathcal{D}) (1 + \mathcal{D}')$$

$$\lambda'' (1 - \mathcal{D}'') = \lambda \lambda' (1 - \mathcal{D}) (1 - \mathcal{D}')$$

Hence—

$$\lambda'' = \lambda \lambda' (1 + \mathcal{D} \mathcal{D}')$$

$$\lambda'' \mathcal{D}'' = \lambda \lambda' (\mathcal{D} + \mathcal{D}')$$

and

$$\mathcal{D}'' = \frac{\mathcal{D} + \mathcal{D}'}{1 + \mathcal{D} \mathcal{D}'}$$

Hence $\mathcal{D}'' = 0$ if $\mathcal{D} + \mathcal{D}' = 0$, or in other words a small mass of iron which placed near the compass on shore will produce a quadrantal deviation $-\mathcal{D}$, will in a similar position on board correct a quadrantal deviation $+\mathcal{D}$.

Second Method (without Computation).

PART IV.

Correct the semicircular deviation by magnets. Place the ship's head on one of the quadrantal points, *i. e.* N.E., S.E., S.W., or N. W. (magnetic). Place the correctors at equal distances to starboard and to port of the compass and at the same level with it, and move them to and from the compass till the needle points correctly. For greater certainty examine the deviation on each of the other quadrantal points.

The correction made by the soft iron correctors is not affected by any change of magnetic latitude of the ship, and when once made should remain perfect at all times.

Second Process.—Capt. Evans' Two Compasses.—When the quadrantal deviation is large its mechanical correction by means of soft iron correctors becomes impracticable from the large amount of iron required, the close proximity to the compass at which it must be placed, and the impossibility of freeing it entirely from permanent magnetism. In that case the following process proposed by Staff-Captain Evans in the *Philosophical Transactions*, 1861, p. 167, may be employed.

Take two compasses of precisely equal strength, the needles being arranged as in the Admiralty compass. These are to be placed as in the usual double binnacle, *i. e.* one to starboard and the other to port, and the place should be as free from iron as possible, so that the forces which act on one compass should as nearly as possible be the same as those which act on the other.

It will easily be seen that these needles produce on each other a negative quadrantal deviation, and will therefore correct a positive quadrantal deviation.*

The distance at which they should be placed from each other in order to correct a quadrantal deviation D on the board may be determined as follows:—

(1.) Place the compasses on shore in a line making an angle of $45^\circ + \frac{1}{2} \lambda D$ with the N. and S. line, move them to and from each other till they produce in each other a deviation λD , measure the distance between them, and place them at this distance from each other on board ship.

Or (2.) Place the compasses on shore in an E. and W. line from each other; turn one of the cards till its needle is directed to the centre of the other card, and move it to and from the other till it produces a deviation whose tangent is $\frac{2}{3} \lambda D$; measure the distance between them, and place them at this distance from each other on board ship.

* This effect of two compasses on each other was carefully observed and correctly explained by Peter Lecount, Midshipman, R.N., so long ago as the year 1820. See *Description of Effects of Iron Bodies on Ships' Compasses*. London, 1820.

The possibility of applying it to correct the quadrantal deviation depends on the law not known when Mr. Airy proposed the correction by soft iron, that in all ordinary cases the coefficient D is positive.

PART IV. Or (3.) The semicircular deviation being corrected and the ship's head placed on a quadrantal course, move the compasses to and from each other till the quadrantal error is corrected.

It must be observed that this correction is not perfect in all latitudes as that by soft iron correctors is. The effective power of each needle in correcting a quadrantal deviation is inversely proportional to the horizontal force of the earth at the place. A quadrantal deviation completely corrected in England would therefore re-appear to nearly half its amount at the magnetic equator; the correction would, however, become again perfect as the vessel went farther south. In lower magnetic latitudes, or more accurately when the horizontal force is greater than in the place of correction, the correction therefore, though insufficient, would be beneficial. In higher magnetic latitudes the quadrantal deviation would be over-corrected. These defects admit of being remedied by a provision for adjusting the mutual distances of the compasses.

CORRECTION OF THE HEELING DEVIATION.

The correction of this error has become a matter of importance in many of the iron-built ships of the present day; happily its amount can be ascertained and a complete mechanical correction applied without the necessity of actually heeling over the ship, a matter of no slight difficulty in the large armoured ships of the present day.

The correction is best made by a vertical magnet placed in the centre of the pillar and immediately under the centre of the compass, the distance at which it ought to be placed below the compass being the only thing to be ascertained. This is done as follows :—

First Method.

Find $\mathfrak{D}$ λ and μ by the methods of Part III. Compute $\lambda \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right)$ and find the angle of which it is the natural tangent.

Then by trials made on shore with a magnet placed east or west of a small compass, and directed endwise to it, find the distance at which it produces a deviation equal to the last-mentioned angle, then place the magnet at that distance below the compass. If the heeling error is to windward, as it usually is, the north end of the magnet should be uppermost. If to leeward, the south end.

Example.

In H.M.S. *Orontes*, Portsmouth, June 1863—

$$\lambda = .875. \quad \mathfrak{D} + \frac{\mu}{\lambda} + 1 = .426 \quad \tan \theta = 2.500.$$

$$.875 \times .426 \times 2.500 = .932 = \tan 43^\circ.$$

Find the distance from a small compass on shore at which the magnet held E. or W. of it and directed towards it will produce a deviation of 43° , and place it at this distance below the compass on board,

If the observation for ascertaining the proper distance of the magnet is made on board, we must use the angle of which the natural tangent is—

$$(\mathfrak{D} + \frac{\mu}{\lambda} - 1) \tan \theta$$

In the *Orontes* this is $\cdot 426 \times 2 \cdot 5 = 1 \cdot 065 = \tan 46^\circ 48'$, and the magnet must be placed at the same distance below the compass at which when placed E. or W. of it, it produces a deviation of $46^\circ 48'$.

Second Method.

In this the observation of vertical form on board, and the adjustment of the vertical magnet are made at the same time, the vertical force on shore having been previously observed.

Let T be the time of ten vibrations of the dipping needle vibrating in an E. or W. plane on shore.

The method is simplest when we can place the ship's head E. or W. (magnetic) in that case, compute—

$$T' = \frac{T}{\sqrt{\lambda(1-\mathfrak{D})}}$$

Place the dipping needle in the position of the compass, the centre of the needle being in the position of the centre of the compass needle, and with its face to N. or S., and cause it to vibrate, and find T' , the time of ten vibrations.

$$\text{If } T' = \frac{T}{\sqrt{\lambda(1-\mathfrak{D})}} \text{ no correction is required.}$$

If not—introduce a vertical magnet immediately below the centre of the compass and move it upwards or downwards till T' , the time of ten vibrations, is exactly $\frac{T}{\sqrt{\lambda(1-\mathfrak{D})}}$, and fix the magnet there; this will correct the heeling error.

Example.

H.M.S. *Orontes*, Portsmouth, June 1863. Head, East magnetic

$$\lambda = \cdot 875.$$

$$1 - \mathfrak{D} = \cdot 904.$$

$$T = 20 \cdot 88.$$

$$\begin{aligned} T' &= \frac{20 \cdot 88}{\sqrt{\cdot 875 \times \cdot 904}} \\ &= \frac{20 \cdot 88}{\sqrt{\cdot 791}} \\ &= \frac{20 \cdot 88}{\cdot 889} \\ &= 23 \cdot 5 \end{aligned}$$

The magnet is therefore to be moved upwards or downwards till the dipping needle in the position of the compass vibrates in an E. and W. plane ten times in $23\frac{1}{2}$ seconds.

* Or $T' = \frac{T}{\sqrt{1+e}}$. If we have not been able to determine λ and $\mathfrak{D}$ by observation, this form will generally be found the most convenient for an estimate.

PART IV. If the ship's head be on a magnetic course ζ , then we must know g and then compute—

$$T' = \frac{T}{\sqrt{\lambda(1-D) + g \cot \theta \cos \zeta}}$$

and proceed as before.

Example.

H.M.S. *Orontes*, Portsmouth, June 1863. Head, S. 26 W. magnetic.

$$\lambda = .875.$$

$$1-D = .904.$$

$$T = 20^{\circ}88.$$

$$g = .041.$$

$$\zeta = 206^{\circ}.$$

$$\cos \zeta = -.899.$$

$$\cot \theta = .400.$$

$$\begin{aligned} T' &= \frac{20^{\circ}88}{\sqrt{.791 - .041 \times .400 \times .899}} \\ &= 23^{\circ}7. \end{aligned}$$

Or the magnet must be moved upwards or downwards till the dipping needle in the position of the compass vibrates ten times in 23.7 seconds.

If we have not the means of determining λ and D the following rules will in the ordinary position of the compass in an iron ship very nearly correct the heeling error.

I. If the quadrantal deviation is not corrected, place the ship's head E. and W., and raise or lower the vertical magnet till nine vertical vibrations of the dipping needle occupy the time of ten vibrations on shore.

II. If the quadrantal deviation is corrected by soft iron.

Place the ship's head E. and W. and raise or lower the vertical magnet till $9\frac{3}{4}$ vertical vibrations of the dipping needle occupy the time of ten vibrations on shore.

In each case, and without a dipping needle, we may nearly correct for heeling error if we adjust a delicate pocket compass so that the needle shall stand horizontal on shore, and then placing it at the position of the centre of the compass on board, raise or lower the vertical magnet till the needle of the pocket compass is nearly horizontal, but the south end dipping a little in north magnetic latitude, the north end dipping a little in south. If the quadrantal deviation has been corrected this dip should be very small or imperceptible.

It is to be observed that the correction for the heeling error is only exact for one latitude, and therefore should be tested on any considerable change of magnetic latitude, either by observing the vertical force or by observing the effect of the heeling of the ship on a north or south course.

PART V.

ON TERRESTRIAL MAGNETISM, AS CONNECTED WITH THE DEVIATION OF THE COMPASS.

I.—*Variation.*

1. THE "Variation" of the compass, with which every seaman is familiar, is known by him as the angle by which the compass needle points to the East or West of the true north. In other words it is the angle between the magnetic meridian and the true meridian at any place. The importance of the knowledge of the variation, and the usual methods of ascertaining its amount at any place by observations of amplitudes, and otherwise, as well as the mode of applying the variation of the compass as a correction in steering, or in laying down a course, are also matters familiar to the seamen, and on which it is not necessary to enlarge. The first chart which is given in this part of the manual is, therefore, one, the value of which the seamen will at once recognize, and the use of which, for the ordinary purposes of navigation, will be obvious to him, without explanation.

2. This chart shows for the present time (1860) the amount and direction of the variation at every part of the navigable globe, by the lines of equal variation (or *isogonic* lines), drawn at convenient intervals on a Mercator's chart.

3. It will be seen that the surface of the globe is divided by these lines into two regions, one, the smaller, in which the variation is westerly, as indicated by the red lines, the other, the larger, in which the variation is easterly, as indicated by the blue lines. As a general rule westerly variation prevails in the Atlantic (except part of the West Indies) and the Indian Oceans, easterly variation in the Pacific.

4. In studying the arrangement of the lines of equal variation, it must be remembered that the information they convey is considerably obscured by their having a relation as well to the pole of the earth as to the magnetic poles. It will be seen that they converge to one point in the northern part of North America (Sir James Ross' magnetic pole), and this indicates a true magnetic pole, or a place towards which the needle points in all

PART V. the adjoining regions. There is a similar convergence to a point in the antarctic region to the southward of Australia, which is the south magnetic pole.

Besides these points of convergence, which indicate the existence of real magnetic poles, it may be observed that there would be a similar convergence of the *isogonic* lines to the North and South poles of the earth, if the lines were laid down on a chart which comprised those poles. This convergence, however, would mislead us, if we inferred from it that there are magnetic poles at the poles of the earth. It merely arises from this, that while the needle moves in a small circle round the pole, the direction of the needle remaining unchanged, the direction of the terrestrial meridian, and therefore the variation, goes through every angle, from 0° to 360° .*

5. This chart, besides furnishing the navigator with the amount of the variation at any place, will be useful as indicating to him those parts of the globe in which the variation changes most rapidly, as shown by the lines being crowded together, and lying in a direction at right angles to the usual tracks of navigation. Of these the most remarkable are the coast of Newfoundland, the gulf and river St. Lawrence, and the seaboard of North America, and the English channel and its approaches.†

6. But it is with the variation as connected with the deviation of the compass that we are at present chiefly concerned.

It will be easily seen that the effect of the deviation is to increase or diminish the *apparent* variation on any course. To increase it if they are both easterly or both westerly; to diminish it if one is easterly and the other westerly. In fact, an observation of amplitude on any course gives us the variation and the deviation on that course combined. So that,

* It has been supposed by some persons that there is a second magnetic pole to the north of Siberia. This, however, appears not to be the case, although there is a *tendency* in the magnetic lines to such a pole, shown, among other things, by the occurrence of a second magnetic "focus" in this part of the globe. It is interesting to observe that if there were a second magnetic pole in the northern hemisphere, there must necessarily be a third of a very peculiar character, such that if four compasses were placed in a circle round it, two would point towards it, and two would point away from it.

† The gulf of St. Lawrence is perhaps of all, that in which greatest attention to the compass is required. This arises not only from the crowding together and direction of the lines of equal variation, and the consequent rapid changes in the amount of variation as the vessel pursues her course, but also from the high dip, combined with a low horizontal intensity, increasing greatly the deviation of the compass of vessels coming from lower magnetic latitudes; and as this takes place in a region in which, from the frequent occurrence of fogs, a vessel must depend often entirely on her compasses, the unusual loss of vessels there, is probably, in some degree at least, to be attributed to unknown, or unallowed for, compass errors.

in order to obtain the variation by astronomical observations, we must first know the deviation on the course on which the observations have been made. This we cannot be sure of in a vessel with large deviations, and after a considerable change of magnetic latitude;* and for this reason a chart which gives the variation without the necessity of observation, is especially useful for the navigation of iron-built ships. Conversely, in order to determine the deviation on the course on which the observation of amplitude has been made, it is necessary to know the variation; and this is the principal object with which Chart No. 1 is here given. It will enable the navigator at any time and at any place to check his Table of deviations by astronomical observations.

II.—Dip.

7. The object of the science of Terrestrial Magnetism, so far as it is a science of observation, is to give us the means of determining at any place and time the direction and amount of the earth's Magnetic Force.

The "Variation" gives us the direction of the force *in* the horizontal plane, but the force does not generally act in the plane, but draws the north end of a freely suspended magnetic needle in a direction which lies below or above the horizontal plane by an angle which is known as the "Dip" or "inclination" of the needle.

8. The direction of the force is horizontal, or the dip is zero, along one line on the globe, not differing very much from the terrestrial equator, and dividing the globe into two nearly equal regions; the northern, in which the north end of the needle is drawn *downwards*, and the southern, in which the north end of the needle is drawn *upwards*. This line is called the magnetic equator by analogy to the terrestrial equator.

This dip increases gradually from the magnetic equator to the magnetic poles, where it amounts to 90° . This is shown in chart No. 2, in which lines of equal dip are laid down,—those of northerly dip, or where the north end of the needle is drawn downwards, by red lines, those of southerly, or where it is drawn upwards, by blue lines.

9. The lines of equal dip, or *isoclinal* lines, as they are sometimes called, have, it will be seen, a certain parallelism to and analogy with the parallels of latitude, and the term "Magnetic latitude" is, therefore, sometimes used to denote position on the earth with reference to the magnetic dip.

The rate at which the magnetic dip increases in going from the magnetic equator to the magnetic poles is not the same as that at which the geographical latitude increases. The dip increases more rapidly in low

* For explanation of the term, see *post*.

PART V. latitudes and less rapidly in high latitudes; thus, near the magnetic equator for a degree of latitude the dip increases about 2° , while near the magnetic poles the ratio is reversed, and the dip increases only about $30'$ for every degree of latitude.*

The use of the chart of the lines of equal dip with reference to the deviation of the compass will be most conveniently described when we have described the third chart, or that of the lines of equal horizontal force.

III.—*Horizontal Force.*

10. The force by which the north end of the needle is drawn in the direction of the dip is not the same over all the globe. It is least near the magnetic equator, and greatest near, but not quite at, the magnetic poles.

These points of maximum intensity are sometimes, but improperly, called "Magnetic poles." This term is more properly confined to the place to which in all the adjoining regions the needle points, and at which the free needle would stand vertically, and the points of maximum intensity are more usually and more conveniently designated as "Magnetic foci."

In the northern hemisphere there are two such foci; and it is believed that there are two corresponding in the southern hemisphere. These foci of magnetic force are of unequal strength; in the northern hemisphere, the strongest, or the American focus, lies to the S.W. of Hudson bay near the great system of the North American lakes; the weaker, or the Siberian focus, may be assumed as in 70° N. and 120° E. In the southern hemisphere the position of the stronger focus may be assumed in 70° S. and 145° E.; and the weaker is probably in about 50° S. and 130° E.

At these foci the force is between two and three times the amount of that at the magnetic equator.†

* These relations arise from this, that the magnetism of the earth resembles in some degree that which would be caused by a small magnet placed near the centre of the earth. Such a magnet would give the relation

$$\tan \text{dip} = 2 \tan \text{lat.};$$

and it is convenient to remember this formula, as giving approximately the dip, remembering of course that the latitude in the formula must be measured, not from the terrestrial equator and poles, but from the magnetic equator and poles. See p. 27, note.

† Expressed in the scale of absolute measure, the values of the force at the foci of the northern hemisphere are respectively 14.21 and 13.30: the values at the foci of the southern hemisphere, 15.50 and 15.00: the probable weakest force on the surface of the globe being represented on the same scale by 6.80. It is convenient to remember that if the force were caused by such a magnet at the earth's centre as we have referred to, the force at the pole would be exactly twice that at the equator, the force in any latitude being proportional to $\sqrt{1 + 3 \sin^2 \text{latitude}}$. See p. 27, note.

11. The force which we have been speaking of is the "Total force," or PART V. that in the direction of the line of dip, but for most purposes it is convenient, instead of considering the earth's force as a single force acting on the direction of the dip, to consider it as composed of two forces; one the "Horizontal force," directed towards the magnetic north at the place, and the other the "Vertical force," acting vertically downwards in the northern (magnetic) hemisphere, and upwards in the southern (magnetic) hemisphere.

The quantities with which we are directly concerned with reference to the deviation of the compass are the dip and horizontal force, and it is consequently of these that charts are here given; but from these, if necessary, the "Total force" and the "Vertical force" may be derived by simple formulæ.*

12. The chart of the horizontal force will show, that the horizontal force, from being a maximum near the magnetic equator, diminishes as we go to the magnetic poles. This arises, not from any diminution of the magnetic force of the earth, for that increases as we go to the poles, but from its direction becoming more and more inclined to the horizon.

13. The knowledge of the dip and horizontal intensity enables us to anticipate the changes which take place in a ship's magnetism and deviation in a manner which has been explained in Part III. It will be sufficient here to observe that the effect of any disturbing cause of fixed amount in producing deviation is (nearly) *inversely* as the directive force of the needle. The permanent magnetism of the hard iron of the ship is a cause of this nature, and the deviation thereby produced is consequently

* The formulæ are—

Let θ be the dip.

H „ horizontal force.

ϕ „ total force.

Z „ vertical force.

then, $\phi = H \sec \theta$

$Z = H \tan \theta$

If we have given, as in some magnetic charts, the total force and the dip then—

$H = \phi \cos \theta$

$Z = \phi \sin \theta$

If we have given the horizontal and vertical forces, then—

$\tan \theta = \frac{Z}{H}$

$\phi = \sqrt{H^2 + Z^2}$

$= H \sec \theta$

$= Z \operatorname{cosec} \theta$

PART V. (nearly) in the *inverse* ratio of the horizontal force. Knowing, therefore, the deviation arising from this cause (which is one of the parts of the semicircular deviation) in a ship in any one part of the globe, we can by means of chart No. 3 estimate its probable amount in any other part of the globe.

It will be observed that in the margin of the chart No. 3, opposite the end of each line of equal horizontal force,* is engraved the value of the reciprocal of the force along that line. These numbers will enable us to ascertain, by a mental interpolation at the geographical position of the ship, the value at that place of the reciprocal of the horizontal force. The multiplier for converting the permanent semicircular deviation at one place to that at another, is therefore the product of the value of the horizontal force at the first place, and of the reciprocal of the horizontal force at the second place. This proportion would be exact if the magnetism of the iron were absolutely permanent. This, however, as explained in Part III., is not strictly so. It is more properly sub-permanent, and such estimates are therefore more or less subject to uncertainty so long as the quality of the magnetism of the particular ship has not been ascertained.

But if the magnitude of the disturbing cause be not itself the same in different parts of the globe, but depends on the earth's magnetism, then, in order to estimate the change that will take place in the deviation, on a change of geographical position, we must take into account the change in the disturbing force as well as the change in the directive force, and the deviation will change as the ratio of these forces to each other changes.

Thus, considering separately the inducing effects of the vertical and horizontal force of the earth, we see that the magnetism induced in the soft iron by the vertical force of the earth is proportional to that force, and does not alter as the ship moves round in azimuth. It therefore produces a deviation, following the same law in any given geographical position, as that produced by the permanent magnetism; and is therefore a part of the semicircular deviation; but when the ship changes its geographical position this part of the semicircular deviation, instead of varying, as the other does, *inversely*, as the horizontal force varies as the

* For convenience in this chart, the value of the horizontal force at Greenwich (and thus also in nearly all the principal naval ports of Great Britain), has been assumed as unity: to convert the values of the horizontal force on this scale, to the scale of absolute measure adopted by English magneticians, they must be multiplied by 3.82; and to convert them to the scale of absolute measure adopted by the continental magneticians they must be multiplied by 1.76.

ratio of the vertical force to the horizontal force, or, in other words, *as the tangent of the dip.** PART V.

The value of this tangent may be ascertained by a table of natural tangents from the value of the dip at the geographical position, as obtained from chart No. 2. It may also be obtained directly by a mental interpolation between the values of the tangent of the dip which are given in the margin of the chart.

The remaining case which we have to consider is the deviation caused by the magnetism induced in the soft iron of the ship by the horizontal force of the earth. Here, the disturbing force being always in the same ratio to the horizontal force, the deviation produced is the same in all parts of the globe. This applies to the "quadrantal deviation," and also to whatever part of the "constant" deviation is a *true* deviation, and does not arise from error of instruments or of observation.

* We may illustrate these statements by a few familiar examples :—

A ship sailing along one of the lines of equal dip is said to remain in the same magnetic latitude; she consequently changes her magnetic latitude most rapidly when her course is at right angles to those lines. Thus a ship from the English channel to the cape of Good Hope would change her magnetic latitude 121° , or from 68° north dip to 53° south dip; but from Auckland, New Zealand, to cape Horn, the magnetic latitude would not be changed more than 2° . Hence in the first case in a wood-built ship, the semicircular deviation may be expected to change in the proportion of the *nat: tangent* of the respective dips, or of $+2.5$ to -1.3 (i.e. a maximum semicircular division of 10° E. would change to $5^\circ 20'$ W.): and in the second case the deviations would remain of the same value from New Zealand to cape Horn.

Again, the same vessel sailing from the English channel to the gulf of St. Lawrence, the dip increasing from 68° to 78° N. or the tangents of the dip increasing in the proportion of $+2.5$ to $+4.7$, the maximum semicircular deviation would increase from 10° to $18^\circ 50'$ or be nearly doubled in amount.

If we take the case of an iron-built ship, the magnetism of which is permanent, we find that in going from the English channel to the cape of Good Hope the horizontal force increases in the proportion of 1.0 at starting, to nearly 2.0 (1.85) on the geographical equator, and then decreases gradually to 1.3 at the cape of Good Hope. The semicircular deviation varying *inversely* as the horizontal force, it follows that a maximum semicircular deviation of 10° in the English channel would be reduced to $5\frac{1}{2}^\circ$ on the Equator and again increase to $7\frac{1}{2}^\circ$ at the cape of Good Hope.

Similarly in going from the English channel to the gulf of St. Lawrence, the horizontal force decreases from 1.0 to 0.65, and the maximum semicircular deviation of 10° would subsequently increase to $16^\circ 45'$.

If we take the case of an iron-built ship (or wood-built steam-ship) the magnetism of which is sub-permanent, so that one part of the semicircular deviation varies *inversely* as the horizontal force, and another part of the semicircular deviation varies as the tangent of the dip; the resulting deviations would probably lie between those of the two examples just given, but the amount of change could only be ascertained by actual observation.

PART V.

IV.—*General Observations.*

14. The representation of the variation, dip, and horizontal intensity, by lines traced on the surface of the globe, implies that these components do not change with the lapse of time. This is not strictly true. Each component is subject to changes which are distinguished as “secular,” “periodical,” and “irregular.”

The secular changes are those which take place in the same direction for a length of time, and which, however slow the rate of change may be, attain a large value in the course of years.

The lines laid down in these charts have been taken generally from observations made between the years 1840 and 1861. The small scale on which they are shown, and the purposes for which the charts are required, hardly admit or require the additional expenditure of labour which would have been requisite to reduce them to a single epoch, although this has been endeavoured, where possible, to be effected. They may be taken, therefore, to represent generally the position of the lines in the year 1860.

As these charts will probably be re-published from time to time, it is deemed also unnecessary to supply the means of adapting them to a different epoch, but it may be useful, as well as interesting, to indicate very generally the changes which are taking place in the arrangement of these lines.

The general change is, that in the northern hemisphere the systems of lines are all sweeping slowly, with more or less change in configuration, from East to West; while in the southern hemisphere, the change appears to be taking place in the opposite direction. The result is that in the British islands the westerly variation is at present diminishing about 7' annually; and on an extensive portion of the seaboard of North America it is increasing at the same rate.

The periodical changes are the “diurnal” and “annual,” depending on the direct and indirect effects of the Sun, and the “lunar,” depending on the Moon. These are, however, so small that they do not require the attention of the navigator.

Besides the secular and periodical changes each component is continually undergoing small irregular fluctuations which occur simultaneously over a large portion of the globe. Occasionally during what are called magnetic storms, which in high latitudes are generally accompanied by brilliant displays of the aurora borealis or aurora australis, these fluctuations attain a considerable amount, but it is only very near the magnetic poles and during magnetic storms that these irregular changes exceed the limit which may be safely disregarded by the navigator. The short duration, too, of the irregular changes removes them still more from the

cognizance of the navigator, who is only concerned with the course which PART V. he has to steer.

It has been sometimes supposed that fogs and peculiar states of the atmosphere affect the compass. There appears, however, to be no ground for this supposition. No ordinary meteorological phenomenon appears to have any appreciable effect on the compass; from this of course must be excepted the occasional effect of lightning in reversing the poles of the needle, or, indeed, rendering it useless.

It is also sometimes supposed that when a ship is near land the nature of the land affects the direction of the compass. The effect of particular rocks on the direction of the needle is very perceptible when a compass is taken on shore, and is in close proximity to volcanic and other igneous rocks; but there seems no ground for believing that this influence is perceptible on board ship.

From these impressions as to the effects of causes which *might* operate, though their effect is in general inappreciable, we must distinguish those misconceptions or delusions which often prevail even among persons who might be expected to know better. Thus though nothing is more certain than that no substance whatever, magnetic or non-magnetic, conducting or non-conducting, electrified or non-electrified, has the slightest influence in intercepting the action of a magnetic body on the needle, yet many schemes have been proposed for correcting the compass by interposition of substances which it is supposed will intercept the action of the iron of a ship on the needle. So, too, it is sometimes supposed that the effect of an iron cargo may be diminished by the interposition of the cargo placed above it, and that the effect of iron stanchions is diminished by covering them with brass or even painting them. All such ideas are entirely groundless.

15. The authorities which have been chiefly consulted in the construction of the charts are the following:—

M. Hansteen, *Magnetismus der Erde*: Christiana, 1819; and Den Magnetiske Inclinations Forandringer, 1855, 1857.

Reports of the British Association for the Advancement of Science, 1835–1837; On the Phenomena of Terrestrial Magnetism, and on the Variations of the Magnetic Intensity, by Major, now General Sabine.

Atlas der Erdmagnetismus, by Gauss and Weber, Leipzig, 1840.

Philosophical Transactions of the Royal Society; Contributions to Terrestrial Magnetism, Nos. I. to XI., 1840–68, by General Sabine, which includes the results of the Antarctic voyage made by Captain Sir James C. Ross in H.M. ships *Erebus* and *Terror*, 1839–43.

Elliott's Magnetic Survey of the Indian Archipelago, *Phil. Trans.*, 1851.

PART V. Reports of the United States Coast Surveys, 1854-7. (Magnetic results.)

— Magnetic observations made by Dr. J. Lamont in North Germany, Belgium, Holland, Denmark, France, Spain, and Portugal, published at Munich, 1858-9.

Magnetic results in India and central Asia, by Messrs Schlagintweit, 1854-7, and also by Messrs. Schaub and Kreil in the Austrian Dominions and Eastern shores of the Mediterranean Sea, 1846-58.

Keith Johnson's Physical Atlas, 2nd edition, 1856; Terrestrial Magnetism, by General Sabine.

Published Observations of the Colonial Magnetic Observatories and in British India, also of those in the Russian Dominions, under the direction of M. Kupffer.

APPENDIX No. 1.

MATHEMATICAL THEORY OF THE DEVIATIONS OF THE COMPASS.

I.

Fundamental Equations.

The whole of the mathematical theory of the deviations of the compass is comprised in the following three equations :—

$$X' = X + aX + bY + cZ + P \dots\dots\dots(1)$$

$$Y' = Y + dX + eY + fZ + Q \dots\dots\dots(2)$$

$$Z' = Z + gX + hY + kZ + R \dots\dots\dots(3)$$

In these equations X, Y, Z , represent the components of the earth's magnetic force drawing the north point of the compass needle to the ship's head, to the starboard side, and to the keel. When the ship is on an even keel, as we shall first suppose, the two last directions are respectively horizontal and vertical.

X', Y', Z' , represent the components of the combined magnetic force of the earth and ship in the same directions.

$a, b, c, d, e, f, g, h, k$, are constant parameters * depending on the amount, arrangement, and capacity for induction of the soft iron of the ship. P, Q, R , are constant parameters depending on the amount, arrangement, and permanent or sub-permanent magnetism of the hard iron of the ship.

These equations were first given by M. Poisson, in the year 1824, in the 5th volume of the Memoirs of the Institute, p. 533.

They result from the hypothesis that the magnetism of the ship is partly the permanent magnetism of hard iron, and partly the transient induced magnetism of soft iron; that the latter is proportional to the intensity of the inducing force, and that the length of the needle is infinitesimally small compared to the distance of the nearest iron, and on this hypothesis they are exact.†

* The term "parameters" is one commonly used by mathematicians to designate the independent constants involved in the solution of a mechanical problem. The term is here introduced because it is convenient to distinguish between the *parameters* $a, b, c, d, e, f, g, h, k, P, Q, R$, which are independent constants having a distinct physical meaning, and the *coefficients* A, B, C, D, E , &c. which are combinations of the parameters and other known quantities, and are introduced merely for convenience of computation.

† These equations may be obtained from the following considerations—

The magnetism induced in the soft iron by a uniform force X produces a force on the north point of the compass, proportional to $X = (\text{say}) lX$ (l being a constant, depending only on the soft iron in the ship). This force does not in general act towards the head, but

APP. 1.

II.

Transformation of the Fundamental Equations.

In order to express these equations in terms of the quantities which are usually given and required, let—

H be the horizontal force of the earth.

H' the horizontal force of the earth and ship.

θ the dip.

ζ the “magnetic course” or the azimuth of the ship’s head measured eastward from the magnetic north.

ζ' the “compass course” or the azimuth of the ship’s head measured eastward from the direction of the disturbed needle.

$\delta = \zeta - \zeta'$ the deviation of the compass.

Then—

$$\begin{aligned} X &= H \cos \zeta & Y &= -H \sin \zeta & Z &= H \tan \theta \\ X' &= H' \cos \zeta' & Y' &= -H' \sin \zeta' \end{aligned}$$

Substituting these values in equations (1), (2), and (3), and dividing (1) and (2) by H , and (3) by Z ,—

$$\frac{H'}{H} \cos \zeta' = (1+a) \cos \zeta - b \sin \zeta + c \tan \theta + \frac{P}{H} \dots (4)$$

$$-\frac{H'}{H} \sin \zeta' = d \cos \zeta - (1+e) \sin \zeta + f \tan \theta + \frac{Q}{H} \dots (5)$$

$$\frac{Z'}{Z} = \frac{g}{\tan \theta} \cos \zeta - \frac{h}{\tan \theta} \sin \zeta + 1 + k + \frac{R}{Z} \dots (6)$$

Equation (4) gives the force of earth and ship to head, equation (5) the force of earth and ship to starboard, each in terms of the earth’s horizontal force as unit, equation (6) the force of earth and ship downwards in terms of the earth’s vertical force as unit.

in some other direction in the ship, of which the direction cosines are (say) $\frac{a}{l}, \frac{d}{l}, \frac{g}{l}$, (a, d, g , being constants, depending only on the soft iron in the ship), and the resolved parts of which force are, therefore, $a X$ to head, $d X$ to starboard, and $g X$ downwards.

Similarly Y produces a force $b Y$ to head, $e Y$ to the starboard, and $h Y$ downwards, and Z produces a force $c Z$ to head, $f Z$ to starboard, and $k Z$ downwards. The hypothesis that the magnetism induced is proportional to the intensity of the inducing force, allows us to combine by simple addition the forces in the three directions which each component of the Earth’s force would produce separately. Adding them, therefore, together and adding to them the force from the permanent magnetism, we get—

$$\text{Disturbing force to head} = X' - X = a X + b Y + c Z + P$$

$$,, \quad ,, \quad \text{starboard} = Y' - Y = d X + e Y + f Z + Q$$

$$,, \quad ,, \quad \text{downwards} = Z' - Z = g X + h Y + k Z + R$$

which are Poisson’s equations.

From these equations we obtain the following :—

Multiplying equation (4) by $\sin \zeta$ and equation (5) by $\cos \zeta$, and adding, we get after some reductions—

$$\frac{H'}{H} \sin \delta = \frac{d-b}{2} + \left(c \tan \theta + \frac{P}{H}\right) \sin \zeta + \left(f \tan \theta + \frac{Q}{H}\right) \cos \zeta \\ + \frac{a-e}{2} \sin 2\zeta + \frac{d+b}{2} \cos 2\zeta \dots (7)$$

Multiplying equation (4) by $\cos \zeta$ and equation (5) by $-\sin \zeta$, and adding, we get after some reductions—

$$\frac{H'}{H} \cos \delta = 1 + \frac{a+e}{2} + \left(c \tan \theta + \frac{P}{H}\right) \cos \zeta - \left(f \tan \theta + \frac{Q}{H}\right) \sin \zeta \\ + \frac{a-e}{2} \cos 2\zeta - \frac{d+b}{2} \sin 2\zeta \dots (8)$$

Equation (7) gives the force of ship to magnetic east in terms of the earth's horizontal force as unit. The mean value of this force is $\frac{d-b}{2}$. If this quantity is + the easterly will predominate over the westerly deviations, if — the westerly will predominate.

Equation (8) gives the force of earth and ship to magnetic north in terms of the same unit. The mean value of this force is $1 + \frac{a+e}{2}$, which in iron built vessels is generally less than 1, and in such ships therefore the mean directive force on the needle is generally diminished.

If we make $1 + \frac{a+e}{2} = \lambda$, and divide (7) and (8) by λ , we have—

$$\frac{H'}{\lambda H} \sin \delta = \frac{1}{\lambda} \frac{d-b}{2} + \frac{1}{\lambda} \left(c \tan \theta + \frac{P}{H}\right) \sin \zeta + \frac{1}{\lambda} \left(f \tan \theta + \frac{Q}{H}\right) \cos \zeta \\ + \frac{1}{\lambda} \frac{a-e}{2} \sin 2\zeta + \frac{1}{\lambda} \frac{d+b}{2} \cos 2\zeta \dots (7a)$$

$$\frac{H'}{\lambda H} \cos \delta = 1 + \frac{1}{\lambda} \left(c \tan \theta + \frac{P}{H}\right) \cos \zeta - \frac{1}{\lambda} \left(f \tan \theta + \frac{Q}{H}\right) \sin \zeta \\ + \frac{1}{\lambda} \frac{a-e}{2} \cos 2\zeta - \frac{1}{\lambda} \frac{d+b}{2} \sin 2\zeta \dots (8a)$$

λH , which is the mean value of $H' \cos \delta$, or of the component of the force of earth and ship directed to the magnetic north, is a quantity we shall have frequently to mention as the “mean force to north.”

In order to simplify these expressions let—

$$\frac{1}{\lambda} \frac{d-b}{2} = \mathfrak{A}; \quad \frac{1}{\lambda} \frac{a-e}{2} = \mathfrak{D}; \quad \frac{1}{\lambda} \frac{d+b}{2} = \mathfrak{E} \\ \frac{1}{\lambda} \left(c \tan \theta + \frac{P}{H}\right) = \mathfrak{B}; \quad \frac{1}{\lambda} \left(f \tan \theta + \frac{Q}{H}\right) = \mathfrak{C}$$

Then equations (7a) and (8a) become—

$$\frac{H'}{\lambda H} \sin \delta = \mathfrak{A} + \mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathfrak{E} \cos 2\zeta \dots (9)$$

$$\frac{H'}{\lambda H} \cos \delta = 1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta \dots (10)$$

APP. 1. Equation (9) gives the force of ship to magnetic east in terms of the "mean force to north" as unit.

Its mean value is $\mathcal{A}$, the magnitude and sign of which will indicate the amount of the predominance of easterly or westerly deviations.

Equation (10) gives the force of earth and ship to magnetic north in terms of the same unit.

Dividing (9) by (10) we get—

$$\tan \delta = \frac{\mathcal{A} + \mathcal{B} \sin \zeta + \mathcal{C} \cos \zeta + \mathcal{D} \sin 2\zeta + \mathcal{E} \cos 2\zeta}{1 + \mathcal{B} \cos \zeta - \mathcal{C} \sin \zeta + \mathcal{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta} \dots (11)$$

Which gives the deviation on any magnetic course when the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, are known.

If in equation (11) we substitute $\frac{\sin \delta}{\cos \delta}$ for $\tan \delta$ and multiply by the denominators we shall find—

$$\sin \delta = \mathcal{A} \cos \delta + \mathcal{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathcal{D} \sin (\zeta + \zeta') + \mathcal{E} \cos (\zeta + \zeta') \dots (12)$$

Or—

$$\sin \delta = \mathcal{A} \cos \delta + \mathcal{B} \sin \zeta' + \mathcal{C} \cos \zeta' + \mathcal{D} \sin (2\zeta' + \delta) + \mathcal{E} \cos (2\zeta' + \delta) \dots (12a)$$

These give the deviations expressed nearly though not wholly in terms of the compass courses.

When the deviations are of moderate amount, say not exceeding 20° , equation (12) may be put under the comparatively simple and convenient form—

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta' \dots (13)$$

in which the deviation is expressed wholly in terms of the compass courses, and this will be sufficiently exact for practical purposes.

It will be seen the $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, are nearly the natural sines of the angles A , B , C , D , E . More exact expressions will be given in the sequel.

III.

Expressions occasionally used.

The following expressions are also sometimes useful :—

$$\frac{X' - X}{H} = a \cos \zeta - b \sin \zeta + \lambda \mathcal{B} \dots (4a)$$

$$\frac{Y' - Y}{H} = d \cos \zeta - e \sin \zeta + \lambda \mathcal{C} \dots (5a)$$

$$\frac{Z' - Z}{H} = g \cos \zeta - h \sin \zeta + \lambda \mathcal{V} \dots (6a)$$

in which $\lambda \mathcal{V} = k \tan \theta + \frac{R}{H}$.

Equation (4a) gives the force of the ship to head, equation (5a) to starboard, and equation (6a) downwards, in terms of the earth's horizontal force as unit.

The same equations divided by λ may be put under the form—

APP. 1.

$$\frac{H' \cos \zeta'}{\lambda H} = (1 + \mathfrak{D}) \cos \zeta + (\mathfrak{A} - \mathfrak{E}) \sin \zeta + \mathfrak{B} \dots (4b)$$

$$-\frac{H' \sin \zeta'}{\lambda H} = (\mathfrak{A} + \mathfrak{E}) \cos \zeta - (1 - \mathfrak{D}) \sin \zeta + \mathfrak{C} \dots (5b)$$

$$\frac{Z'}{Z} = \frac{g}{\tan \theta} \cos \zeta - \frac{h}{\tan \theta} \sin \zeta + \mu \dots (6b)$$

in which $\mu = 1 + k + \frac{R}{Z}$.

Equation (4b) gives the force of earth and ship to head in terms of the "mean force to north" as unit; equation (5b) the force of earth and ship to starboard in the same unit; and equation (6b) the force of earth and ship downwards in terms of the earth's vertical force as unit.

It will be seen that μ is the mean value of $\frac{Z'}{Z}$ or the mean force downward of earth and ship in terms of earth's vertical force as unit.

$\mu - 1 = k + \frac{R}{Z}$ is therefore the mean force downward of ship in terms of the same unit, and

$(\mu - 1) \tan \theta = k \tan \theta + \frac{R}{H} = \lambda V$ is the mean force downward of the ship in terms of the earth's horizontal force as unit.

When, as is generally the case, $\mathfrak{A}$ and $\mathfrak{E}$ are very small, the following expression for $\sin \delta$ derived from (12) is very nearly exact:—

$$\sin \delta = \frac{1}{1 - \mathfrak{D} \cos 2\zeta'} \left\{ \mathfrak{A} + \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin 2\zeta' + \mathfrak{E} \cos 2\zeta' \right\} \dots (13a)$$

When the soft iron is symmetrically arranged on each side of the fore and aft vertical section—

$$b=0 \quad d=0 \quad f=0 \quad h=0$$

Whence—

$$\mathfrak{A}=0 \quad \mathfrak{E}=0$$

$$\tan \delta = \frac{\mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta}{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta}$$

$$\sin \delta = \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin (\zeta' + \zeta')$$

$$= \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin (2\zeta' + \delta)$$

$$\delta = \mathfrak{B} \sin \zeta' + \mathfrak{C} \cos \zeta' + \mathfrak{D} \sin 2\zeta' \text{ approximately.}$$

IV.

On the several Parts of which the Deviation is composed.

When the deviation is small the several parts of which it is composed are—

1. A , the constant deviation.
2. $B \sin \zeta' + C \cos \zeta'$, the semicircular deviation.
3. $D \sin 2\zeta' + E \cos 2\zeta'$, the quadrantal deviation.

APP. 1.

If $\frac{C}{B} = \tan \alpha$, then—

$$B \sin \zeta' + C \cos \zeta' = \sqrt{B^2 + C^2} \sin (\zeta' + \alpha).$$

$\sqrt{B^2 + C^2}$ is the maximum of semicircular deviation, α is the starboard angle ; this form of expression is often useful.

So if $\frac{E}{D} = \tan \beta$ —

$$D \sin 2\zeta' + E \cos 2\zeta' = \sqrt{D^2 + E^2} \sin (2\zeta' + \beta).$$

As E , and consequently β , are generally so small that they may be neglected, this expression is not often used, and $D \sin 2\zeta'$ is considered as the quadrantal deviation, and D as the maximum of quadrantal deviation.

When the deviations are large we can no longer consider the total deviation as made up of the sum of the constant, the semicircular, and the quadrantal deviations ; but nevertheless it is still convenient to define and distinguish these parts.

The maximum semicircular deviation may in this case be considered to be the angle whose sine is $\sqrt{B^2 + C^2}$, and the starboard angle as the angle whose tangent is $\frac{C}{B}$.

So the maximum of quadrantal deviation will be the angle of which the sine is D or $\sqrt{D^2 + E^2}$, according as E is or is not zero.

V.

On the Parameters $a, b, c, d, e, f, g, h, k, P, Q, R$.

There is a physical representation of Poisson's fundamental equations so simple, and which gives us so great a power of estimating the effect on the compass of different arrangements of iron in a ship, as well as of tracing to their causes any peculiarities in the observed deviation, that it is of great importance to make ourselves masters of it.

If an infinitely thin straight rod of soft iron be magnetized by the induction of the earth, the effect will be the same as if each end became a pole having an intensity proportional to the component of the earth's force resolved in the direction of the rod, and to the section and capacity for induction of the rod.

Let us now suppose nine soft iron rods placed as in Plate I. It will be seen that for each rod we must distinguish the two cases, that in which its coefficient is +, and that in which it is -. It will also be seen that in the three cases, viz., $-a, -e, -k$, in which the rod passes through the compass we may consider both ends as acting, but that in other cases it is

convenient to consider only the action of the near end, and that the far end is at an infinite distance. APP. 1.

The rod a , it will be observed, can only be magnetized by the component X , b only by Y , and c only by Z ; and if we call aX , bY , and cZ the force with which these rods attract the north end of the needle, and if we suppose, as we are at liberty to do, the rods being imaginary, that they exercise no action on one another, a , b , and c will produce a force to head

$$=aX+bY+cZ.$$

So d , e , and f will produce a force to starboard

$$=dX+eY+fZ,$$

and g , h , and k will produce a force to keel

$$=gX+hY+kZ.$$

By comparing these results with Poisson's formula we see that for the soft iron of the ship, however complicated its arrangement may be, we may substitute the nine soft iron rods.

Of these rods five, viz., a , c , e , g , k , are symmetrically placed with reference to the fore and after midship section; the other four, viz., b , d , f , h , are unsymmetrically placed. If, therefore, we may, as in most cases, suppose the soft iron to be symmetrically placed with regard to that plane, we may suppose b , d , f , $h=0$.

The quantities P , Q , R in the general equations may be conveniently represented by three bar magnets placed in fixed positions in the ship, $+P$ attracting the north end of the compass needle to the head, $+Q$ to the starboard, and $+R$ to keel.

VI.

On the Coefficients λ , $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, u .

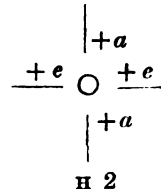
I.—*On the Coefficient λ .*

$$\lambda=1+\frac{a+e}{2}$$

We see from equation (8) that λ is the mean value of $\frac{H'}{H} \cos \delta$, or of the proportion of the force to north of earth and ship to the earth's horizontal force. It is therefore the "mean force to north" in terms of the earth's horizontal force as unit.

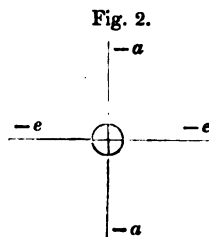
Such an arrangement of the supposed soft iron rods as Fig. 1 gives λ greater than unity and increases the "mean force to north."

Fig. 1.



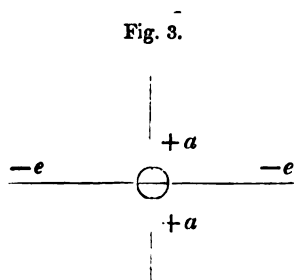
APP. 1.

Such an arrangement as Fig. 2 gives λ less than unity and diminishes the "mean force to north."

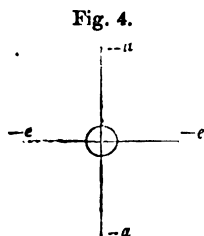
II.—On the Coefficient $\mathfrak{D}$.

$$\mathfrak{D} = \frac{1}{\lambda} \frac{a - e}{2}$$

$\mathfrak{D}$ is the principal coefficient of the quadrantal deviation; it is, as will be seen, closely connected with λ . A positive value, which $\mathfrak{D}$ almost without exception has, may be caused by the arrangement



in which both the fore-and-aft and the transverse iron concur in giving the positive value, or by an arrangement



in which the positive value is given by the excess of $-e$ over $-a$, it being observed that, *ceteris paribus*, a short rod in the position in the figure, has more effect than a longer rod. The values of a and e which give $\mathfrak{D}$ may be obtained, if we know both $\mathfrak{D}$ and λ , by the formulæ—

$$1 + a = \lambda(1 + \mathfrak{D})$$

$$1 + e = \lambda(1 - \mathfrak{D})$$

We thus see that the value of $\mathfrak{D}$ when it is + (as it is in all ordinary cases) may be diminished by soft iron rods placed as in the figure APP. 1.



with the additional advantage of increasing the directive force.*

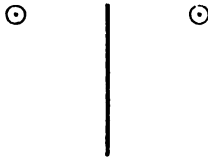
III.—On the Coefficients $\mathfrak{A}$ and $\mathfrak{C}$.

$$\mathfrak{A} = \frac{1}{\lambda} \frac{d-b}{2} \qquad \mathfrak{C} = \frac{1}{\lambda} \frac{d+b}{2}$$

The constant part of the deviation and the secondary and smaller part of the quadrantal deviation are, it will be seen, closely connected. Each arises, and can only arise, from horizontal induction in soft iron placed *unsymmetrically*, and each is, therefore, zero when the soft iron is symmetrically arranged on each side of the principal fore and aft section of a ship, and where the ship is on an even keel, each being, therefore, in general very small.

A case, in which large values of $\mathfrak{A}$ and $\mathfrak{C}$ have been observed, is of this kind, where the rod represents the iron spindle of a steering wheel.

Fig. 7.



Which, it will be easily seen, will give + $\mathfrak{A}$ and + $\mathfrak{C}$ in the starboard compass, and - $\mathfrak{A}$ and - $\mathfrak{C}$ in the port compass.

* The precise amount of effect cannot in general be determined except by observation, but there is one exception, viz., where the soft iron correctors are spheres or spherical

 shells (not very thin). In this case if p be the radius of the spheres, r the distance of the centre of the spheres from the centre of the compass, they will produce

$$\lambda = 1 + \frac{p^3}{r^3}$$

$$\lambda \mathfrak{D} = -3 \frac{p^3}{r^3}$$

Thus if $r = 3p$ $\lambda = 1 + \frac{p^3}{27} = 1.037$

$$\lambda \mathfrak{D} = -\frac{1}{9} = -.111$$

$$\mathfrak{D} = -.107$$

$$\mathfrak{D} = -6^\circ 10'$$

See *Phil. Trans.* 1865, p. 319.

APP. I. Large values have also been observed in turret ships of war, where the exigencies of navigation have required a steering compass to be placed out of the central line and near a turret. Large values are also observed when a ship heels over, for reasons which will be more clearly seen hereafter.

IV.—On the Coefficient $\mathfrak{B}$.

$$\mathfrak{B} = \frac{1}{\lambda} \left(c \tan \theta + \frac{P}{H} \right)$$

This is the principal coefficient of the semicircular deviation.

$\lambda \mathfrak{B}$ is the mean or polar force to head of the ship in terms of the earth's horizontal force as unit. $\mathfrak{B}$ is the same force in terms of the "mean force to north" as unit.

The part $\frac{c}{\lambda} \tan \theta$ arises from vertical induction on soft iron before or abaft the compass, and varies as the tangent of the dip. The part $\frac{P}{\lambda H}$ arises from permanent magnetism of the hard iron, and varies inversely as the earth's horizontal force.

V.—On the Coefficient $\mathfrak{C}$.

$$\mathfrak{C} = \frac{1}{\lambda} \left(f \tan \theta + \frac{Q}{H} \right)$$

This is the other part of the semicircular deviation.

$\lambda \mathfrak{C}$ is the mean or polar force to starboard of the ship in terms of the earth's horizontal force as unit; $\mathfrak{C}$ is the same force in terms of the "mean force to north" as unit.

The part $\frac{f}{\lambda} \tan \theta$ depends on the soft iron unsymmetrically arranged. It therefore is probably generally very small, and the principal part, $\frac{Q}{\lambda H}$, is probably that which gives $\mathfrak{C}$ nearly all its value. It depends on the magnetism of the hard iron, and varies inversely as the earth's horizontal force.

As $\lambda \mathfrak{B}$ is the mean or polar force of the ship to head, and $\lambda \mathfrak{C}$ the mean or polar force to starboard, $\lambda \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$ is the mean or polar horizontal force; and it acts in a direction making an angle α with the line to the ship's head where $\tan \alpha = \frac{\mathfrak{C}}{\mathfrak{B}}$

$\lambda \mathfrak{B}$ might be counteracted by a fore and aft magnet with its centre in a transverse vertical plane passing through the compass.

$\lambda \mathfrak{C}$ might be counteracted by a transverse magnet with its centre in a fore and aft plane passing through the compass.

$\lambda\sqrt{B^2+C^2}$ or in other words both λB and λC might be corrected by a single magnet with its centre immediately below the compass, its axis being horizontal, and making an angle α with the fore and aft line. APP. 1.

VI.—On the Coefficient μ .

$$\mu = 1 + k + \frac{R}{Z}$$

μ is that part of the vertical magnetism which is made up of (1) the earth's vertical force; (2) the vertical force caused by vertical induction in the soft iron of the ship; and (3) the vertical force caused by vertical magnetism; all in terms of the earth's vertical force as unit.

$$\mu - 1 = k + \frac{R}{Z}$$

is the ship's vertical force in terms of the earth's vertical force as unit.

$$(\mu - 1) \tan \theta = k \tan \theta + \frac{R}{H}$$

is the ship's vertical force in terms of the earth's horizontal force as unit.

In the ordinary positions of the standard compass this force in ships built in the northern hemisphere acts downwards; but it will be easily seen that rods as in the figure would produce a force upwards; and hence between decks in iron-built ships, and particularly armour-plated ships, the force generally acts upwards.*

VII.

On the changes which take place in the Parameters and Coefficients.

If the hypothesis from which Poisson's equations result, viz., that the magnetism of the ship is partly permanent and partly transient, were absolutely true, the equations would be exact, and the parameters $a, b, c, d, e, f, g, h, k, P, Q, R$, as well as the coefficients $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \lambda$, would be constant, while the coefficients $\mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mu$ would only change on a change of geographical position, by reason of their involving the dip and horizontal force. And though the hypothesis is not absolutely true, the so-called permanent magnetism being only sub-permanent, and the so-called transient magnetism requiring some time to change, the equations may still be con-

* Two spheres or spherical shells, using the same notation as before, would produce

$$-k = 2 \frac{p^3}{r^3}$$

Such correctors will therefore assist in correcting a heeling error to windward. In fact, in England it may be shown that for every degree of quadrantal deviation which they correct they will correct about $5\frac{1}{2}'$ of the heeling error for each degree of heel. The result will be the same with cylinders, and generally with all bodies for which $k = a$, post, p. 147.

APP. 1. sidered as exact, provided we consider the parameters not as absolutely constant, but as subject to variations.

As regards P, Q, R , their original values in iron-built ships depend on the position of the ship in building, and express an attraction of the north end of the needle to the part which was south in building. The values generally diminish when a ship has been launched and allowed to take up different positions. These changes make it necessary to have frequent re-determinations of the values of B and C .

*[As regards the parameters representing the effect of the soft iron, those which represent the three vertical soft iron rods, viz., c, f, h , probably do not change sensibly, as they do not change their inclination to the lines of force, without a change of geographical position; a change which necessarily occupies so much time that the masses of soft iron which they represent have time to receive their full charge of induced magnetism, notwithstanding any slowness in so doing arising from the imperfect softness of the iron. The parameters b, d, h are so small that any change in them may be neglected. The parameters a, e, g are, however, not only large in amount, but as the ship swings they are successively presented at all angles to the lines of force, and it is in them that any slowness of the soft iron in receiving or parting with its full charge of induced magnetism shows itself. The effect of such slowness may be expected, in the ordinary process of swinging a ship rapidly, to show itself, 1st, by a diminution of the value of a, e , and g , and therefore of $1 - \lambda$ and $\mathfrak{D}$; and 2ndly, by the values $a \cos \zeta, -e \sin \zeta$, and $g \cos \zeta$ in equations 4, 5, and 6, being somewhat in arrear of those due to the actual position of the ship. This may be represented by using $\zeta + s$ for ζ ; s being a small angle, the amount of which probably depends on the rapidity of swinging, and which will be + if the ship is swung to the left, — if swung to the right.

And as—

$$\begin{aligned} +a \cos (\zeta+s) &= +a \cos \zeta - sa \sin \zeta \\ -e \sin (\zeta+s) &= -se \cos \zeta - e \sin \zeta \\ +g \cos (\zeta+s) &= +g \cos \zeta - sg \sin \zeta \end{aligned}$$

we are, in fact, introducing into equations 4a, 5a, and 6a, a—

$$b = +sa$$

$$d = -se$$

$$h = +sg$$

the first two introduce—

$$\mathfrak{A} = -s \frac{a+e}{2\lambda} = +s \left(\frac{1}{\lambda} - 1 \right)$$

$$\mathfrak{C} = +s \frac{a-e}{2\lambda} = +s \mathfrak{D}$$

* This may be omitted by the student.

Hence the error introduced by the direction of swinging is—

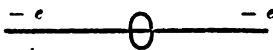
APP. 1.

$$\begin{aligned} & -s \frac{a+e}{2\lambda} + s \frac{a-e}{2\lambda} \cos 2\zeta' \\ & = -\frac{sa}{\lambda} \sin^2 \zeta' - \frac{se}{\lambda} \cos^2 \zeta' \end{aligned}$$

The meaning of which will be seen if we suppose the ship to be on one of the compass cardinal points. Thus at N. and S. the error is—

$$-\frac{se}{\lambda}$$

that is to say, when the rod— e turning from left to right is in an E. and W. position—



its starboard extremity having come from the north still repels, its port extremity having come from the south still attracts, the north end of the needle.

This error may be considerable as $-\frac{e}{\lambda}$ is frequently from $\cdot 300$ to $\cdot 500$.

At E. and W. (compass) the error is

$$-\frac{sa}{\lambda}$$

which is generally much smaller as $\frac{a}{\lambda}$ seldom exceeds $\cdot 100$.

As $\frac{1}{\lambda} - 1$ and $\mathfrak{D}$ are $+$ in all ordinary cases, we get the noticeable result that when an iron-built ship is swung to the left there are introduced a $+$ $\mathfrak{A}$ and $+$ $\mathfrak{E}$, and when swung to the right a $-$ $\mathfrak{A}$ and $-$ $\mathfrak{E}$.

In the case of iron-built ships swung at Greenhithe the value of $s\mathfrak{D}$ appears to be on an average about $30'$. The value of $s\left(\frac{1}{\lambda} - 1\right)$ is, as might be expected, somewhat larger.

These results tend to show that in many cases, though the observations of deviation give values of $\mathfrak{A}$ and $\mathfrak{E}$, it is better to make them zero in the expressions.

They also show that in correcting a compass by Mr. Airy's method it is proper at N. and S. to leave a little error uncorrected, *i.e.*, if the ship is swinging to the right to leave a little westerly deviation; if to the left, to leave a little easterly deviation. M. Gaussin, to whom this observation is due, leaves about $1^\circ 30'$ at N. and S., and about $30'$ at E. and W.]

VIII.

Determination of the Coefficients A, B, C, D, E.

We have seen that when the deviation is of moderate amount, not exceeding, say 20° , it may be expressed with sufficient exactness by the formula—

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta' \quad (13)$$

APP. I. We shall now show how A, B, C, D, E are to be determined by observations of deviation, made with the ship on different compass courses.

Let $\delta_0, \delta_1, \delta_2, \dots, \delta_{31}$ be the deviations observed with the ship's head successively on the 32 compass courses, N., N.b.E., N.N.E. . . . N.b.W., the $S_1, S_2, \dots, S_7$ being, as in Part III., the natural sines of the rhumbs or of the angles $11\frac{1}{4}^\circ, 22\frac{1}{2}^\circ, \dots, 78\frac{3}{4}^\circ$. We have the following 32 equations from which to determine A, B, C, D, E:—

$$\begin{aligned}\delta_0 &= A && + C && + E \\ \delta_1 &= A + BS_1 + CS_1 + DS_2 + ES_6 \\ \delta_2 &= A + BS_2 + CS_2 + DS_4 + ES_4 \\ \delta_3 &= A + BS_3 + CS_3 + DS_6 + ES_2 \\ \delta_4 &= A + BS_4 + CS_4 + D \\ \delta_5 &= A + BS_5 + CS_5 + DS_6 - ES_2 \\ &\vdots \\ &\vdots \\ &\vdots \\ \delta_{31} &= A - BS_1 + CS_7 - DS_2 + ES_6\end{aligned}$$

To obtain from these equations the most probable values of A, B, C, D, E, we must reduce them to five by the method of least squares, which fortunately furnishes in this case not only the best but the easiest method of obtaining the coefficients.

The process is the following:—First add all the equations together; this gives—

$$\delta_0 + \delta_1 + \delta_2 + \dots + \delta_{31} = 32 A$$

Multiplying each equation by the coefficient of the B contained in it we have—

$$\delta_1 S_1 + \delta_2 S_2 + \delta_3 S_3 + \&c. \dots \dots \dots = 16 B$$

So likewise—

$$\delta_0 + \delta_1 S_7 + \delta_2 S_6 + \&c. \dots \dots \dots = 16 C$$

$$\delta_1 S_2 + \delta_2 S_4 + \delta_3 S_6 + \&c. \dots \dots \dots = 16 D$$

$$\delta_0 + \delta_1 S_6 + \delta_2 S_4 + \&c. \dots \dots \dots = 16 E$$

These equations are the algebraical expressions of the tabular forms for computing the equations given in Part III., p. 36, and the computation is facilitated by means of the tables of the products of every five minutes of arc from 5° to $34^\circ 55'$ given in pp. 90–99.

If we observe on the 16 principal points only we have—

$$\delta_0 + \delta_2 + \delta_4 + \&c. \dots \dots \dots = 16 A$$

$$\delta_2 S_2 + \delta_4 S_4 + \&c. \dots \dots \dots = 8 B$$

$$\delta_0 + \delta_2 S_6 + \delta_4 S_4 + \&c. \dots \dots \dots = 8 C$$

$$\delta_2 S_4 + \delta_4 + \&c. \dots \dots \dots = 8 D$$

$$\delta_0 + \delta_2 S_4 - \delta_4 S_4 + \&c. \dots \dots \dots = 8 E$$

The corresponding tabular forms are obtained by taking every second line of the form for 32 points and halving the divisors. APP. 1.

If we observe on the eight principal points only we have—

$$\begin{aligned}\delta_0 + \delta_4 + \delta_8 + \delta_{12} + \&c. \dots\dots\dots &= 5 A \\ \delta_4 S_4 + \delta_8 + \delta_{12} S_4 - \delta_{20} S_4 - \delta_{24} - \delta_{28} S_4 \dots\dots\dots &= 4 B \\ \delta_0 + \delta_4 S_4 - \delta_{12} S_4 - \delta_{16} - \delta_{20} S_4 + \delta_{28} S_4 \dots\dots\dots &= 4 C \\ \delta_4 - \delta_{12} + \delta_{20} - \delta_{28} \dots\dots\dots &= 4 D \\ \delta_0 - \delta_4 + \delta_{16} - \delta_{24} \dots\dots\dots &= 4 E\end{aligned}$$

The corresponding tabular forms are obtained by taking every fourth line of the form for 32 points and one-fourth of the divisors.

It will be observed that we may in the same way obtain the coefficients from the 16 deviations $\delta_1, \delta_3, \&c.$, or from the eight deviations $\delta_1, \delta_5, \delta_9, \&c.$, or from the eight deviations $\delta_2, \delta_6, \delta_{10}, \&c.$, or from the eight deviations $\delta_3, \delta_7, \delta_{11}, \&c.$ The mode of obtaining these several sets of values from the complete tabular form for 32 observations, and the use to be made of them, has been pointed out in Part III., p. 39, note.

We shall find in the sequel that by a slight modification of the form we may obtain A, B, C, E, from the deviations on the four cardinal compass courses, N., E., S., W.; A, B, C, D, from the deviations on the four quadrantal compass courses N.E., S.E., S.W., N.W.

IX.

Deduction of the approximate Coefficients A, B, C, D, E from the exact Coefficients A, B, C, D, E, and vice versa.

From the expression—

$$\sin \delta = A \cos \delta + B \sin \zeta' + C \cos \zeta' + D \sin (2\zeta' + \delta) + E \cos (2\zeta' + \delta)$$

we may by a somewhat laborious process of expansions and substitutions obtain an expression for δ of the form—

$$\begin{aligned}\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta' \quad (14) \\ + F \sin 3\zeta' + G \cos 3\zeta' + H \sin 4\zeta' + K \cos 4\zeta' \\ + L \sin 5\zeta' + M \cos 5\zeta' + N \sin 6\zeta' + \&c.\end{aligned}$$

and if we consider B, C, D to be small quantities of the first order, and A and E to be small quantities of the second order, and if we go to terms of the third order, we shall find—

$$\begin{aligned}A &= A \\ B &= B \left(1 - \frac{D}{2} + \frac{B^2}{8} + \frac{C^2}{8} + \frac{D^2}{4} \right) - \frac{EC}{2} \quad (15) \\ C &= C \left(1 + \frac{D}{2} + \frac{B^2}{8} + \frac{C^2}{8} + \frac{D^2}{4} \right) - \frac{EB}{2} \\ D &= D \\ E &= E + AD\end{aligned}$$

APP. I.

$$F = \frac{BD - CG}{2} - \frac{B^3}{24} + \frac{B^2C}{8} - \frac{1}{8}BD^2$$

$$G = \frac{CD + BG}{2} + \frac{C^3}{24} - \frac{CB^2}{8} + \frac{1}{8}CD^2$$

$$H = \frac{D^3}{2}$$

$$K = DG$$

$$L = \frac{3}{8}BD^2$$

$$M = \frac{3}{8}CD^2$$

$$N = \frac{1}{8}D^3$$

The succeeding coefficients would be all of the fourth and higher orders. From these expressions are obtained by successive substitutions*—

$$A = \sin A \quad (16)$$

$$B = B \left\{ 1 + \frac{D}{2} - \frac{B^2}{8} - \frac{C^2}{8} \right\} + \frac{CE}{2}$$

$$= \sin B \left\{ 1 + \frac{1}{2} \sin D + \frac{1}{12} \text{versin } B - \frac{1}{4} \text{versin } C \right\} + \frac{1}{2} \sin C \sin E$$

$$C = C \left\{ 1 - \frac{D}{2} - \frac{B^2}{8} - \frac{C^2}{8} \right\} + \frac{BE}{2}$$

$$= \sin C \left\{ 1 - \frac{1}{2} \sin D + \frac{1}{12} \text{versin } C - \frac{1}{4} \text{versin } B \right\} + \frac{1}{2} \sin B \sin E$$

$$D = D = \sin D \left(1 + \frac{1}{8} \text{versin } D \right)$$

$$E = E - AD = \sin E - \sin A \sin D$$

$$F = B \left\{ \frac{D}{2} - \frac{B^2}{24} + \frac{C^2}{8} - \frac{D^2}{8} \right\} - \frac{EC}{2}$$

$$G = C \left\{ \frac{D}{2} + \frac{C^2}{24} - \frac{B^2}{8} + \frac{D^2}{8} \right\} + \frac{EB}{2}$$

$$H = \frac{1}{2} D^2$$

$$K = DE$$

$$L = \frac{3}{8} BD^2$$

$$M = \frac{3}{8} CD^2$$

$$N = \frac{1}{8} D^3$$

* These expressions give us the means of determining on what number of equidistant points the deviation must be observed to give the approximate coefficients with any required degree of exactness. Thus if we wish that they should be exact to terms of the second order inclusive, as eight and eight only of the coefficients A, B, C, D, E, F, G, H, do not exceed the second order, we must know the deviation on the eight principal points, in order that they should be exact to terms of the third order inclusive, as twelve and twelve only of the coefficients A, B, C, &c. do not exceed the third order, we must have the deviation on twelve equidistant points at least.

† Whenever expressions involving the quantities A, B, C, D, E, &c., are not homogeneous, *i.e.*, when they involve squares, products, &c., as well as the first powers of these quantities, as in the expressions $H = \frac{1}{2}D^2$ $K = DE$, we must suppose them expressed not in degrees or minutes, but in terms of the arc, which is equal to radius, or the arc of $57^\circ.3$. Or, if we please, we may take one power in each term in degrees and minutes, and the others in terms of the arc of $57^\circ.3$ as unit, or if the quantities be small we may substitute for them their natural sines. Thus we may express the above equations as follows :

$$H = D \sin D \quad K = E \sin D$$

X.

APP. 1.

Deduction of the approximate Coefficients A_1, B_1, C_1, D_1, E_1 from the exact Coefficients $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E}$, and vice versa.

From the expression—

$$\tan \delta = \frac{\mathfrak{A} + \mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathfrak{E} \cos 2\zeta}{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta}$$

we may by a comparatively easy and expeditious process* obtain an expression for δ in the form—

$$\begin{aligned} \delta = & A_1 + B_1 \sin \zeta + C_1 \cos \zeta + D_1 \sin 2\zeta + E_1 \cos 2\zeta \\ & + F_1 \sin 3\zeta + G_1 \cos 3\zeta + H_1 \sin 4\zeta + K_1 \cos 4\zeta \\ & + L_1 \sin 5\zeta + M_1 \cos 5\zeta + N_1 \sin 5\zeta + \&c. \end{aligned} \quad (17)$$

* The expansion of δ in terms of sines and cosines of multiples of ζ' does not appear to be capable of being obtained in any simple mode. The expansion of δ in terms of sines and cosines of multiples of ζ may be obtained much more easily as follows :—

Multiplying equation (9) by $\sqrt{-1}$ and adding to equation (10), we have—

$$\begin{aligned} \frac{H'}{\lambda H} (\cos \delta + \sqrt{-1} \sin \delta) &= \frac{H'}{\lambda H} e^{\delta \sqrt{-1}} = 1 + \mathfrak{A} \sqrt{-1} \\ &+ (\mathfrak{B} + \mathfrak{C} \sqrt{-1}) (\cos \zeta + \sqrt{-1} \sin \zeta) \\ &+ (\mathfrak{D} + \mathfrak{E} \sqrt{-1}) (\cos 2\zeta + \sqrt{-1} \sin 2\zeta) \end{aligned}$$

and taking the Napierian logarithms on both sides, we have at once—

δ = terms multiplied by $\sqrt{-1}$ in

$$\begin{aligned} &\mathfrak{A} \sqrt{-1} + (\mathfrak{B} + \mathfrak{C} \sqrt{-1}) (\cos \zeta + \sqrt{-1} \sin \zeta) + (\mathfrak{D} + \mathfrak{E} \sqrt{-1}) \\ &\quad (\cos 2\zeta + \sqrt{-1} \sin 2\zeta) \\ &- \frac{1}{2} \{ \mathfrak{A} \sqrt{-1} + (\mathfrak{B} + \mathfrak{C} \sqrt{-1}) (\cos \zeta + \sqrt{-1} \sin \zeta) + (\mathfrak{D} + \mathfrak{E} \sqrt{-1}) \\ &\quad (\cos 2\zeta + \sqrt{-1} \sin 2\zeta) \}^2 \\ &+ \frac{1}{6} \{ \mathfrak{A} \sqrt{-1} + (\mathfrak{B} + \mathfrak{C} \sqrt{-1}) (\cos \zeta + \sqrt{-1} \sin \zeta) + (\mathfrak{D} + \mathfrak{E} \sqrt{-1}) \\ &\quad (\cos 2\zeta + \sqrt{-1} \sin 2\zeta) \}^3 \end{aligned}$$

The expansions here indicated are very easily performed, as the troublesome process of converting powers and products of sines and cosines into sines and cosines of multiple arcs is not required.

Equation (14) is, it will be seen, of the form $\delta = f(\zeta - \delta)$, and equation (17) of the form $\delta = F(\zeta' + \delta)$. Either of these may be readily obtained from the other by the following form of Lagrange's theorem :—

$$\begin{aligned} \delta &= f(\zeta - \delta) \\ &= f'(\zeta) - \frac{1}{2} \left(f''(\zeta) \right)' + \frac{1}{2 \cdot 3} \left(f'''(\zeta) \right)'' - \&c. \\ \delta &= F(\zeta' + \delta) \\ &= F'(\zeta') + \frac{1}{2} \left(F''(\zeta') \right)' + \frac{1}{2 \cdot 3} \left(F'''(\zeta') \right)'' + \&c. \end{aligned}$$

APP. 1. In which to quantities of the third order inclusive—

$$\begin{aligned}
 A_1 &= \mathfrak{A} & (18) \\
 B_1 &= \mathfrak{B} + \mathfrak{A} \mathfrak{C} \\
 C_1 &= \mathfrak{C} - \mathfrak{A} \mathfrak{B} \\
 D_1 &= \mathfrak{D} - \frac{1}{2}(\mathfrak{B}^2 - \mathfrak{C}^2) \\
 E_1 &= \mathfrak{E} - \mathfrak{B} \mathfrak{C} - \mathfrak{A} \mathfrak{D} \\
 F_1 &= -\mathfrak{B} \mathfrak{D} + \mathfrak{C} \mathfrak{E} + \frac{1}{2} \mathfrak{B}^3 - \mathfrak{B} \mathfrak{C}^2 \\
 G_1 &= -\mathfrak{B} \mathfrak{E} - \mathfrak{C} \mathfrak{D} - \frac{1}{2} \mathfrak{C}^3 + \mathfrak{C} \mathfrak{B}^2 \\
 H_1 &= -\frac{1}{2} \mathfrak{D}^2 + (\mathfrak{B}^2 - \mathfrak{C}^2) \mathfrak{D} \\
 K_1 &= -\mathfrak{D} \mathfrak{E} + 2 \mathfrak{B} \mathfrak{C} \mathfrak{D} \\
 L_1 &= \mathfrak{B} \mathfrak{D}^2 \\
 M_1 &= \mathfrak{C} \mathfrak{D}^2 \\
 N_1 &= \frac{1}{2} \mathfrak{D}^3
 \end{aligned}$$

The succeeding coefficients would be of the fourth and higher orders.

The coefficients A_1, B_1, C_1, D_1, E_1 are derived from observations of deviation made on the 32, 16, or 8 principal *magnetic* points, exactly as A, B, C, D, E are derived from observations of deviation on the *compass* points.

From these expressions we obtain by successive substitution—

$$\begin{aligned}
 \mathfrak{A} &= A_1 = \sin A & (19) \\
 \mathfrak{B} &= B_1 - A_1 C_1 = \sin B_1 (1 + \frac{1}{2} \text{versin } B_1) - \sin A_1 \sin C_1 \\
 \mathfrak{C} &= C_1 + A_1 B_1 = \sin C_1 (1 + \frac{1}{2} \text{versin } C_1) + \sin A_1 \sin B_1 \\
 \mathfrak{D} &= D_1 + \frac{1}{2}(B_1^2 - C_1^2) \\
 &= \sin D_1 (1 + \frac{1}{2} \text{versin } D_1) + \text{versin } B_1 - \text{versin } C_1 \\
 \mathfrak{E} &= E_1 + B_1 C_1 + A_1 D_1 \\
 &= \sin E_1 + \sin B_1 \sin C_1 + \sin A_1 \sin D_1 \\
 F_1 &= -B_1 D_1 + C_1 E_1 - \frac{1}{2} B_1^3 - \frac{1}{2} B_1 C_1^2 \\
 G_1 &= -C_1 D_1 + B_1 E_1 + \frac{1}{2} C_1^3 + \frac{1}{2} C_1 B_1^2 \\
 H_1 &= -\frac{1}{2} D_1^2 + \frac{1}{2} D_1 B_1^2 - \frac{1}{2} D_1 C_1^2 \\
 K_1 &= -D_1 E_1 + 2 B_1 C_1 D_1 \\
 L_1 &= B_1 D_1^2 \\
 M_1 &= C_1 D_1^2 \\
 N_1 &= \frac{1}{2} D_1^3
 \end{aligned}$$

XI.

Deduction of A, B, C, D, E from A_1, B_1, C_1, D_1, E_1 , and vice versâ.

We may also deduce A, B, C, D, E from A_1, B_1, C_1, D_1, E_1 , and vice versâ, by the following formulæ, which are correct as far as terms of the third order inclusive :—

$$\begin{aligned}
 A &= A_1 \\
 B &= B_1 \left\{ 1 - \frac{1}{2} \sin D_1 - \frac{1}{4} \text{versin } B_1 + \frac{3}{4} \text{versin } C_1 + \frac{1}{2} \text{versin } D_1 \right\} - \left(A_1 + \frac{E_1}{2} \right) \sin C_1 & (20)
 \end{aligned}$$

$$C = C_1 \left\{ 1 + \frac{1}{2} \sin D_1 + \frac{3}{4} \text{versin } B_1 - \frac{1}{4} \text{versin } C_1 + \frac{1}{2} \text{versin } D_1 \right\} + \left(A_1 - \frac{E_1}{2} \right) \sin B_1 \quad \text{APP. 1.}$$

$$D = D_1 + \frac{1}{2} B_1 \sin B_1 - \frac{1}{2} C_1 \sin C_1$$

$$E = E_1 + B_1 \sin C_1 + 2 A_1 \sin D_1$$

$$A_1 = A$$

(21)

$$B_1 = B \left\{ 1 + \frac{1}{2} \sin D - \frac{1}{4} \text{versin } B - \frac{1}{4} \text{versin } C \right\} + \left(A + \frac{E}{2} \right) \sin C$$

$$C_1 = C \left\{ 1 - \frac{1}{2} \sin D - \frac{1}{4} \text{versin } B - \frac{1}{4} \text{versin } C \right\} - \left(A - \frac{E}{2} \right) \sin B$$

$$D_1 = D \left\{ 1 - \text{versin } B - \text{versin } C \right\} - \frac{1}{2} B \sin B + \frac{1}{2} C \sin C$$

$$E_1 = E - B \sin C - 2 A \sin D$$

XII.

On the degree of approximation of the different formulæ.

The following expressions will be found useful as showing the degree of correctness with which the coefficients are obtained from deviations observed on a greater or less number of equidistant points.

Let $\Sigma^n \delta \sin \zeta'$ represent the sum of the values of $\delta \sin \zeta'$ for the n equidistant values $\zeta', \zeta' + \frac{360^\circ}{n}, \zeta' + 2 \frac{360}{n}$ &c. $\zeta' + (n-1) \frac{360}{n}$

Then going as far as terms of the third order—

$$\frac{1}{4} \Sigma^4 \delta = A + H \sin 4\zeta' + K \cos 4\zeta'$$

$$\frac{1}{4} \Sigma^4 \delta \sin \zeta' = B + (G - M) \sin 4\zeta' + (L - F) \cos 4\zeta'$$

$$\frac{1}{2} \Sigma^4 \delta \cos \zeta' = C + (F + L) \sin 4\zeta' + (G + M) \cos 4\zeta'$$

$$\frac{1}{2} \Sigma^4 \delta \sin 2\zeta' = D + E \sin 4\zeta' + (N - D) \cos 4\zeta' - N \cos 8\zeta'$$

$$\frac{1}{2} \Sigma^4 \delta \cos 2\zeta' = E + (D + N) \sin 4\zeta' + E \cos 4\zeta' + N \sin 8\zeta'$$

$$\frac{1}{4} \Sigma^8 \delta = A$$

$$\frac{1}{4} \Sigma^8 \delta \sin \zeta' = B$$

$$\frac{1}{4} \Sigma^8 \delta \cos \zeta' = C$$

$$\frac{1}{4} \Sigma^8 \delta \sin 2\zeta' = D - N \cos 8\zeta'$$

$$\frac{1}{4} \Sigma^8 \delta \cos 2\zeta' = E + N \sin 8\zeta'$$

If we take 12 or 16 points, the values of the coefficients are correct to terms of the third order.

If the deviations are given on four points only the results are different, according as these are the cardinal or the quadrantal points. Let A', B', C', D', E' , be the values of $\frac{1}{4} \Sigma^4 \delta$, $\frac{1}{4} \Sigma^4 \delta \sin \zeta'$, $\frac{1}{2} \Sigma^4 \delta \cos \zeta'$, $\frac{1}{2} \Sigma^4 \delta \sin 2\zeta'$, and $\frac{1}{2} \Sigma^4 \delta \cos 2\zeta'$, obtained from the cardinal, and A'', B'', C'', D'', E'' , from the quadrantal points, then going to terms of the second order—

$$A' = A$$

$$A'' = A$$

$$B' = B \left(1 - \frac{D}{2} \right)$$

$$B'' = B \left(1 + \frac{D}{2} \right)$$

$$C' = C \left(1 + \frac{D}{2} \right)$$

$$C'' = C \left(1 - \frac{D}{2} \right)$$

$$D' = 0$$

$$D'' = 2 D$$

$$E' = 2 E$$

$$E'' = 0.$$

APP. 1. Comparing these, it will be seen that the results from the quadrantal points are the most accurate, subject to this, that we must take for D half the value that would be given by the usual expression. In fact, in going to terms of the second order, we have from the cardinal points—

$$\mathfrak{B} = B' (1 + D) \quad \mathfrak{C} = C' (1 - D)$$

and from the quadrantal points—

$$\mathfrak{B} = B'' \quad \mathfrak{C} = C''$$

the values of B and C derived from the four quadrantal points are, therefore, even more near to the *exact* values than those derived from eight points. Observations on the four quadrantal points, if carefully made with good instruments, should, therefore, give very exact values of the coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$.

If we go to terms of the third order, we get from the quadrantal points—

$$\mathfrak{B} = B'' (1 - \frac{1}{12} B''^2 - \frac{1}{4} C''^2 + \frac{1}{2} D^2)$$

$$\mathfrak{C} = C'' (1 - \frac{1}{12} C''^2 - \frac{1}{4} B''^2 + \frac{1}{2} D^2)$$

or,

$$\mathfrak{B} = \sin B'' \left\{ 1 + \frac{1}{8} \text{versin } B'' - \frac{1}{2} \text{versin } C'' + \text{versin } D \right\}$$

$$\mathfrak{C} = \sin C'' \left\{ 1 + \frac{1}{8} \text{versin } C'' - \frac{1}{2} \text{versin } B'' + \text{versin } D \right\}$$

which should in this case be used instead of the expressions (16).

We also get

$$\frac{1}{2} D'' = D - N = D - \frac{1}{3} D^3$$

and therefore approximately

$$D = \frac{1}{2} D'' + \frac{1}{2} \left(\frac{1}{2} D'' \right)^3$$

and

$$\mathfrak{D} = D = \tan \left(\frac{1}{2} D'' \right) = \frac{1}{2} D''$$

so that, in this case, instead of the expression $\mathfrak{D} = D$ (16), we ought to take $\mathfrak{D} = \tan \frac{1}{2} D''$.

XIII.

Direct determination of the exact Coefficients from Observations on Eight or a smaller number of Points by disturbed Compass.

I. Preliminary Equations.

We may obtain values of $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, directly from observations on the eight principal points by the disturbed compass, or even on fewer points by equation (12), which gives—

$$\sin \delta_0 = \mathfrak{A} \cos \delta_0 + \mathfrak{C} + \mathfrak{D} \sin \delta_0 + \mathfrak{E} \cos \delta_0 \quad (22)$$

$$\sin \delta_4 = \mathfrak{A} \cos \delta_4 + \mathfrak{B} S_4 + \mathfrak{C} S + \mathfrak{D} \cos \delta_4 - \mathfrak{E} \sin \delta_4$$

$$\sin \delta_8 = \mathfrak{A} \cos \delta_8 + \mathfrak{B} - \mathfrak{D} \sin \delta_8 - \mathfrak{E} \cos \delta_8$$

$$\sin \delta_{12} = \mathfrak{A} \cos \delta_{12} + \mathfrak{B} S_4 - S_4 - \mathfrak{D} \cos \delta_{12} + \mathfrak{E} \sin \delta_{12}$$

$$\sin \delta_{16} = \mathfrak{A} \cos \delta_{16} - \mathfrak{C} + \mathfrak{D} \sin \delta_{16} + \mathfrak{E} \cos \delta_{16}$$

$$\sin \delta_{20} = \mathfrak{A} \cos \delta_{20} - \mathfrak{B} S_4 + \mathfrak{C} S_4 + \mathfrak{D} \cos \delta_{20} - \mathfrak{E} \sin \delta_{20}$$

$$\sin \delta_{24} = \mathfrak{A} \cos \delta_{24} - \mathfrak{B} - \mathfrak{D} \sin \delta_{24} - \mathfrak{E} \cos \delta_{24}$$

$$\sin \delta_{28} = \mathfrak{A} \cos \delta_{28} - \mathfrak{B} S_4 + \mathfrak{C} S_4 - \mathfrak{D} \cos \delta_{28} + \mathfrak{E} \sin \delta_{28}$$

These may be grouped thus—

$$\begin{aligned}
 \sin \delta_4 (1 + \mathcal{G}) &= (\mathcal{M} + \mathcal{D}) \cos \delta_4 + \mathcal{B} S_4 + \mathcal{C} S_1 \\
 \sin \delta_{20} (1 + \mathcal{G}) &= (\mathcal{M} + \mathcal{D}) \cos \delta_{20} - \mathcal{B} S_4 - \mathcal{C} S_1 \\
 \sin \delta_{12} (1 - \mathcal{G}) &= (\mathcal{M} - \mathcal{D}) \cos \delta_{12} + \mathcal{B} S_4 - \mathcal{C} S_1 \\
 \sin \delta_{28} (1 - \mathcal{G}) &= (\mathcal{M} - \mathcal{D}) \cos \delta_{28} - \mathcal{B} S_4 + \mathcal{C} S_1 \\
 \sin \delta_0 (1 - \mathcal{D}) &= (\mathcal{M} + \mathcal{G}) \cos \delta_0 + \mathcal{G} \\
 \sin \delta_{16} (1 - \mathcal{D}) &= (\mathcal{M} + \mathcal{G}) \cos \delta_{16} - \mathcal{G} \\
 \sin \delta_8 (1 + \mathcal{D}) &= (\mathcal{M} - \mathcal{G}) \cos \delta_8 + \mathcal{G} \\
 \sin \delta_{24} (1 + \mathcal{D}) &= (\mathcal{M} - \mathcal{G}) \cos \delta_{24} - \mathcal{G}
 \end{aligned} \tag{23}$$

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and these again thus—

$$\begin{aligned}
 \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) (1 + \mathcal{G}) &= (\mathcal{M} + \mathcal{D}) \frac{1}{2}(\cos \delta_4 + \cos \delta_{20}) \\
 \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) (1 - \mathcal{G}) &= (\mathcal{M} - \mathcal{D}) \frac{1}{2}(\cos \delta_{12} + \cos \delta_{28}) \\
 S_4 \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) (1 + \mathcal{G}) &= (\mathcal{M} + \mathcal{D}) S_4 \frac{1}{2}(\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2}\mathcal{B} + \frac{1}{2}\mathcal{C} \\
 S_1 \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) (1 - \mathcal{G}) &= (\mathcal{M} - \mathcal{D}) S_1 \frac{1}{2}(\cos \delta_{12} - \cos \delta_{28}) + \frac{1}{2}\mathcal{B} - \frac{1}{2}\mathcal{C} \\
 \frac{1}{2}(\sin \delta_0 + \sin \delta_{16}) (1 - \mathcal{D}) &= (\mathcal{M} + \mathcal{G}) \frac{1}{2}(\cos \delta_0 + \cos \delta_{16}) \\
 \frac{1}{2}(\sin \delta_8 + \sin \delta_{24}) (1 + \mathcal{D}) &= (\mathcal{M} - \mathcal{G}) \frac{1}{2}(\cos \delta_8 + \cos \delta_{24}) \\
 \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) (1 - \mathcal{D}) &= (\mathcal{M} + \mathcal{G}) \frac{1}{2}(\cos \delta_0 - \cos \delta_{16}) + \mathcal{G} \\
 \frac{1}{2}(\sin \delta_8 - \sin \delta_{24}) (1 + \mathcal{D}) &= (\mathcal{M} - \mathcal{G}) \frac{1}{2}(\cos \delta_8 - \cos \delta_{24}) + \mathcal{G}
 \end{aligned} \tag{24}$$

II. Determination of the Coefficients $\mathcal{M}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, from Deviations observed on the Four Quadrantal Compass Points.

From these we derive the following expressions, which are exact :—

$$\begin{aligned}
 \mathcal{M} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} + \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 &+ \mathcal{G} \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} - \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 \mathcal{D} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} - \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 &+ \mathcal{G} \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} + \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 \mathcal{B} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
 &+ \mathcal{G} S_1 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
 &- \mathcal{M} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
 &- \mathcal{D} S_1 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
 \mathcal{C} &= S_1 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
 &+ \mathcal{G} S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\
 &- \mathcal{M} S_1 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
 &- \mathcal{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\}
 \end{aligned}$$

Or going to terms of the third order:—

$$\begin{aligned}
 \mathcal{M} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} + \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} + \mathcal{D} \mathcal{G} \\
 \mathcal{D} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_4 + \sin \delta_{20})}{(\cos \delta_4 + \cos \delta_{20})} - \frac{1}{2} \frac{(\sin \delta_{12} + \sin \delta_{28})}{(\cos \delta_{12} + \cos \delta_{28})} \right\} \\
 \mathcal{B} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathcal{C} \mathcal{G} \\
 &- \mathcal{D} S_1 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\} \\
 \mathcal{C} &= S_1 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathcal{B} \mathcal{G} \\
 &- \mathcal{D} S_4 \left\{ \frac{1}{2} (\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2} (\cos \delta_{12} - \cos \delta_{28}) \right\}
 \end{aligned}$$

APP. 1. Or going to terms of the second order :—

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} (\sin \delta_4 + \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} + \sin \delta_{28}) \right\} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{1}{2} (\sin \delta_4 + \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} + \sin \delta_{28}) \right\} \\ \mathfrak{B} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\} \\ \mathfrak{C} &= S_4 \left\{ \frac{1}{2} (\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2} (\sin \delta_{12} - \sin \delta_{28}) \right\}\end{aligned}$$

And going to terms of the first order, we have as before

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} (\delta_4 + \delta_{20}) + \frac{1}{2} (\delta_{12} + \delta_{28}) \right\} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{1}{2} (\delta_4 + \delta_{20}) - \frac{1}{2} (\delta_{12} + \delta_{28}) \right\} \\ \mathfrak{B} &= S_4 \left\{ \frac{1}{2} (\delta_4 - \delta_{20}) + \frac{1}{2} (\delta_{12} - \delta_{28}) \right\} \\ \mathfrak{C} &= S_4 \left\{ \frac{1}{2} (\delta_4 - \delta_{20}) - \frac{1}{2} (\delta_{12} - \delta_{28}) \right\}\end{aligned}$$

III. Determination of the Coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{E}$, from Deviations observed on the Four Cardinal Compass Points.

From the preliminary equations we derive the following expressions, which are exact :—

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} + \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} \\ &\quad - \mathfrak{D} \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} - \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} - \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} \\ &\quad - \mathfrak{A} \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} + \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} \\ \mathfrak{B} &= (1 + \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) - (\mathfrak{A} - \mathfrak{E}) \frac{1}{2} (\cos \delta_8 - \cos \delta_{24}) \\ \mathfrak{C} &= (1 - \mathfrak{D}) \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) - (\mathfrak{A} + \mathfrak{E}) \frac{1}{2} (\cos \delta_0 - \cos \delta_{16})\end{aligned}$$

Or going to terms of the third order :—

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} + \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} - \mathfrak{D} \mathfrak{E} \\ \mathfrak{E} &= \frac{1}{2} \left\{ \frac{1}{2} \frac{(\sin \delta_0 + \sin \delta_{16})}{(\cos \delta_0 + \cos \delta_{16})} - \frac{1}{2} \frac{(\sin \delta_8 + \sin \delta_{24})}{(\cos \delta_8 + \cos \delta_{24})} \right\} - \mathfrak{D} \mathfrak{A} \\ \mathfrak{B} &= (1 + \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) \\ \mathfrak{C} &= (1 - \mathfrak{D}) \frac{1}{2} (\sin \delta_0 - \sin \delta_{16})\end{aligned}$$

Or going to terms of the second order :—

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} (\sin \delta_0 + \sin \delta_{16}) + \frac{1}{2} (\sin \delta_8 + \sin \delta_{24}) \right\} \\ \mathfrak{E} &= \frac{1}{2} \left\{ \frac{1}{2} (\sin \delta_0 + \sin \delta_{16}) - \frac{1}{2} (\sin \delta_8 + \sin \delta_{24}) \right\} \\ \mathfrak{B} &= (1 + \mathfrak{D}) \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) \\ \mathfrak{C} &= (1 - \mathfrak{D}) \frac{1}{2} (\sin \delta_0 - \sin \delta_{16})\end{aligned}$$

And going to terms of the first order, we have as before,

$$\begin{aligned}\mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2} (\delta_0 + \delta_{16}) + \frac{1}{2} (\delta_8 + \delta_{24}) \right\} \\ \mathfrak{E} &= \frac{1}{2} \left\{ \frac{1}{2} (\delta_0 + \delta_{16}) - \frac{1}{2} (\delta_8 + \delta_{24}) \right\} \\ \mathfrak{B} &= \frac{1}{2} (\delta_8 - \delta_{24}) \\ \mathfrak{C} &= \frac{1}{2} (\delta_0 - \delta_{16})\end{aligned}$$

IV. *Determination of the Coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, from the Deviations observed on the Eight Principal Compass Points.* APP. 1.

If we have the deviations on the eight principal compass points the exact expressions will be the mean of the exact expressions given above for the four quadrantal compass points, and for the four cardinal compass points. It is therefore unnecessary to repeat them.

In taking them to the third order only some simplifications are introduced, and the following are the expressions:—

$$\begin{aligned} 2 \mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) + \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) \right. \\ &\quad \left. + \frac{1}{2}(\sin \delta_0 + \sin \delta_{16}) + \frac{1}{2}(\sin \delta_8 + \sin \delta_{24}) \right\} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) - \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) \right\} \\ \mathfrak{E} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_0 + \sin \delta_{16}) - \frac{1}{2}(\sin \delta_8 + \sin \delta_{24}) \right\} - \mathfrak{D} \mathfrak{A} \\ 2 \mathfrak{B} &= S_4 \left\{ \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathfrak{C} \mathfrak{E} \\ &\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2}(\cos \delta_4 - \cos \delta_{20}) - \frac{1}{2}(\cos \delta_{12} - \cos \delta_{28}) \right\} \\ &\quad + (1 + \mathfrak{D}) \frac{1}{2}(\sin \delta_8 - \sin \delta_{24}) \\ 2 \mathfrak{C} &= S_4 \left\{ \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) \right\} + \mathfrak{B} \mathfrak{E} \\ &\quad - \mathfrak{D} S_4 \left\{ \frac{1}{2}(\cos \delta_4 - \cos \delta_{20}) + \frac{1}{2}(\cos \delta_{12} - \cos \delta_{28}) \right\} \\ &\quad + (1 - \mathfrak{D}) \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) \end{aligned}$$

Going to terms of the second order, we have—

$$\begin{aligned} 2 \mathfrak{A} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) + \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) \right. \\ &\quad \left. + \frac{1}{2}(\sin \delta_0 + \sin \delta_{16}) + \frac{1}{2}(\sin \delta_8 + \sin \delta_{24}) \right\} \\ \mathfrak{D} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_4 + \sin \delta_{20}) - \frac{1}{2}(\sin \delta_{12} + \sin \delta_{28}) \right\} \\ \mathfrak{E} &= \frac{1}{2} \left\{ \frac{1}{2}(\sin \delta_0 + \sin \delta_{16}) - \frac{1}{2}(\sin \delta_8 + \sin \delta_{24}) \right\} \\ 2 \mathfrak{B} &= S_4 \left\{ \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) + \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) \right\} \\ &\quad + (1 + \mathfrak{D}) \frac{1}{2}(\sin \delta_8 - \sin \delta_{24}) \\ 2 \mathfrak{C} &= S_4 \left\{ \frac{1}{2}(\sin \delta_4 - \sin \delta_{20}) - \frac{1}{2}(\sin \delta_{12} - \sin \delta_{28}) \right\} \\ &\quad + (1 - \mathfrak{D}) \frac{1}{2}(\sin \delta_0 - \sin \delta_{16}) \end{aligned}$$

Going to terms of the first order, we have the formulæ which are expressed in the tabular forms in Part III.

It will be seen on inspection of these forms that they give considerable facilities for the arithmetical operations. Thus the first line of $\mathfrak{A}$ with one sign changed gives $\mathfrak{D}$; the second line of $\mathfrak{A}$ with one sign changed gives the first part of $\mathfrak{E}$, and the two first lines of $\mathfrak{B}$ and $\mathfrak{C}$ are the same with one change of sign.

It will also be observed that though in some cases the unknown coefficients enter into the right-hand side of the equations, yet when they do so they are multiplied by very small factors, so that their approximate values may be used.

APP. 1. *V. Determination of the Coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, from Deviations observed in one quadrant or semicircle.*

From the formulæ No. (22) we may derive the coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, from observations made on a part only of the horizon. This is owing to the circumstances that the coefficients $\mathfrak{A}$ and $\mathfrak{E}$ are always so small when the compass is amidships and the ship on an even keel, that they may in case of need be neglected, and we may then derive $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ from three of the equations (22). In combining these equations care should be taken that the coefficient of the quantity sought should be as large as possible.

The following formulæ are obtained in this way, and give the means of determining the coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, from observations made in one quadrant or semicircle.

It will be observed that in the expressions it has been thought desirable to retain the terms involving $\mathfrak{A}$ and $\mathfrak{E}$, but rather for the purpose of showing the error which is introduced by omitting them, than for the purpose of their being retained in the practical application of the formulæ.

1. N.E. Quadrant.

$$\begin{aligned}\mathfrak{D}\{S_1 \cos \delta_1 - \tfrac{1}{2}(\sin \delta_0 - \sin \delta_2)\} &= S_1 \sin \delta_1 - \tfrac{1}{2}(\sin \delta_0 + \sin \delta_2) \\ &\quad + \mathfrak{A}\{-S_1 \cos \delta_1 + \tfrac{1}{2}(\cos \delta_0 + \cos \delta_2)\} \\ &\quad + \mathfrak{E}\{+S_1 \sin \delta_1 + \tfrac{1}{2}(\cos \delta_0 - \cos \delta_2)\} \\ \mathfrak{B} &= \sin \delta_0 (1 + \mathfrak{D}) - (\mathfrak{A} - \mathfrak{E}) \cos \delta_0 \\ \mathfrak{C} &= \sin \delta_0 (1 - \mathfrak{D}) - (\mathfrak{A} + \mathfrak{E}) \cos \delta_0\end{aligned}$$

2. S.E. Quadrant.

$$\begin{aligned}\mathfrak{D}\{S_1 \cos \delta_{12} - \tfrac{1}{2}(\sin \delta_8 - \sin \delta_{16})\} &= -S_1 \sin \delta_{12} + \tfrac{1}{2}(\sin \delta_8 + \sin \delta_{16}) \\ &\quad + \mathfrak{A}\{+S_1 \cos \delta_{12} - \tfrac{1}{2}(\cos \delta_8 + \cos \delta_{16})\} \\ &\quad + \mathfrak{E}\{+S_1 \sin \delta_{12} + \tfrac{1}{2}(\cos \delta_8 - \cos \delta_{16})\} \\ \mathfrak{B} &= +\sin \delta_8 (1 + \mathfrak{D}) - (\mathfrak{A} - \mathfrak{E}) \cos \delta_8 \\ \mathfrak{C} &= -\sin \delta_{16} (1 - \mathfrak{D}) + (\mathfrak{A} + \mathfrak{E}) \cos \delta_{16}\end{aligned}$$

3. S.W. Quadrant.

$$\begin{aligned}\mathfrak{D}\{S_1 \cos \delta_{20} - \tfrac{1}{2}(\sin \delta_{16} - \sin \delta_{24})\} &= S_1 \sin \delta_{20} - \tfrac{1}{2}(\sin \delta_{16} + \sin \delta_{24}) \\ &\quad + \mathfrak{A}\{-S_1 \cos \delta_{20} + \tfrac{1}{2}(\cos \delta_{16} + \cos \delta_{24})\} \\ &\quad + \mathfrak{E}\{+S_1 \sin \delta_{20} - \tfrac{1}{2}(\cos \delta_{16} - \cos \delta_{24})\} \\ \mathfrak{B} &= -\sin \delta_{24} (1 + \mathfrak{D}) + (\mathfrak{A} - \mathfrak{E}) \cos \delta_{24} \\ \mathfrak{C} &= -\sin \delta_{16} (1 - \mathfrak{D}) + (\mathfrak{A} + \mathfrak{E}) \cos \delta_{16}\end{aligned}$$

4. N.W. Quadrant.

$$\begin{aligned}\mathfrak{D}\{S_1 \cos \delta_{28} + \tfrac{1}{2}(\sin \delta_0 - \sin \delta_{24})\} &= -S_1 \sin \delta_{28} + \tfrac{1}{2}(\sin \delta_0 + \sin \delta_{24}) \\ &\quad + \mathfrak{A}\{+S_1 \cos \delta_{28} - \tfrac{1}{2}(\cos \delta_0 + \cos \delta_{24})\} \\ &\quad + \mathfrak{E}\{+S_1 \sin \delta_{28} - \tfrac{1}{2}(\cos \delta_0 - \cos \delta_{24})\} \\ \mathfrak{B} &= -\sin \delta_{24} (1 + \mathfrak{D}) + (\mathfrak{A} - \mathfrak{E}) \cos \delta_{24} \\ \mathfrak{C} &= \sin \delta_0 (1 - \mathfrak{D}) - (\mathfrak{A} + \mathfrak{E}) \cos \delta_0\end{aligned}$$

5. N.E.S. *Semicircle*.

$$\begin{aligned}
 \mathfrak{D} \{ \cos \delta_1 + \cos \delta_{12} - S_1 (\sin \delta_0 - \sin \delta_{16}) \} \\
 &= \sin \delta_1 - \sin \delta_{12} - S_4 (\sin \delta_0 - \sin \delta_{16}) \\
 &\quad - \mathfrak{A} \{ \cos \delta_1 - \cos \delta_{12} - S_1 (\cos \delta_0 - \cos \delta_{16}) \} \\
 &\quad + \mathfrak{G} \{ \sin \delta_1 + \sin \delta_{12} + S_1 (\cos \delta_0 - \cos \delta_{16}) \} \\
 \mathfrak{B} &= \sin \delta_0 (1 + \mathfrak{D}) - (\mathfrak{A} - \mathfrak{G}) \cos \delta_0 \\
 \mathfrak{C} &= \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) (1 - \mathfrak{D}) - (\mathfrak{A} + \mathfrak{G}) \frac{1}{2} (\cos \delta_0 - \cos \delta_{16})
 \end{aligned}$$

6. N.W.S. *Semicircle*.

$$\begin{aligned}
 \mathfrak{D} \{ \cos \delta_{20} + \cos \delta_{28} + S_1 (\sin \delta_0 - \sin \delta_{16}) \} \\
 &= \sin \delta_{20} - \sin \delta_{28} + S_4 (\sin \delta_0 - \sin \delta_{16}) \\
 &\quad - \mathfrak{A} \{ \cos \delta_{20} - \cos \delta_{28} + S_1 (\cos \delta_0 - \cos \delta_{16}) \} \\
 &\quad + \mathfrak{G} \{ \sin \delta_{20} + \sin \delta_{28} - S_1 (\cos \delta_0 - \cos \delta_{16}) \} \\
 \mathfrak{B} &= -\sin \delta_{24} (1 + \mathfrak{D}) + (\mathfrak{A} - \mathfrak{G}) \cos \delta_{24} \\
 \mathfrak{C} &= \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) (1 - \mathfrak{D}) - (\mathfrak{A} + \mathfrak{G}) \frac{1}{2} (\cos \delta_0 - \cos \delta_{16})
 \end{aligned}$$

7. W.N.E. *Semicircle*.

$$\begin{aligned}
 \mathfrak{D} \{ \cos \delta_1 + \cos \delta_{24} + S_1 (\sin \delta_0 - \sin \delta_{24}) \} \\
 &= \sin \delta_1 - \sin \delta_{28} - S_4 (\sin \delta_0 - \sin \delta_{24}) \\
 &\quad - \mathfrak{A} \{ \cos \delta_1 - \cos \delta_{28} - S_1 (\cos \delta_0 - \cos \delta_{24}) \} \\
 &\quad + \mathfrak{G} \{ \sin \delta_1 + \sin \delta_{28} - S_1 (\cos \delta_0 - \cos \delta_{24}) \} \\
 \mathfrak{B} &= \frac{1}{2} (\sin \delta_0 - \sin \delta_{24}) (1 + \mathfrak{D}) - (\mathfrak{A} - \mathfrak{G}) \frac{1}{2} (\cos \delta_0 - \cos \delta_{24}) \\
 \mathfrak{C} &= \sin \delta_0 (1 - \mathfrak{D}) - (\mathfrak{A} + \mathfrak{G}) \cos \delta_0
 \end{aligned}$$

8. E.S.W. *Semicircle*.

$$\begin{aligned}
 \mathfrak{D} \{ \cos \delta_{12} + \cos \delta_{20} - S_1 (\sin \delta_0 - \sin \delta_{24}) \} \\
 &= \sin \delta_{20} - \sin \delta_{12} + S_4 (\sin \delta_0 - \sin \delta_{24}) \\
 &\quad + \mathfrak{A} \{ \cos \delta_{12} - \cos \delta_{20} - S_1 (\cos \delta_0 - \cos \delta_{24}) \} \\
 &\quad + \mathfrak{G} \{ \sin \delta_{12} + \sin \delta_{20} + S_4 (\cos \delta_0 - \cos \delta_{24}) \} \\
 \mathfrak{B} &= \frac{1}{2} (\sin \delta_0 - \sin \delta_{24}) (1 + \mathfrak{D}) - (\mathfrak{A} - \mathfrak{G}) \frac{1}{2} (\cos \delta_0 - \cos \delta_{24}) \\
 \mathfrak{C} &= -\sin \delta_{16} (1 - \mathfrak{D}) + (\mathfrak{A} + \mathfrak{G}) \cos \delta_{16}
 \end{aligned}$$

VI. *Determination of the Coefficients* $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, *from deviations observed on any three courses* $\mathfrak{A}$ *and* $\mathfrak{G}$ *by zero.*

When the deviations $\delta_1, \delta_2, \delta_3$ on any three compass courses $\zeta'_1, \zeta'_2, \zeta'_3$ are given we may find $\mathfrak{D}$, $\mathfrak{B}$, and $\mathfrak{C}$ by the following formulæ :—

$$\begin{aligned}
 \mathfrak{D} &= \frac{\sin(\zeta'_2 - \zeta'_1) \sin \delta_1 + \sin(360 + \zeta'_1 - \zeta'_2) \sin \delta_2 + \sin(\zeta'_2 - \zeta'_1) \sin \delta_3}{\sin(\zeta'_2 - \zeta'_1) \sin(2\zeta'_1 + \delta_1) + \sin(360 + \zeta'_1 - \zeta'_2) \sin(2\zeta'_2 + \delta_2) + \sin(\zeta'_2 - \zeta'_1) \sin(2\zeta'_3 + \delta_3)} \\
 \mathfrak{B} &= - \frac{\sin \delta_1 - \mathfrak{D} \sin(2\zeta'_1 + \delta_1)}{\sin(\zeta'_2 - \zeta'_1)} \cos \zeta'_2 + \frac{\sin \delta_2 - \mathfrak{D} \sin(2\zeta'_2 + \delta_2)}{\sin(\zeta'_2 - \zeta'_1)} \cos \zeta'_1 \\
 \mathfrak{C} &= + \frac{\sin \delta_1 - \mathfrak{D} \sin(2\zeta'_1 + \delta_1)}{\sin(\zeta'_2 - \zeta'_1)} \sin \zeta'_2 - \frac{\sin \delta_2 - \mathfrak{D} \sin(2\zeta'_2 + \delta_2)}{\sin(\zeta'_2 - \zeta'_1)} \sin \zeta'_1
 \end{aligned}$$

In these equations we must take care that no two of the points are opposite each other.

APP. 1.

XIV.

Determination of the Coefficients B and C from any number of observations, D being supposed known and A and E being supposed zero.

1. As the quantity D does not change its value with a change of geographical position or the lapse of time, it is often better to make use of the value of D , carefully determined at what may be called a base station, than the value actually derived from observations made under less favourable circumstances. The same may be said of A and E , when they have real values; but as they may always be considered zero for compasses in the midship line, when the soft iron is symmetrically distributed, they will be considered as zero whenever the formulæ are thereby materially simplified.

In this case B and C are the only unknown quantities, and they may be determined from a smaller number of observations, and also more easily from a larger number of observations, than they can be when D is also to be found. It is, therefore, important to be furnished with formulæ for this purpose.

If A and E are zero, equation (12) may be put under the form—

$$B \sin \zeta' + C \cos \zeta' = \sin \delta - D \sin (2\zeta' + \delta)$$

2. If the deviations are observed on any two compass azimuths ζ'_1 and ζ'_2 , we have—

$$B \sin \zeta'_1 + C \cos \zeta'_1 = \sin \delta_1 - D \sin (2\zeta'_1 + \delta_1)$$

$$B \sin \zeta'_2 + C \cos \zeta'_2 = \sin \delta_2 - D \sin (2\zeta'_2 + \delta_2)$$

Whence—

$$B = -\frac{\{\sin \delta_1 - D \sin (2\zeta'_1 + \delta_1)\} \cos \zeta'_2 + \{\sin \delta_2 - D \sin (2\zeta'_2 + \delta_2)\} \cos \zeta'_1}{\sin (\zeta'_2 - \zeta'_1)}$$

$$C = \frac{+\{\sin \delta_1 - D \sin (2\zeta'_1 + \delta_1)\} \sin \zeta'_2 - \{\sin \delta_2 - D \sin (2\zeta'_2 + \delta_2)\} \cos \zeta'_2}{\sin (\zeta'_2 - \zeta'_1)}$$

These equations cannot of course be used if the compass courses are nearly together or nearly opposite to each other.

3. If the observations are made on two adjacent cardinal compass courses, the expressions are very much simplified, as we have—

$$\text{Compass North} + C = \sin \delta_0 (1 - D)$$

$$,, \quad \text{East} + B = \sin \delta_8 (1 + D)$$

$$,, \quad \text{South} - C = \sin \delta_{16} (1 - D)$$

$$,, \quad \text{West} - B = \sin \delta_{24} (1 + D)$$

4. If we have observations on the four cardinal compass courses the best expressions to use are—

$$B = \frac{1}{2} (\sin \delta_8 - \sin \delta_{24}) (1 + D)$$

$$C = \frac{1}{2} (\sin \delta_0 - \sin \delta_{16}) (1 - D)$$

5. If the observations are made on the quadrantal compass courses—

$$+ B S_1 + C S_1 = \sin \delta_1 - D \cos \delta_1$$

$$+ B S_1 - C S_1 = \sin \delta_{12} + D \cos \delta_{12}$$

$$- B S_1 - C S_1 = \sin \delta_{20} - D \cos \delta_{20}$$

$$- B S_1 + C S_1 = \sin \delta_{28} + D \cos \delta_{28}$$

Whence—

$$\begin{aligned} + \mathfrak{B} + \mathfrak{C} &= \mathfrak{S}_4 (\sin \delta_4 - \sin \delta_{20}) - \mathfrak{T} \mathfrak{S}_4 (\cos \delta_4 - \cos \delta_{20}) \\ + \mathfrak{B} - \mathfrak{C} &= \mathfrak{S}_4 (\sin \delta_{12} - \sin \delta_{28}) + \mathfrak{T} \mathfrak{S}_4 (\cos \delta_{12} - \cos \delta_{28}) \end{aligned}$$

From which $\mathfrak{B}$ and $\mathfrak{C}$ are of course at once determined.

6. If the observations, more than two in number, are irregularly distributed we have the advantage of being able to apply the method of least squares without requiring excessive labour.

For this purpose let—

$$\Delta = \sin \delta - \mathfrak{D} \sin (2\zeta' + \delta)$$

or if $\mathfrak{U}$ and $\mathfrak{G}$ are not zero and are known, let—

$$\Delta = \sin \delta - \mathfrak{U} \cos \delta - \mathfrak{D} \sin (2\zeta' + \delta) - \mathfrak{G} \cos (2\zeta' + \delta)$$

and suppose we have observed the deviations on the compass courses $\zeta'_1 \zeta'_2 \dots \zeta'_n$, we have—

$$\begin{aligned} \Delta_1 &= \mathfrak{B} \sin \zeta'_1 + \mathfrak{C} \cos \zeta'_1 \\ \Delta_2 &= \mathfrak{B} \sin \zeta'_2 + \mathfrak{C} \cos \zeta'_2 \\ &\vdots \\ \Delta_n &= \mathfrak{B} \sin \zeta'_n + \mathfrak{C} \cos \zeta'_n \end{aligned}$$

whence—

$$\begin{aligned} \Sigma \Delta \sin \zeta' &= \mathfrak{B} \Sigma \sin^2 \zeta' + \mathfrak{C} \Sigma \sin \zeta' \cos \zeta' \\ &= \frac{1}{2} \mathfrak{B} (n - \Sigma \cos 2\zeta') + \frac{1}{2} \mathfrak{C} \Sigma \sin 2\zeta' \end{aligned}$$

so—

$$\begin{aligned} \Sigma \Delta \cos \zeta' &= \mathfrak{B} \Sigma \sin \zeta' \cos \zeta' + \mathfrak{C} \Sigma \cos^2 \zeta' \\ &= \frac{1}{2} \mathfrak{B} \Sigma \sin 2\zeta' + \frac{1}{2} \mathfrak{C} (n + \Sigma \cos 2\zeta') \end{aligned}$$

From this we get—

$$\begin{aligned} \mathfrak{B} &= 2 \frac{\frac{1}{2} \Sigma \Delta \sin \zeta' + \frac{1}{2} \Sigma \Delta \sin \zeta'_n \Sigma \cos 2\zeta' - \frac{1}{2} \Sigma \Delta \cos \zeta'_n \Sigma \sin 2\zeta'}{1 - (\frac{1}{2} \Sigma \sin \zeta')^2 - (\frac{1}{2} \Sigma \cos 2\zeta')^2} \\ \mathfrak{C} &= 2 \frac{\frac{1}{2} \Sigma \Delta \cos \zeta' - \frac{1}{2} \Sigma \Delta \cos \zeta'_n \Sigma \cos 2\zeta' - \frac{1}{2} \Sigma \Delta \sin \zeta'_n \Sigma \sin 2\zeta'}{1 - (\frac{1}{2} \Sigma \sin \zeta')^2 - (\frac{1}{2} \Sigma \cos 2\zeta')^2} \end{aligned}$$

In Part III., p. 62, we have given a tabular form for facilitating these calculations.

7. If in any vessel $\frac{\mathfrak{C}}{\mathfrak{B}} = \tan \alpha$ is known from observations made in any latitude, and if in that vessel α be known not to change on a change of latitude, then we can determine $\sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$, and therefore $\mathfrak{B}$ and $\mathfrak{C}$, from observing the deviation on one point on which there is a considerable deviation: for,

$$\begin{aligned} \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} &= \frac{\sin \delta - \mathfrak{D} \sin (2\zeta' + \delta)}{\sin (\zeta' + \alpha)} \\ \mathfrak{B} &= \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} \cos \alpha \\ \mathfrak{C} &= \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2} \sin \alpha \end{aligned}$$

APP. I.

APP. 1.

XV.

Determination of the Coefficients from Observations of Deviation and Horizontal Force.

The horizontal force may be conveniently measured by observing the time occupied by a horizontal needle in making a certain number of vibrations—say ten. Let T be the time of the vibrations on shore, T' on board, then—

$$\frac{H'}{H} = \frac{T^2}{T'^2}$$

And equations (4) and (5) may be put under the form.

$$\begin{aligned} + \frac{H'}{H} \cos \zeta' - \cos \zeta &= a \cos \zeta - b \sin \zeta + \lambda \mathfrak{B} \\ - \frac{H'}{H} \sin \zeta' + \sin \zeta &= d \cos \zeta - e \sin \zeta + \lambda \mathfrak{C} \end{aligned}$$

From these formulæ, if we have observations on the 32, 16, or 8 principal *magnetic* points, we may find a , b , $\lambda \mathfrak{B}$, d , e , and $\lambda \mathfrak{C}$.

And hence—

$$\begin{aligned} \lambda &= 1 + \frac{a+b}{2} \\ \lambda \mathfrak{D} &= \frac{a-e}{2} \\ \lambda \mathfrak{A} &= \frac{d-b}{2} \\ \lambda \mathfrak{E} &= \frac{d+b}{2} \end{aligned}$$

Or from the equation—

$$\frac{H'}{H} \sin \delta = \lambda \mathfrak{A} + \lambda \mathfrak{B} \sin \zeta + \lambda \mathfrak{C} \cos \zeta + \lambda \mathfrak{D} \sin 2\zeta + \lambda \mathfrak{E} \cos 2\zeta$$

we may determine $\lambda \mathfrak{A}$, $\lambda \mathfrak{B}$, $\lambda \mathfrak{C}$, $\lambda \mathfrak{D}$, $\lambda \mathfrak{E}$;

And from—

$$\frac{H'}{H} \cos \delta = \lambda + \lambda \mathfrak{B} \cos \zeta - \lambda \mathfrak{C} \sin \zeta + \lambda \mathfrak{D} \sin 2\zeta - \lambda \mathfrak{E} \cos 2\zeta$$

we may determine λ by taking the mean of all the values.

This method of determining $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$ can, however, be rarely useful, as the difficulty of obtaining accurate observations of force is much greater than that of obtaining equally accurate observations of deviation and the principal use of observations of horizontal force is two-fold.

1. When a sufficient number of observations of deviation are made to enable us to find $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ from the deviations, then the observation of horizontal force enables us to determine λ .

2. When observations of deviation are not sufficient in number to enable us to determine $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, we may supplement them by observations of horizontal force. APP. I.

1. *Determination of λ from one or more Observations of Horizontal Force, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, being known.*

From equation—

$$\lambda = \frac{H'}{H} \frac{\cos \delta}{1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta}$$

λ may be determined by one observation of force on board and one on shore.

If we have observations of horizontal force on two or more azimuths we may take the mean of the values of λ so given. If, however, the observations happen to be equally distributed round the circle the expression is simplified, and we then have—

$$\lambda = \frac{1}{n} \sum \frac{H'}{H} \cos \delta$$

or λ is the mean of the values of $\frac{H'}{H} \cos \delta$

2. *Determination of $\mathfrak{B}$ and $\mathfrak{C}$ from Observations of Deviation and Horizontal Force on one Azimuth, λ and $\mathfrak{D}$ being known or estimated, $\mathfrak{A}$ and $\mathfrak{E}$ neglected.*

As λ and $\mathfrak{D}$ do not change with a change of geographical position their values may be determined at a base station and afterwards used in other localities, or if not known the value of λ as well as $\mathfrak{D}$ may be estimated, and it is desirable to be furnished with formulæ adapted for this case.

From formulæ 4*b*, 5*b*, we have when $\mathfrak{A}$ and $\mathfrak{E}$ are zero—

$$\mathfrak{B} = + \frac{1}{\lambda} \frac{H'}{H} \cos \zeta' - (1 + \mathfrak{D}) \cos \zeta$$

$$\mathfrak{C} = - \frac{1}{\lambda} \frac{H'}{H} \sin \zeta' + (1 - \mathfrak{D}) \sin \zeta$$

Or—

$$\lambda \mathfrak{B} = + \frac{H'}{H} \cos \zeta' - (1 + a) \cos \zeta$$

$$\lambda \mathfrak{C} = - \frac{H'}{H} \sin \zeta' + (1 + e) \sin \zeta$$

These equations are given in both forms as it is sometimes more easy to estimate λ and $\mathfrak{D}$, sometimes a and e .

These formulæ have been found extremely useful in finding approximate values of the coefficients when a ship is on the stocks or in dock, and also when for scientific purposes we desire to trace the history of the changes which take place in the magnetism of a ship in circumstances in which

APP. 1. she cannot be swung. A table of the values of $\mathfrak{D}$, λ , a and e , for ships of different classes, is given in Part III., p. 73, by the aid of which estimates of the value of $\mathfrak{D}$ and λ or of a and e may be made.

3. *Determination of a and e (or $\mathfrak{D}$ and λ), and $\mathfrak{B}$ and $\mathfrak{C}$, from Observations of Deviation and Horizontal Force made on Two Courses $\mathfrak{A}$ and $\mathfrak{E}$ being neglected.*

We have in this case—

$$\lambda \mathfrak{B} = + \frac{H'_1}{H} \cos \zeta'_1 - (1+a) \cos \zeta_1$$

$$\lambda \mathfrak{B} = + \frac{H'_2}{H} \cos \zeta'_2 - (1+a) \cos \zeta_2$$

$$\lambda \mathfrak{C} = - \frac{H'_1}{H} \sin \zeta'_1 + (1+e) \sin \zeta_1$$

$$\lambda \mathfrak{C} = - \frac{H'_2}{H} \sin \zeta'_2 + (1+e) \sin \zeta_2$$

Whence—

$$1+a = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 - \frac{H'_2}{H} \cos \zeta'_2 \right\}}{\frac{1}{2} (\cos \zeta_1 - \cos \zeta_2)}$$

$$1+e = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 - \frac{H'_2}{H} \sin \zeta'_2 \right\}}{\frac{1}{2} (\sin \zeta_1 - \sin \zeta_2)}$$

$$\lambda \mathfrak{B} = + \frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 + \frac{H'_2}{H} \cos \zeta'_2 \right\} - (1+a) \frac{1}{2} (\cos \zeta_1 + \cos \zeta_2)$$

$$\lambda \mathfrak{C} = - \frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 + \frac{H'_2}{H} \sin \zeta'_2 \right\} - (1+e) \frac{1}{2} (\sin \zeta_1 + \sin \zeta_2)$$

And—

$$\lambda = \frac{1}{2} \{ (1+a) + (1+e) \}$$

$$\lambda \mathfrak{D} = \frac{1}{2} \{ (1+a) - (1+e) \}$$

If the magnetic courses are diametrically opposite $\cos \zeta_1 + \cos \zeta_2 = 0$
 $\sin \zeta_1 + \sin \zeta_2 = 0$ and we have—

$$1+a = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 - \frac{H'_2}{H} \cos \zeta'_2 \right\}}{\cos \zeta_1}$$

$$1+e = \frac{\frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 - \frac{H'_2}{H} \sin \zeta'_2 \right\}}{\sin \zeta_1}$$

$$\lambda \mathfrak{B} = \frac{1}{2} \left\{ \frac{H'_1}{H} \cos \zeta'_1 + \frac{H'_2}{H} \cos \zeta'_2 \right\}$$

$$\lambda \mathfrak{C} = - \frac{1}{2} \left\{ \frac{H'_1}{H} \sin \zeta'_1 + \frac{H'_2}{H} \sin \zeta'_2 \right\}$$

In these formulæ it is to be observed that unless we wish to determine λ it is not necessary to determine H , and that we may express H'_1 and H'_2 in any unit.

These formulæ may be used when *very good* observations of force and deviation can be obtained on two points, as, for instance, when a ship is lying in a rapid tideway, or before and after shifting position in a dock. The values of λ and $\mathfrak{D}$ derived from these equations will furnish a test of the exactness of the observations, because λ and $\mathfrak{D}$ may always be estimated with a considerable degree of approximation. If the values derived from the equations should appear to be impossible values, it will be better to assume more probable values of λ and $\mathfrak{D}$, and then from the four equations derive the two values of $\mathfrak{B}$ and $\mathfrak{C}$, and take the mean of the values so determined.

It must be observed that these formulæ fail when the two points are at equal distances from one of the cardinal points: if at equal distances from N. and S., because the expression for $1+a$ takes the form $\frac{0}{0}$ and gives no result; if at equal distances from E. and W., because the expression for $1+e$ takes the form $\frac{0}{0}$. We must therefore not take the two points on or near two opposite cardinal points, or two adjacent quadrantal points. The best points to select are two adjacent cardinal points, or two opposite quadrantal points, and in such cases very good results may be obtained from careful observations.*

XVI.

HEELING DEVIATION.

Let the ship heel to starboard and let i be the angle of heel.

Let X, Y, Z, represent as before the components of the earth's magnetic force to head, to starboard (in the plane of the horizon), and vertically downwards.

Let Y_i Z_i represent the components of the earth's magnetic force to starboard (in a plane parallel to the deck), and to keel.

Let Y'_i Z'_i represent the components of the combined magnetic force of earth and ship in the same directions, and

* The following expression derived from the above equations may be useful in finding the horizontal force at sea when, from the variation being unknown, or from the want of astronomical observations, ζ_1 and ζ_2 cannot be determined separately, but when by observation of a distant object $\zeta_1 - \zeta_2$ is known, λ and $\mathfrak{D}$ or a and e being supposed to be known.

$$H = \frac{\sqrt{(1-\mathfrak{D})^2 (\mathcal{H}'_1 \cos \zeta'_1 - \mathcal{H}'_2 \cos \zeta'_2)^2 + (1+\mathfrak{D})^2 (\mathcal{H}'_2 \sin \zeta'_2 - \mathcal{H}'_1 \sin \zeta'_1)^2}}{2\lambda(1-\mathfrak{D}^2) \sin \frac{\zeta_2 - \zeta_1}{2}}$$

$$= \frac{\sqrt{(1+e)^2 (\mathcal{H}'_1 \cos \zeta'_1 - \mathcal{H}'_2 \cos \zeta'_2)^2 + (1+a)^2 (\mathcal{H}'_2 \sin \zeta'_2 - \mathcal{H}'_1 \sin \zeta'_1)^2}}{2(1+a)(1+e) \sin \frac{\zeta_2 - \zeta_1}{2}}$$

APP. 1. — Let X' , Y' , Z' , represent the components of the combined magnetic force of earth and ship in the three first-mentioned directions.

Then it is required to obtain expressions for X' , Y' , Z' , in terms of X , Y , Z , corresponding to equations 1, 2, 3.

From the common formulæ for the transformation of coordinates and from the circumstance that equations 1, 2, 3, are applicable whenever the forces are resolved in the same directions with reference to the ship, we have—

$$\begin{aligned} Y_i &= Y \cos i + Z \sin i \\ Z_i &= Z \cos i - Y \sin i \\ X' &= X + aX + bY_i + cZ_i + P \\ Y'_i &= Y_i + dX + eY_i + fZ_i + Q \\ Z'_i &= Z_i + gX + hY_i + kZ_i + R \\ Y' &= Y'_i \cos i - Z'_i \sin i \\ Z' &= Z'_i \cos i + Y'_i \sin i \end{aligned}$$

Eliminating Y_i , Z_i , Y'_i , Z'_i , and putting $1 - \sin^2 i$ for $\cos^2 i$ we have—

$$\begin{aligned} X' &= X + aX + (b \cos i - c \sin i) Y + (c \cos i + b \sin i) Z + P \\ Y' &= Y + (d \cos i - g \sin i) X \\ &\quad + \{e - (f + h) \cos i \sin i - (e - k) \sin^2 i\} Y \\ &\quad + \{f + (e - k) \cos i \sin i - (f + h) \sin^2 i\} Z \\ &\quad + Q \cos i - R \sin i \\ Z' &= Z + (g \cos i + d \sin i) X \\ &\quad + \{h + (e - k) \cos i \sin i - (f + h) \sin^2 i\} Y \\ &\quad + \{k + (f + h) \cos i \sin i + (e - k) \sin^2 i\} Z \\ &\quad + R \cos i + Q \sin i \end{aligned}$$

These are of the form :—

$$\begin{aligned} X' &= X + a_i X + b_i Y + c_i Z + P_i \\ Y' &= Y + d_i X + e_i Y + f_i Z + Q_i \\ Z' &= Z + g_i X + h_i Y + k_i Z + R_i \end{aligned}$$

If,—

$$\begin{aligned} a_i &= a \\ b_i &= b \cos i - c \sin i \\ c_i &= c \cos i + b \sin i \\ d_i &= d \cos i - g \sin i \\ e_i &= e - (f + h) \cos i \sin i - (e - k) \sin^2 i \\ f_i &= f + (e - k) \cos i \sin i - (f + h) \sin^2 i \\ g_i &= g \cos i + d \sin i \\ h_i &= h + (e - k) \cos i \sin i - (f + h) \sin^2 i \\ k_i &= k + (f + h) \cos i \sin i + (e - k) \sin^2 i \\ P_i &= P \\ Q_i &= Q \cos i - R \sin i \\ R_i &= R \cos i + Q \sin i \end{aligned}$$

And if $\lambda_i, \mathfrak{D}_i, \mathfrak{N}_i, \mathfrak{G}_i, \mathfrak{B}_i, \mathfrak{C}_i, \mathfrak{V}_i$, represent the altered values of $\lambda, \mathfrak{D}$, &c., APP. 1.

$$\begin{aligned}\lambda_i &= \lambda - \frac{f+h}{2} \cos i \sin i - \frac{e-k}{2} \sin^2 i \\ \lambda_i \mathfrak{D}_i &= \lambda \mathfrak{D} + \frac{f+h}{2} \cos i \sin i + \frac{e-k}{2} \sin^2 i \\ \lambda_i \mathfrak{N}_i &= \lambda \mathfrak{N} \cos i + \frac{c-g}{2} \sin i \\ \lambda_i \mathfrak{G}_i &= \lambda \mathfrak{G} \cos i - \frac{c+g}{2} \sin i \\ \lambda_i \mathfrak{B}_i &= \lambda \mathfrak{B} + \left\{ b \sin i - c \operatorname{versin} i \right\} \tan \theta \\ \lambda_i \mathfrak{C}_i &= \lambda \mathfrak{C} + \left\{ (e-k) \cos i \sin i - \frac{R}{Z} \sin i - (f+h) \sin^2 i \right\} \tan \theta + \frac{Q}{H} \operatorname{versin} i \\ \lambda_i \mathfrak{V}_i &= \lambda \mathfrak{V} + \left\{ (f+h) \cos i \sin i - \frac{R}{Z} \operatorname{versin} i + (e-k) \sin^2 i \right\} \tan \theta + \frac{Q}{H} \sin i.\end{aligned}$$

If the soft iron be symmetrically arranged on each side of the fore and aft midship line,—

$$b=0, \quad d=0, \quad f=0, \quad h=0, \quad \mathfrak{N}=0, \quad \mathfrak{G}=0;$$

and if i be so small that its square and higher powers may be neglected—

$$\sin i = i, \quad \cos i = 1, \quad \operatorname{versin} i = 0 \quad \sin^2 i = 0;$$

And—

$$\begin{aligned}\lambda_i &= \lambda \\ \mathfrak{D}_i &= \mathfrak{D} \\ \mathfrak{N}_i &= + \frac{c-g}{2\lambda} i \\ \mathfrak{G}_i &= - \frac{c+g}{2\lambda} i \\ \mathfrak{B}_i &= \mathfrak{B} \\ \mathfrak{C}_i &= \mathfrak{C} + \frac{1}{\lambda} \left(e-k - \frac{R}{Z} \right) \tan \theta i \\ &= \mathfrak{C} + J i * \\ \mathfrak{V}_i &= \mathfrak{V} + \frac{Q}{\lambda H} i\end{aligned}$$

Hence if δ represent the deviation for a given compass course ζ' , when the ship is on even keel, δ , the deviation for the same compass course when the ship heels i to starboard.

$$\begin{aligned}\delta_i &= \delta + \frac{c-g}{2\lambda} i + J i \cos \zeta' - \frac{c+g}{2\lambda} i \cos 2\zeta' \\ &= \delta + J i \cos \zeta' + \frac{c}{\lambda} i \sin^2 \zeta' - \frac{g}{\lambda} i \cos^2 \zeta'\end{aligned}$$

* In the last edition this expression was printed $C - \chi i$; $J = -\chi$ is used in this edition as it is more in analogy with the other uses of $+$ and $-$, that a $+$ quantity should indicate a deviation of the north end of the needle to starboard when the heel is to starboard. $-J = \chi$ is therefore the heeling coefficient to *windward*.

APP. 1. Here we see that the effect of heeling besides altering the $\mathcal{C}$ is to introduce a constant term, and a quadrantal term of the $\mathcal{E}$ type, and this because the heeling of the ship puts an end to the symmetry of arrangement, on each side of the fore and aft line, which causes the $\mathcal{A}$ and $\mathcal{C}$ to vanish when the ship is on even keel. And in fact, if we consider the position the nine soft iron rods take up when the ship heels, and their projections on the horizontal plane, we shall see that the c introduces a $-b = ci$, and g introduces a $d = -gi$, and it is these that give rise to the $\mathcal{A}$, and $\mathcal{E}$. c and g are generally + and do not often exceed .100 each. If we assume $\frac{c}{\lambda} = .100$ and $\frac{g}{\lambda} = .100$, we see the first will introduce a heeling error to east when the ship heels to starboard and to west when she heels to port, and which is a maximum when the ship's head is east or west, and is then about $\frac{1}{10}$ th of the angle of heel, or 6' for each degree of heel. This error will disappear if the compass be in a position in which no part of the semicircular deviation is caused by soft iron. $\frac{g}{\lambda}$ will introduce a heeling error to west when the ship heels to starboard, and to east when she heels to port, which will be a maximum when her head is north or south, and will then be about $\frac{1}{10}$ th of the angle of heel, or 6' for each degree of heel. By considering the position of the rod g , it will be seen that the deviation caused by it disappears when the compass is near the middle of the ship, it will be greatest when the compass is near the stern, and will act in the opposite direction to that which has been described when the compass is near the bow. These errors may be safely neglected in ordinary cases in the navigation of a ship.

But the case is very different with the alteration of $\mathcal{C}$ or the heeling error represented by $Ji \cos \zeta'$. When J is $-$, as it usually is with compasses on the upper deck, this represents a deviation of the north point of the compass to windward; when J is $+$, as it often is in compasses on the main deck of an iron ship, it represents a deviation of the north point of the compass to leeward. This deviation is of course a maximum when a ship's head is north or south, and is zero when it is east or west.

The expression,—

$$J = \frac{1}{\lambda} \left(e - k - \frac{R}{Z} \right) \tan \theta$$

may be put under the several forms following (in which the signs on both sides have been changed in order that the positive values of the quantities on the right hand side may indicate, what is the usual case in northern latitudes, a deviation of the *north* end of the needle to *windward*).

$$-J = \frac{1}{\lambda} \left\{ (-e + k) \tan \theta + \frac{R}{H} \right\} \quad . \quad . \quad . \quad (1)$$

$$= -\frac{eZ}{\lambda H} + \frac{kZ + R}{\lambda H} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= \left(\mathfrak{D} + \frac{1}{\lambda} - 1 \right) \tan \theta + \frac{\mu - 1}{\lambda} \tan \theta \quad . \quad . \quad . \quad (3)$$

$$= \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$= \frac{\mu - 1 - e}{\lambda} \tan \theta \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In these expressions—

— J is the heeling *coefficient* to windward ;

— Ji being the heeling *error* to windward.

— J is therefore generally a fractional number: thus for the standard compass of the *Warrior* $-J = 1 \cdot 82$, indicating a heeling deviation of $1 \cdot 82$ degrees, or $1^\circ 49'$ for each degree of heel. It is in the last form that the heeling coefficient is generally tabulated.

The different forms for $-J$ are useful for different purposes.

$$-J = \frac{1}{\lambda} \left\{ (-e + k) \tan \theta + \frac{R}{H} \right\} \quad . \quad . \quad . \quad (1)$$

shows most conveniently the changes which may be expected in $-J$ on a change of magnetic latitude.

At the usual position of a standard compass in an iron ship built in England, $-e$, $+k$, and $+R$ are positive; $\tan. \theta$ is $+$ in the northern and $-$ in the southern magnetic hemisphere; λ and H are always $+$. Hence in general the heeling error will be to windward till the ship has penetrated so far into south magnetic latitudes that $(-e + k) \tan. \theta$ is greater than $\frac{R}{H}$.

$$-J = -\frac{e Z}{\lambda H} + \frac{k Z + R}{\lambda H} \quad . \quad . \quad . \quad . \quad (2)$$

shows the causes of the two parts of which the heeling error is made up.

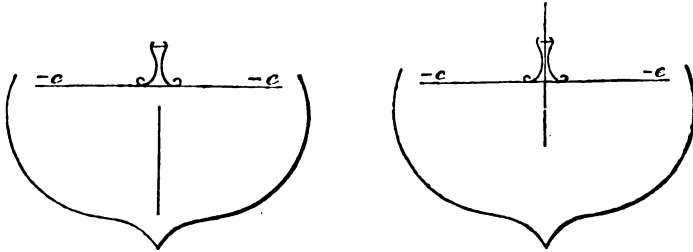
— $\frac{e Z}{\lambda H} i$ expresses the effect of the earth's vertical force, inducing magnetism in horizontal iron represented by the rod $-e$, inclined at an angle i to the horizon, and acting against a directive force λH .

$\frac{k Z + R}{\lambda H} i$ expresses the effect of the earth's vertical force, inducing magnetism in vertical iron represented by the rod $+k$, and of the permanent force to keel of the ship (R); each acting at an angle i from the vertical and against a directive force λH .

As $-e$ is always $+$ in an uncorrected compass, the first part produces, in such compasses, a heeling error to windward in north and to leeward in south magnetic latitudes.

The second part produces a heeling error to windward or to leeward according as $k Z + R$ is $+$ or $-$, i.e., according as the ship's vertical force

APP. 1. acts downwards, as represented in the first figure in the margin, or upwards, as represented in the second.



$$-J = \left(\mathfrak{D} + \frac{1}{\lambda} - 1 \right) \tan \theta + \frac{\mu - 1}{\lambda} \tan \theta \quad . \quad . \quad . \quad (3)$$

is the most convenient form for computing separately the two parts of the heeling coefficient.

$$\left(\mathfrak{D} + \frac{1}{\lambda} - 1 \right) \tan \theta = -\frac{e Z}{\lambda H}$$

is the part which arises from horizontal induction in soft iron.

$$\frac{\mu - 1}{\lambda} \tan \theta = \frac{k Z + R}{\lambda H}$$

is the part which arises from vertical induction in soft iron and vertical permanent magnetism.

$$-J = \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta \quad . \quad . \quad . \quad (4)$$

is the most convenient form for computing the whole heeling coefficient to windward. It may also be put under the form—

$$-J = \frac{\mu - 1 - e}{\lambda} \tan \theta \quad . \quad . \quad . \quad (5)$$

which may occasionally be found more convenient when some of the quantities involved have to be estimated.

In order that there may be no semicircular heeling error we must have—

$$\mathfrak{D} + \frac{\mu}{\lambda} - 1 = 0$$

or—

$$\begin{aligned} \mu &= \lambda (1 - \mathfrak{D}) \\ &= 1 + e \end{aligned}$$

or—

$$\mu - 1 = e.$$

In other words we must have—

$$\begin{aligned} \mu &= \frac{\text{Mean vertical force of earth and ship}}{\text{Vertical force of earth}} = \lambda (1 - \mathfrak{D}) \\ &= 1 + e \end{aligned}$$

or—

$$\mu - 1 = \frac{\text{Mean vertical force of ship}}{\text{Vertical force of earth}} = e.$$

As e is always negative when the quadrantal deviation has not been corrected mechanically and is less than unity, these expressions show that when the quadrantal deviation has not been corrected, the mean vertical force at the compass must be less than the vertical force on shore, or if T be the time of ten vibrations of a dipping needle vibrating in an E. and W. plane on shore, and T' on board with the ship's head E. or W., we must, in order that there may be no heeling error, have T' greater than T .*

Formulae for the Mechanical Correction of the Heeling Error.—This correction it will be seen may be effected by altering the vertical force so as to reduce the heeling coefficient to zero. This may be done by introducing a vertical magnet immediately below the compass; the position of this magnet is the point to be determined.

There are two cases for which formulæ are required.

CASE I.

When the Observation of Vertical Force and the placing the Magnet are performed at different times.

Let R' be the force downwards exerted by the magnet when in position, $-J$ will become—

$$-J = 0 = \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta - \frac{R'}{\lambda H}$$

whence—

$$\frac{R'}{\lambda H} = \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$$

or—

$$\frac{R'}{H} = \lambda \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$$

R' depends partly on the force of the magnet, partly on the distance below the compass at which it is placed, and when the magnet is given this distance may be found:—

* Mr. Towson has observed (Practical Information for Masters and Mates, pp. 36 and 111,) that in an uncompensated compass when the vertical force on board is nine-fourteenths of that on shore, the compass will not be considerably affected by heeling. This gives for the ships observed by Mr. Towson—

$$\left. \begin{aligned} 1 + e &= \frac{9}{14} \\ &= \cdot 643 \\ e &= -\cdot 357 \end{aligned} \right\} \text{approximately.}$$

This it will be seen from the Table in p. 73 is a higher value of $-e$ than is usually found at the position of compasses in H.M. ships, but not higher than is found at the steering and main-deck compasses of iron-built armour-plated ships having iron decks.

APP. 1. 1. *On shore* by placing it east or west of a compass, its axis being directed towards the compass, and moving it to or from the compass till it produces a deviation of which the natural tangent is—

$$\lambda \left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$$

2. *On board* by placing it in the same way till it produces a deviation of which the natural tangent is—

$$\left(\mathfrak{D} + \frac{\mu}{\lambda} - 1 \right) \tan \theta$$

The distance of the magnet from the centre of the compass in either of these observations is the distance which the magnet should be placed below the compass to correct the heeling error.

CASE II.

When the Observation of Vertical Force and the fixing the Magnet are performed at the same time.

In this case we move the vertical magnet, which is placed below the compass,* up and down till the vertical vibrations of the dipping needle take place in the proper time, and this is ascertained as follows:—

Equation (6 b) gives us, λ being zero as the compass is supposed to be amidship—

$$\frac{Z'}{Z} = \frac{T^2}{T'^2} = \mu + \frac{g}{\tan \theta} \cos \zeta$$

whence—

$$T' = \frac{T}{\sqrt{\mu + g \cot \theta \cos \zeta}}$$

T being the time of ten vibrations of the dipping needle in an E. and W. plane on shore, T' on board.

But when the heeling error is corrected—

$$\mu = \lambda (1 - \mathfrak{D}) = 1 + e$$

and therefore—

$$T' = \frac{T}{\sqrt{\lambda (1 - \mathfrak{D}) + g \cot \theta \cos \zeta}} \\ = \frac{T}{\sqrt{1 + e + g \cot \theta \cos \zeta}}$$

If the ship's head is E. or W. (magnetic) $\cos \zeta = 0$, and—

$$T' = \frac{T}{\sqrt{\lambda (1 - \mathfrak{D})}} \\ = \frac{T}{\sqrt{1 + e}}$$

* Instead of placing the magnet below the compass we might place it at the same level, inverting its poles; but in that case for the same distance a magnet of double power would be required, and any error in the verticality of the magnet would produce a deviation four times as great as a similar error when the magnet is below the compass.

If we have not the means of determining λ and $\mathfrak{D}$ we must estimate e , in doing which the table in page 73 will help us. For the standard compass of an iron-built ship we may take $e = -.200$, $1+e = .800$, $\sqrt{1+e} = .900$, whence the mean vertical force ought to be reduced to $\frac{9}{10}$, and the number of vertical vibrations of the dipping needle in a given time to $\frac{9}{10}$ of that on shore. APP. 1.

If the quadrantal deviation has been corrected by Mr. Airy's method, $\mathfrak{D} = 0$ or $e = a$, this may be taken as $-.050$ nearly at the standard compass of an iron-built ship, whence from such a ship the vertical force ought to be reduced to $\frac{19}{20}$, and the number of vertical vibrations of the dipping needle in a given time to $\frac{19}{20}$ of that on shore.

Effect of the Mechanical Correction of the Quadrantal Deviation by Transverse Soft Iron on the Heeling Error.

The amount of the heeling deviation to windward for each degree of heel which will be corrected by the correction of one degree of quadrantal deviation by soft iron correctors will be found to be very nearly—

$$\frac{2 \cdot \tan \theta \text{ degrees}}{57 \cdot 3}$$

or—

$$2 \cdot 1 \tan \theta \text{ minutes ;}$$

in the south of England this is about $5\frac{1}{4}$ minutes of heeling deviation for each degree of heel corrected by the correction of one degree of quadrantal deviation.

Comparison of Observation with Theory.—This comparison may be made—

- I. By swinging the ship on two angles of heel and computing the coefficients.
- II. By observing the deviation with the ship's head on the same compass point, on two angles of heel.
- III. By observing the deviation with the ship's head on the same magnetic point, on two angles of heel.

Of these methods the first is the best where practicable. The second and third require g to be known. One of the angles of heel may be zero, or the ship may be on an even keel, but it is better to have observations on one angle of heel to starboard and one to port.

I. Let i, i' , be the two angles of heel ; $\mathfrak{A}, \mathfrak{C}, \mathfrak{E}, \mathfrak{A}', \mathfrak{C}', \mathfrak{E}'$, the altered values of $\mathfrak{A}, \mathfrak{C}$, and $\mathfrak{E}$.

APP. 1.

Then—

$$J = + \frac{C_i - C_{i'}}{i - i'}$$

$$\frac{c - g}{2\lambda} = + \frac{\alpha_i - \alpha_{i'}}{i - i'}$$

$$\frac{c + g}{2\lambda} = - \frac{\beta_i - \beta_{i'}}{i - i'}$$

Whence—

$$\frac{c}{\lambda} = \frac{1}{i - i'} \left\{ + \alpha_i - \alpha_{i'} - \beta_i + \beta_{i'} \right\}$$

$$\frac{g}{\lambda} = \frac{1}{i - i'} \left\{ - \alpha_i + \alpha_{i'} - \beta_i + \beta_{i'} \right\}$$

II. If the ship be heeled over, and its head then brought to the previous *compass course*, so that $\zeta'_i = \zeta'$, $\zeta_i = \zeta + \delta_i - \delta$, we shall find—

$$J \cos \zeta' = + \frac{\sin \delta_i - \sin \delta_{i'}}{\sin i - \sin i'} \left\{ 1 - \mathfrak{D} \cos (\zeta + \zeta') \right\}$$

$$- \frac{c}{\lambda} \sin \zeta \sin \zeta' + \frac{g}{\lambda} \cos \zeta \cos \zeta'$$

hence at north $+J$, and at south $-J$

$$= + \frac{\sin \delta_i - \sin \delta_{i'}}{\sin i - \sin i'} (1 - \mathfrak{D}) + \frac{g}{\lambda}$$

and at east and west—

$$\frac{c}{\lambda} = \frac{\sin \delta_i - \sin \delta_{i'}}{\sin i - \sin i'} (1 + \mathfrak{D})$$

which gives the means of determining c .

III. If the ship be heeled over without altering her *magnetic course*, so that $\zeta_i = \zeta$, $\zeta'_i = \zeta' + \delta - \delta_i$, we shall find—

$$J \cos \zeta' = + \frac{\sin \delta_i - \sin \delta_{i'}}{\sin i - \sin i'} \left\{ 1 + \mathfrak{B} \cos \zeta' - C \sin \zeta' + \mathfrak{D} \cos (\zeta + \zeta') \right\}$$

$$- \frac{c}{\lambda} \sin \zeta \cos \zeta' + \frac{g}{\lambda} \cos \zeta \cos \zeta'$$

Determination of the Value of μ and g .

1. Equidistant Azimuths.

The value of μ , g , and h is derived from the equation—

$$\frac{Z'}{Z} = \frac{g}{\tan \theta} \cos \zeta - \frac{h}{\tan \theta} \sin \zeta + \mu \dots (6b)$$

If we have observations of $\frac{Z'}{Z}$ on any number (*i. e.*, two or more) equidistant magnetic courses, μ is the mean of all the values of $\frac{Z'}{Z}$ so observed.

If these observations are made on 4, 8, 16, or 32 equidistant courses APP. 1.
it will be seen by comparing the formulæ that we may obtain $\frac{g}{\tan \theta}$ and
 $-\frac{h}{\tan \theta}$ by the same tabular forms by which B and C are obtained from
observations of deviation on a like number of compass courses.

The circumstance that μ is the mean of two values of $\frac{Z'}{Z}$ observed on two
opposite azimuths furnishes an easy mode of obtaining it with great accu-
racy when a ship is lying in a tideway. In this case, by making one
observation during flood tide, the other during ebb, steering the ship if
necessary by the rudder, the value may generally be determined with great
accuracy.

2. Two Azimuths not equidistant.

If we have not observations of $\frac{Z'}{Z}$ on equidistant azimuths, it will be
best to suppose h to be zero, we have then—

$$\mu = \frac{Z'}{Z} - \frac{g}{\tan \theta} \cos \zeta$$

By this we may determine μ by one observation if g be known, or if
the observation be made with the ship's head east or west (magnetic).

If we have observations of $\frac{Z'}{Z}$ on any two magnetic courses ζ_1 and ζ_2 ,
then we have—

$$\mu = \frac{\frac{1}{2} \left\{ \frac{Z'_1}{Z} \cos \zeta_2 - \frac{Z'_2}{Z} \cos \zeta_1 \right\}}{\frac{1}{2} \{ \cos \zeta_2 - \cos \zeta_1 \}}$$

and—

$$\frac{g}{\tan \theta} = \frac{\frac{1}{2} \left\{ \frac{Z'_1}{Z} - \frac{Z'_2}{Z} \right\}}{\frac{1}{2} \{ \cos \zeta_1 - \cos \zeta_2 \}}$$

3. Three or more Azimuths not equidistant.

If we have observations on three or more azimuths, then from the
equations—

$$\mu = \frac{Z'_1}{Z} - \frac{g}{\tan \theta} \cos \zeta_1$$

$$\mu = \frac{Z'_2}{Z} - \frac{g}{\tan \theta} \cos \zeta_2$$

⋮

$$\mu = \frac{Z'_n}{Z} - \frac{g}{\tan \theta} \cos \zeta_n$$

APP. 1. We have by the method of least squares for the most probable values—

$$u = \frac{\sum \frac{Z'}{Z} \sum \cos^2 \zeta - \sum \frac{Z'}{Z} \cos \zeta \sum \cos \zeta}{n \sum \cos^2 \zeta - (\sum \cos \zeta)^2}$$

$$\frac{g}{\tan \theta} = \frac{n \sum \frac{Z'}{Z} \cos \zeta - \sum \frac{Z'}{Z} \sum \cos \zeta}{n \sum \cos^2 \zeta - (\sum \cos \zeta)^2}$$

where—

$$\sum \frac{Z'}{Z} = \frac{Z'_1}{Z} + \frac{Z'_2}{Z} \dots + \frac{Z'_n}{Z}$$

and so of the others.

4. Graphic Method.

We may also find the most probable value of μ and g graphically with great ease as follows :—

Take a horizontal line of any convenient length, marked N. at the left hand, S. at the right; on this as diameter describe a circle marked E. at the top, W. at the bottom; suppose it divided as a compass card.

Take on the circle the magnetic course on which each observation is made, and through each such course draw a vertical line. Let the least value of $\frac{Z'}{Z}$ be indicated by a point on the horizontal line, and lay down the excess of each of the other observed values on the vertical line passing through the course to which that value belongs as a vertical ordinate on any convenient scale, marking the end with a cross or dot. Then draw a straight line passing as nearly as possible through all crosses or dots.

The ordinate to this line at E. or W. or the mean of the ordinates at N. and S. added to the least value of $\frac{Z'}{Z}$ will give μ ; half the difference of the ordinates at N. and S. will give $\frac{g}{\tan \theta}$, which will be + if the ordinate at N. is the greatest.

If h has a sensible value the crosses or dots will lie on an elongated ellipse of which $\frac{h}{\tan \theta}$ will be the vertical semi-diameter.

APPENDIX No. 2.

GRAPHIC REPRESENTATIONS OF THE AMOUNT AND DIRECTION OF THE FORCES WHICH ACT ON THE MAGNETIC NEEDLE.

These representations are designated by the general name of "Dylogram," a contraction for "Dynamo-gonio-gram," or "Force and Angle Diagram." There are several distinct forms of dylogram, each of which has its peculiar advantages, they are,—

- I.—DYLOGRAM No. I., or the LIMAÇON of PASCAL.*
- II.—The ELLIPSE AND CIRCLE.
- III.—DYLOGRAM No. II., or the TWO CIRCLES.
- IV.—The STEERING DYLOGRAM.

Dylogram No. I. is the most easily understood of these, and it is that which shows most clearly the effect of the different parts of which the deviation is composed. It is specially adapted to those cases in which $\mathfrak{A}$ and $\mathfrak{C}$ have sensible values, and to those cases in which observations of horizontal force, as well as of deviation, are made; and when constructed it gives most readily the deviations on the different courses. The construction, however, occupies some time, and a separate figure is required for each case.

The ellipse and circle is principally useful as a necessary step to Dylogram No. II., but there are certain cases to which it is specially applicable; the principal of which are those in which a large number of observations of force and deviation on different azimuths are made; and also when it is desired to show to the eye the comparison of the magnetic state of different ships, and of different compasses in the same ship, not only as regards deviation, but also as regards directive force.

Dylogram No. II. is only of practical use in those cases in which $\mathfrak{A}$ and $\mathfrak{C}$ may be considered zero. It is specially adapted to cases in which observations of deviation alone are made, and in such cases it enables us from a few observations to arrive with great ease at a probable estimate of the value of $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$. It has also the advantage of being easily and rapidly constructed, and of giving on one diagram a comparative view of the magnetic condition of different ships, of different compasses in the same ship, and of the same compass at different times.

The steering dylogram is merely a device for making use of Dylogram No. II. for the purpose of steering a ship.

* So called by Roberval; Memoires de l'Academie, Tome VI., p. 42. This curve is of some celebrity in the history of mathematics. It is at once a conchoid of the circle, an epitrochoid, and a Cartesian oval. When $p=q$ it gives, as was shown by Pascal, a solution of the problem of the trisection of an angle; when $p=2q$ it is a caustic by reflexion of the circle.

APP. 2.

DYGOGRAM No. I.

Is the geometrical construction of equations 9 and 10 of Appendix I.

Let O be the origin of co-ordinates. Let the axis of x or OX be directed to the magnetic north, the axis of y or OY to the magnetic east. Let ζ be the magnetic azimuth of the ship's head measured from north to east. Let R be a point of which the co-ordinates are,—

$$OL = x = 1 + \mathfrak{B} \cos \zeta - \mathfrak{C} \sin \zeta + \mathfrak{D} \cos 2\zeta - \mathfrak{E} \sin 2\zeta \dots (10)$$

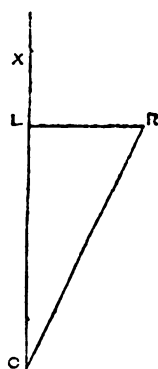
$$LR = y = \mathfrak{A} + \mathfrak{B} \sin \zeta + \mathfrak{C} \cos \zeta + \mathfrak{D} \sin 2\zeta + \mathfrak{E} \cos 2\zeta \dots (9)$$

Then we see from equations 10 and 9 that $OL = x = \frac{H'}{\lambda H} \cos \delta$ represents the force of earth and ship to north in terms of the "mean force to north," or λH , as unit, $LR = y = \frac{H'}{\lambda H} \sin \delta$ represents the force of earth and ship to east in terms of the same unit. $OR = \sqrt{x^2 + y^2} = \frac{H'}{\lambda H}$ represents the whole horizontal force of earth and ship in terms of the same unit.

XOR represents the deviation = $\zeta - \zeta' = \delta$.

If we give ζ every value from 0° to 360° , i.e., if we suppose the ship's head to move in azimuth about the point O, the point R will trace out the curve which we have called Dygogram No. I., and which is in fact, as will be seen in the sequel, an epitrochoid.

Fig. 1.



To construct Dygogram No. 1, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, being known.

First (Colongue's) method.*

From O draw OP upwards, OP being any length conveniently divisible by scale into 100 parts and by estimate into 1000 parts, as 5 inches, $3\frac{1}{8}$ inches or 100 millimetres. In the figures OP is 25 millimetres.

From P draw PA = $\mathfrak{A}$ to the right if $\mathfrak{A}$ be +, to the left if —.

„ A „ AE' = $\mathfrak{E}$ to the right if $\mathfrak{E}$ be +, to the left if —.

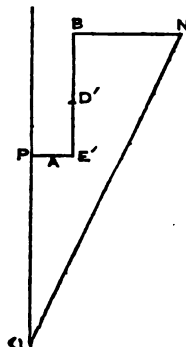
„ E' „ E'D' = $\mathfrak{D}$ upwards if $\mathfrak{D}$ be, as it generally is, +.

„ D' „ D'B = $\mathfrak{B}$ upwards if $\mathfrak{B}$ be +, downwards if —.

„ B „ BN = $\mathfrak{C}$ to the right if $\mathfrak{C}$ be +, to the left if —.

Join ON.

Fig. 2.



* This modification of the method of the 1st and 2nd editions is due to Lieut. Colongue, R.I.N., of the Imperial Compass Observatory, Cronstadt. See the other method, post, p. 166.

It will easily be seen that N is the point in the dygogram corresponding to $\zeta = 0$ or the north (magnetic) course; ON represents the force on that course; PON the deviation.

To find the other points in the dygogram we proceed as follows :—

With centre A and radius AD' describe a circle, which may be called for distinction the generating circle. Join ND' and produce it to S so that D'S = D'N. S it will easily be seen is the point on the dygogram corresponding to $\zeta = 180$ or to the south (magnetic) course.*

The line NS will intersect the generating circle in a second point Q, which is a point of great importance in the dygogram, and is called the *pole* of the dygogram.

Take a straight edge of paper of length NS, and mark its centre. Move it so that its centre moves on the circumference of the generating circle, the edge always passing through the pole Q. The curve traced out by its extremities is the dygogram, which may be easily drawn by points.

When the dygogram is drawn it is desirable for the sake of clearness to draw the line NS, and also a line from W to E at right angles to it, and likewise passing through Q. These two lines indicate with unmistakable clearness the properties of the point Q, and the direction in which the radius vector corresponding to any other magnetic course must be drawn.

To find the point on the dygogram corresponding to any given magnetic course.

The dygogram being constructed, from Q draw QR, cutting the dygogram in R, and making the angle $NQR = \zeta$.

Fig. 3.

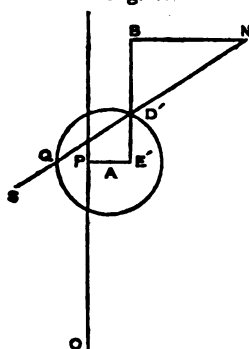
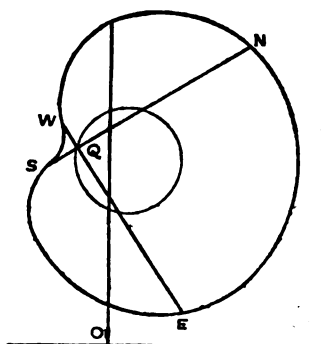


Fig. 4.



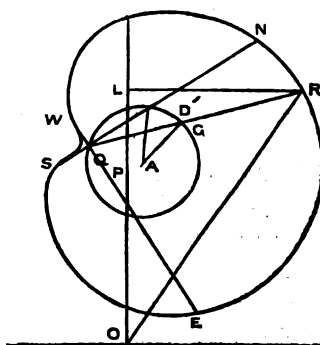
* For this and the following constructions it is convenient to have, in addition to an ivory scale properly divided, a small scale known as an "off-set" used by draughtsmen to set off perpendiculars to any line. A protractor, an ivory scale about six inches long, and one or better two off-sets about two and three inches long, together with a pair of pen compasses, are the instruments required.

APP. 2. Then R is the point in the dygogram corresponding to a magnetic azimuth ζ . OR is the directive force on that azimuth, on the same scale on which OP represents the mean force to north; POR is the deviation.

Demonstration.

Let QR cut the generating circle in G and join AD', AG.
From R let fall on OP a perpendicular RL.

Fig. 5.



Then by Euclid, Book III., prop. 20, $D'AG = 2D'QG = 2\zeta$. R is therefore the point obtained by turning the line AD' through an angle 2ζ , while the line D'N, which may be supposed jointed to the extremity of AD', turns through an angle ζ , and consequently it will easily be seen that,—

$$OL = 1 + B \cos \zeta - C \sin \zeta + D \cos 2\zeta - E \sin 2\zeta$$

$$LR = A + B \sin \zeta + C \sin \zeta + D \sin 2\zeta + E \cos 2\zeta$$

R is therefore the point on the dygogram corresponding to the magnetic azimuth ζ .

To obtain from Dygogram No. I. the Deviations on the 32 Magnetic Courses.

Having constructed the dygogram, draw through Q lines $QR_1, QR_2, \&c.$, making angles of $11^\circ 15'$ with QN and with each other; and join $ON_1, OR_1, OR_2, \&c.$; then the angles PON, POR_1, POR_2 , are the deviations on the N., N. by E., $\&c.$, magnetic courses.

$ON, OR_1, OR_2, \&c.$, represent the directive force of earth and ship while the ship's head is on these courses, on the same scale on which $OP = \lambda H$ is the "mean force to north."

To obtain the Deviations on the 32 Compass Courses.

APP. 2.

This may in general be most conveniently done by obtaining the deviations on the *magnetic* courses, laying them down on the plain lines of the Napier curve, drawing the curve, and then reading off the deviations on the compass courses from the dotted lines.

Approximate values of the deviation on any compass course ζ' may be found as follows:—

Take a point R' on the dygogram, such that $NQR' = \zeta'$, and draw $R'R''$ parallel to PO , cutting the arc of the circle, drawn with centre O through P , in R'' , then it will be seen that—

$$\sin POR'' = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'$$

POR'' is therefore very nearly the deviation for the *compass* course ζ' .

If instead of drawing the arc of the circle through P we draw it with O as centre and radius $OP - AD' \cos 2\zeta' = 1 - \sqrt{D^2 + E^2} \cos 2\zeta'$, then as will be seen from equation (13a) the angle obtained will be extremely near the deviation on the given compass course.*

* The following constructions are due to Lieut. Colongue.

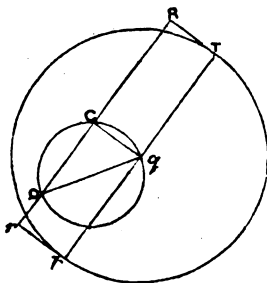
To construct the Dygogram No. I. by means of the Polar Circle.

Construction.—From Q , the pole, draw Qq , a diameter of the generating circle.

With q as centre and radius $= \sqrt{B^2 + C^2}$ describe a circle. This circle, from its representing the effect of polar magnetism, may be called the polar circle.

Then the dygogram is the locus of the intersection of a tangent to the polar circle with a perpendicular let fall on it from the pole.

Thus, in the figure R, r are points on the dygogram. This is evident when we draw the line qG .



To obtain the Deviation on a given Compass Course.

I. When the Dygogram has been constructed.

From Q , the pole, draw QM in the direction of the given compass course, cutting OX in M .

APP. 2.

Independent determination of the point R for any given course ζ .

First method.—Construct the dygogram so far as to draw the generating circle, and the line QD'N.

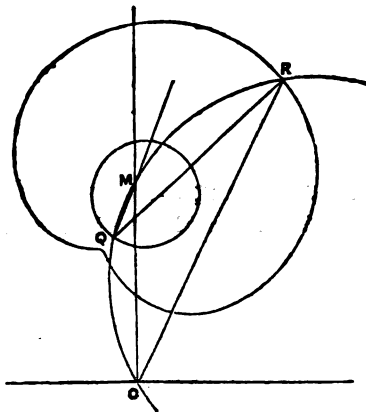
Draw a circle OQM, cutting the dygogram in R.

Join QR, OR.

Then MQR is the deviation required.

Demonstration.

MOR = MQR is the deviation on the *magnetic* course QR, and therefore QM is the compass course corresponding to the magnetic course QR, and vice versa.

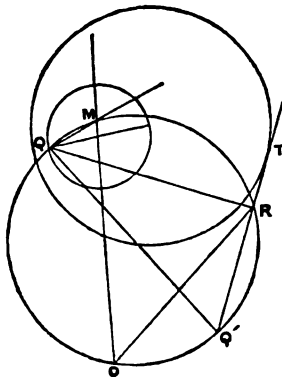


II. When the Dygogram is not constructed.

Draw QM the compass course, the polar circle and the circle OQM as in the last case.

From Q', the extremity of the diameter QQ', draw Q'T a tangent to the polar circle, cutting the circle OQM in R, and join QR, OR.

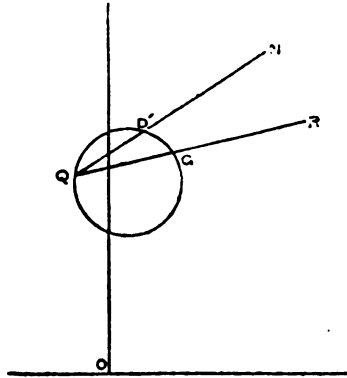
Then R is a point on the dygogram, and MQR = MOR is the deviation on the compass course QM and also on the magnetic course QR.



Then draw QGR such that $NQR = \zeta$, $GR = D'N$.
 R is the point required.

APP. 2.

Fig. 6.

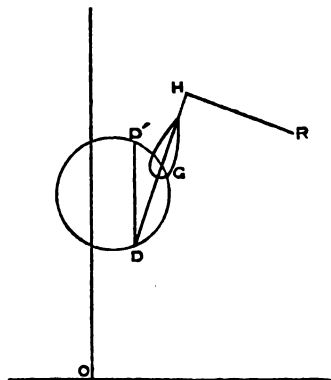


Second method.—Construct the dygogram so far as to draw the generating circle, and to find the point D' .

Draw $D'D$, vertically cutting the circle in D , and draw DGH in the direction of the ship's course and cutting the circle in G , i.e., making $D'DG = \zeta$; make $GH = \mathfrak{B}$, and draw $HR = \mathfrak{C}$ perpendicular to DH .*

R is the point required.

Fig. 7.



* We observe, once for all that, in all cases in which $\mathfrak{B}$ and $\mathfrak{C}$ are directed to be drawn, care must be taken to draw them in the right direction, viz., $\mathfrak{B}$ to head if +, to stern if -, and $\mathfrak{C}$ to starboard if +, to port if -.

APP. 2. *Third Method.*—Construct the dygogram so far as to draw the generating circle, and to find the point D. Draw a diameter D F.

From F draw $FB=B$ in the direction of the ship's course, and from D draw $DC=C$ at right angles to the course, and complete the rectangle by drawing BR, CR.

R is the point required.

The lines DGH, FG are drawn in the figure merely to show the connexion of the second and third methods.

Simplification of the Dygogram when $\mathcal{A}$ and $\mathcal{C}$ are Zero.

When $\mathcal{A}$ and $\mathcal{C}$ are zero the centre of the generating circle is at the point P, its radius is $\mathcal{D}$, and the line SQN passes through D', the intersection of the generating circle with OP produced, and the line WQE passes through D, the intersection of the generating circle with OP. The construction of the dygogram and the determination of the deviation for the 32 magnetic points is therefore somewhat simplified, as is also the first of the methods just given for the independent determination of the point R. The second and third methods for the independent determination of the point R are still more simplified and become the following :—

Second Method ($\mathcal{A}$ and $\mathcal{C}$ being zero),
—With P as centre and radius $\mathcal{D}$ draw the circle, cutting OP in D and OP produced in D', and draw DGH in the direction of the ship's course, cutting the circle in G and make $GH=B$, and draw $HR=C$ perpendicular to DH.

R is the point required.

Fig. 8.

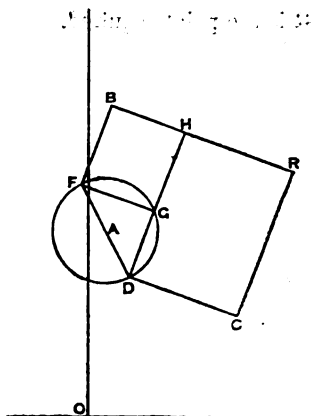


Fig. 9.

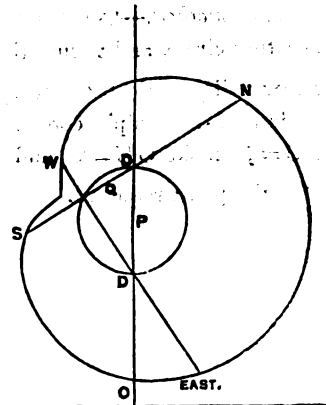
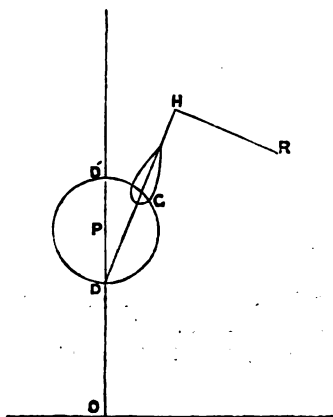
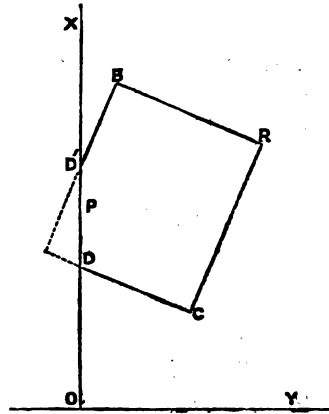


Fig. 10.



Third Method ($\mathcal{M}$ and $\mathcal{C}$ being zero).—On OP take the point $\mathcal{D}$, and on OP produced the point $\mathcal{D}'$, such that $PD = PD' = \mathcal{D}$. From $\mathcal{D}'$ draw $\mathcal{D}'B = \mathcal{B}$ in the direction of the course, and from $\mathcal{D}$ draw $DC = \mathcal{C}$ at right angles to the course, and complete the rectangle by drawing BR, CR. R is the point required.

Fig. 11.



Cor. I.—If with DD' as diameter we draw the generating circle, and if we also draw the circle of which BC is diameter, the circles will intersect in the point of intersection of BD' and CD , and also in Q the pole of the dygogram.

Cor. II.—If we make the same construction for a second angle ζ' , and get the points B', C', R' , we shall easily see that if from U the point of intersection of RC and $R'C'$ we draw UD it will bisect the angle CUC'.

So if from V, the point of intersection of $RV, R'V'$, we draw $D'V$ it will bisect the angle BVB' . This corollary will be useful to us hereafter.

By means of Dygogram No. I. we may conveniently solve the following—
PROBLEMS ON THE DETERMINATION OF THE CO-EFFICIENTS, FROM OBSERVATIONS MADE ON A SMALL NUMBER OF COURSES.

PROBLEM I.—Given $\mathcal{M}, \mathcal{C}, \mathcal{D}, \lambda$, to determine $\mathcal{B}$ and $\mathcal{C}$, from Observations of Deviation and Horizontal Force made on one Course.

Construct the dygogram, so far as to draw the generating circle, and the line $D'D$.

Observe ζ, δ , and $\frac{H'}{H}$.

Draw DGH, the magnetic course, cutting the generating circle in G.

Draw $OR = \frac{1}{\lambda} \frac{H'}{H}$ in the direction of the disturbed needle, *i.e.*, making $POR = \delta$.

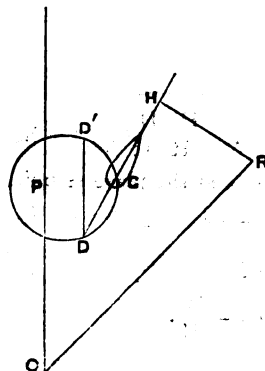
Draw RH perpendicular to FH.

Then $GH = \mathcal{B}$

$HR = \mathcal{C}$

An outline ship drawn about the point G will prevent any ambiguity as to the sign of $\mathcal{B}$ and $\mathcal{C}$.

Fig. 12.



APP. 2. This construction is most useful in enabling us to obtain approximately the values of $\mathfrak{B}$ and $\mathfrak{C}$ when a ship is on the stocks or in dock. In that case we generally do not know $\mathfrak{A}$ or $\mathfrak{E}$ and therefore assume them to be zero, which somewhat simplifies the construction. $\mathfrak{D}$ and λ though generally not known may be estimated with sufficient exactness. As the construction for this particular case is fully given in Part III., it is unnecessary to go more fully into it here.

PROBLEM 2.—Given $\mathfrak{A}$, $\mathfrak{E}$, $\mathfrak{D}$ to find $\mathfrak{B}$ to $\mathfrak{C}$ from Observations of Deviation made on two Courses.

Construct the dygogram so far as to draw the generating circle and the line $D'D$.

Observe the two magnetic courses and the corresponding deviations.

Draw DGT , $DG'T'$ the two magnetic courses; G , G' being the points of intersection with the generating circle.

Draw the two directions of the disturbed needle OT , OT' , meeting the courses in T and T' .

On GT take $GT'' = G'T$ and draw $T''R$ making the angle $GT''R = G'T'O$ intersecting OT in R ; R is a point in the dygogram, and drawing RH perpendicular to GT , we have—

$$\mathfrak{B} = GH$$

$$\mathfrak{C} = HT$$

This construction fails if the two compass courses are opposite.

When $\mathfrak{A}$ and $\mathfrak{E}$ are zero this problem is most conveniently solved by dygogram No. 2., see p. 29, *post*.

PROBLEM 3.—Given the Deviation and the Force on two Courses, $\mathfrak{A}$ and $\mathfrak{E}$ being zero, to find λ , $\mathfrak{D}$, $\mathfrak{B}$, $\mathfrak{C}$.

Let ζ_1 , ζ_2 be the two magnetic courses.

Let δ_1 , δ_2 be the deviations.

Let $\frac{H'_1}{H}$, $\frac{H'_2}{H}$ be the horizontal forces in terms of the earth's horizontal force at the place as unit.

Find the points R_1 , R_2 , which will be points on the dygogram and which may be called datum points, by taking $POR_1 = \delta_1$, $OR_1 = \frac{H'_1}{H}$, $POR_2 = \delta_2$, $OR_2 = \frac{H'_2}{H}$.

There are two cases which must be distinguished.

(α .) *When the two magnetic courses are diametrically opposite.*

APP. 2.

Join $R_1 R_2$ and bisect $R_1 R_2$ in G ; G will be a point in the generating circle.

Through G draw GD parallel to the keel intersecting OP in D , and draw GD' perpendicular to GD , intersecting OP in D' and bisect DD' in P , and from R_1 let fall on DG the perpendicular $R_1 H$, then

$$OP = \lambda \quad PD = PD' = \lambda \mathfrak{D}$$

$$GH = \lambda \mathfrak{B} \quad HR_1 = \lambda \mathfrak{C}$$

The signs of $\mathfrak{B}$ and $\mathfrak{C}$ will be seen without difficulty if we draw an outline ship about the point G .

$\mathfrak{D}$ being positive, D' should always lie above D . If it do not we may suspect error in the observations or in the construction.

If the two magnetic courses are N. and S. the middle point of $R_1 R_2$ will be D' , and in that case we cannot determine D and therefore cannot find λ and $\mathfrak{D}$. We can only obtain $\lambda \mathfrak{B}$ $\lambda \mathfrak{C}$ and $OD' = \lambda(1 + \mathfrak{D}) = 1 + a$. In order therefore to determine the four co-efficients required, we must know either λ and $\mathfrak{D}$ or e independently.

If the two magnetic courses are E. and W. the middle point of $R_1 R_2$ will be D , and in that case we cannot determine D' and therefore cannot find λ and $\mathfrak{D}$. We can only obtain $\lambda \mathfrak{B}$ $\lambda \mathfrak{C}$ and $OD = \lambda(1 - \mathfrak{D}) = 1 + e$. In order therefore to determine the four co-efficients required we must know either λ and $\mathfrak{D}$ or a independently.

(β .) *When the two magnetic courses are not diametrically opposite.*

Find $R_1 R_2$ the datum points.

Through $R_1 R_2$ draw lines parallel to the keel meeting in U , and lines perpendicular to the keel meeting in V .

Through V draw VW parallel to the line which bisects the angle made by the magnetic courses, and through U draw UW perpendicular to VW and meeting it in W .

Let VW intersect OP in D' .

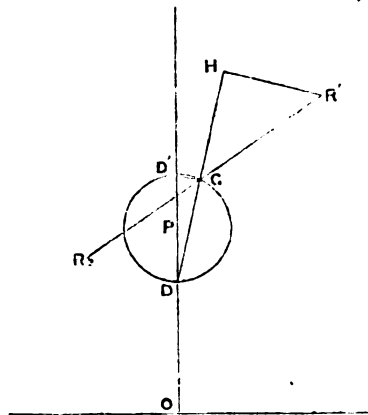
Let UW intersect OP in D .

Bisect DD' in P .

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Fig. 13.



APP. 2. From D' let fall a perpendicular $D'B$ or $D'B'$ on R_1V or R_2V (the perpendiculars will be of the same length).

From D let fall a perpendicular DC or DC' on R_1U or R_2U (the perpendiculars will be of the same length).

Then—

$$OP = \lambda$$

$$PD = PD' = \lambda \mathfrak{D}$$

$$D'B = D'B' = \lambda \mathfrak{B}$$

$$DC = DC' = \lambda \mathfrak{C}$$

DD' is the diameter of the generating circle.

If we draw the circle of which UV is a diameter, it will intersect the generating circle in W , and in a second point Q , which will be the pole of the dygogram.

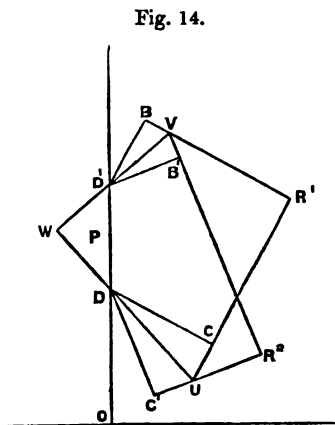


Fig. 14.

The demonstration of this construction will be evident from Cor. II., page 9.

The points R_1, R_2, W , will lie on the circumference of the last-mentioned circle, and as UW is perpendicular to the line bisecting R_1UR_2 , it will be seen that W bisects the arc R_1WR_2 and that therefore if we draw R_1W, R_2W , these lines are of the same length.

The construction fails when the magnetic courses are equidistant from any cardinal point for the following reasons:—

1. When equidistant from N . or S . it will be seen that $R_1D = R_2D$ and $R_1DR_2 = \text{difference of magnetic azimuths} = R_1WR_2$. W will therefore coincide with D and we cannot determine D' . Hence we may find $\lambda \mathfrak{C}$ and $\lambda(1 - \mathfrak{D})$ or $1 + e$, but not $\lambda \mathfrak{B}$ or $\lambda(1 + \mathfrak{D})$ or $1 + a$.

2. When equidistant from E . or W . $R_1D' = R_2D'$ and $R_1D'R_2 = \text{difference of magnetic azimuths} = R_1WR_2$. W will therefore coincide with D' and we cannot determine D , hence we may find $\lambda \mathfrak{B}$ and $\lambda(1 + \mathfrak{D})$ or $1 + a$, but not $\lambda \mathfrak{C}$ or $\lambda(1 - \mathfrak{D})$ or $1 + e$.

Corollary.—If $\mathfrak{H}$ and $\mathfrak{E}$ are not zero then instead of finding $\mathfrak{D}$ by the intersection of a line with OP we find a point D by the intersection of the same line with a line parallel to OP but at a distance $\mathfrak{H} + \mathfrak{E}$ to the right of it. And instead of finding D' by the intersection of a line with OP produced we find a point F by the intersection of the same line with a line at a distance $\mathfrak{H} - \mathfrak{E}$ to the right of it. The points D, F , so found are the extremities of a diameter of the generating circle which may then be drawn and the perpendiculars from D on the courses will give $\mathfrak{C}$, the perpendiculars from F on the lines at right angles to the courses will give $\mathfrak{B}$.

PROBLEM 4.—*Given the Deviation and Force on three Courses to find λ , $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$.*

APP. 2.

CONSTRUCTION.—Lay down the three datum points and through them draw the three magnetic courses and the perpendiculars to them.

Find the centre and radius of the circle which touches the three courses, and the centre and radius of the circle which touches the three perpendiculars.

The line joining these centres is a diameter of the generating circle which is therefore given.

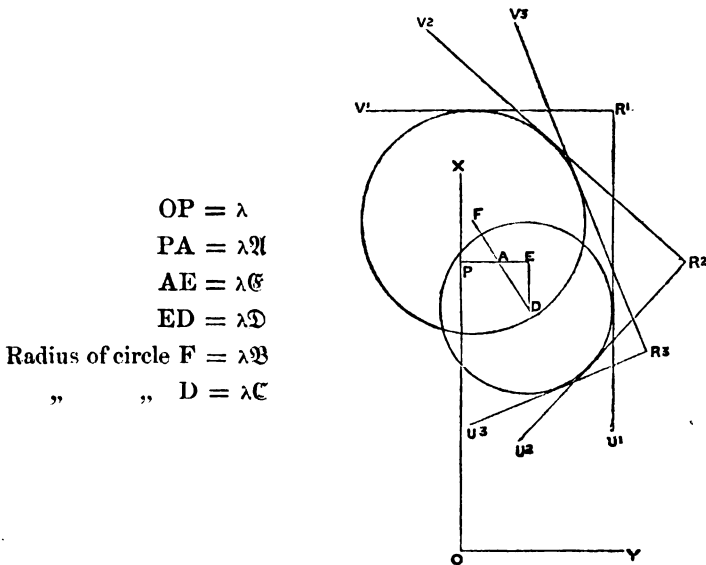
Half the sum of the distance of the centres from the axis of Y is λ . Half the difference is $\lambda\mathcal{D}$.

Half the sum of the distance of the centres from the axis of X is $\lambda\mathcal{A}$. Half the difference is $\lambda\mathcal{E}$.

The radius of the circle which touches the courses is $\lambda\mathcal{C}$, the radius of the circle which touches the perpendiculars is $\lambda\mathcal{B}$. The signs of these quantities is best determined by actually constructing the dygogram or supposing it constructed.

This process will be understood without further explanation from the figure, in which R_1U_1 , R_2U_2 , R_3U_3 represent the courses, R_1V_1 , R_2V_2 , R_3V_3 represent the perpendiculars.

Fig. 15.



This remarkable construction was first given under a somewhat different form by Lieut. Colongue.

APP. 2.

SECOND MODE OF CONSTRUCTING THE DYGOGRAM NO. I.

In finding the position of the point N or of any point R by construction we may of course draw $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, in any order we please, and so draw the dygogram in $5 \times 4 \times 3 \times 2 = 120$ different ways.

It is sometimes convenient to construct it as follows :—

Having found the point A as before lay off $AB = \mathcal{B}$, upwards if $\mathcal{B}$ is +, downwards if —; and from B lay off $BC = \mathcal{C}$, to the right if $\mathcal{C}$ be +, to the left if —.

With A as centre and radius AC, describe a circle and divide its circumference into 32 equal parts, beginning at C and marking the point, $C_1, C_2, \dots, C_{31}$.

From C draw $CD = \mathcal{D}$ upwards ($\mathcal{D}$ being +).

„ C_1 „ $C_1D_1 = \mathcal{D} 22\frac{1}{2}^\circ$ to the right of CD.

„ C_2 „ $C_2D_2 = \mathcal{D} 45^\circ$ „ „

and so on.

From D draw $DN = \mathcal{E}$ at right angles to CD, to the right if $\mathcal{E}$ be +, to the left if —; so from D_1 draw $D_1E_1 = \mathcal{E}$ at right angles to CD, and proceed in the same way with the points D_2, D_3 , &c. Finally draw ON, OE_1, OE_2 , &c. Then the angle PON is the deviation when the ship's head is N. (magnetic), POE_1 when the ship's head is N. by E., &c.

Also the length of the lines ON, OE_1, OE_2 , &c., represents the directive force on the needle on those points in terms of the mean force to N. as unit.

This was the construction of the dygogram given in the first and second editions of the Manual, and its application to H.M.S. Warrior is shown in plate (3). It is perhaps more easily understood than the method first given, and shows more clearly the effect of the different parts of the deviation on different courses, and the connexion of this with other forms of dygogram, but is on the whole much less convenient.

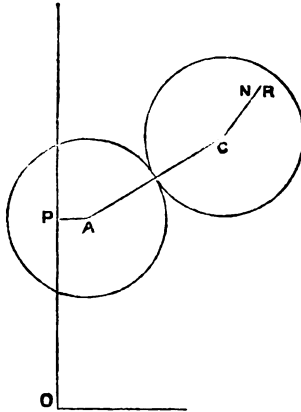
It will be seen that the dygogram may be considered as described by the extremity of a jointed rod A, C, E, of which the joint $CE = \sqrt{\mathcal{D}^2 + \mathcal{E}^2}$, revolves with twice the velocity of $AC = \sqrt{\mathcal{B}^2 + \mathcal{C}^2}$. This should be borne in mind when we come to consider the connexion between dygogram No. 1, and dygogram No. 2. We may also from this see the following mode of constructing the dygogram mechanically.

MECHANICAL CONSTRUCTION OF DYGOGRAM NO. I.

With A as centre and radius $= \frac{1}{2}\sqrt{\mathcal{B}^2 + \mathcal{C}^2}$ describe a circle and let another circle of the same radius roll on it. A point R at a distance

$\sqrt{\mathfrak{D}^3 + \mathfrak{E}^2}$ from the centre of the rolling circle will trace out the dygo-gram.*

Fig. 19.



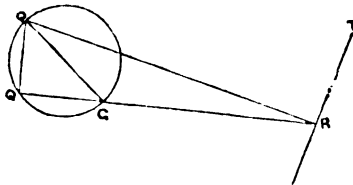
* The following problems on the tangent of the dygogram are of too much interest to be wholly omitted, though of too rare practical use to have a place in the text.

PROBLEM I.—*To draw a Tangent to the Dygogram at a Point R.*

CONSTRUCTION.—Let Q be the pole of the generating circle.

Join QR, cutting the generating circle in G the magnetic course.

Find Q' , a point on the generating circle, by drawing QQ' a perpendicular to QR or GQ' a diameter, and join $Q'R$ and draw RT perpendicular to it. Then $Q'R$ is a normal to the dygogram, TR a tangent.



Analytical Demonstration.

The polar equation to the dygogram being—

$$r = p + 2q \cos \theta$$

the tangent of the angle between the normal and radius vector—

$$= - \frac{1}{r} \frac{dr}{d\theta}$$

$$= \frac{2q \sin \theta}{r}$$

$$= \frac{Q'Q}{QR}$$

$= \tan Q'RQ$; whence $Q'R$ is a normal and RT is a tangent.

APP. 2.

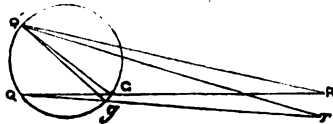
I.—ELLIPSE AND CIRCLE.

[This may be omitted by the Student.]

In dygogram No. 1 we supposed the axes to be fixed relatively to the horizon, and the ship to turn round from left to right. In the following

Geometrical Demonstration.

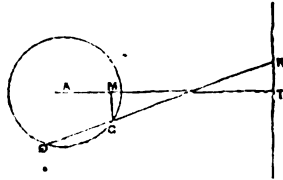
In the last figure take a point g on the generating circle at an infinitesimal distance, from G and draw Qgr , making $gr = GR$, so that r is a point on the dygogram, and join $Q'g$ $Q'r$.



Then $Q'g$ differs from $Q'G$ by an infinitesimal of the second order, and by the property of the circle $Q'GR = Q'gr$. Hence $Q'r = Q'R$ to terms of the first order inclusive, and therefore $Q'R$ is a normal to the dygogram at R . Q.E.D.

We may arrive at the same conclusion by drawing the circle $QQ'G$ and the line $Q'R$ in fig. 19. It will be found that $Q'R$ intersects the fixed and moving circles in their point of contact, and is therefore a normal to the locus of R .

Corollary.—If from the centre of the generating circle we let fall AT a perpendicular on the tangent, and from G the magnetic course let fall a perpendicular GM on AT , then $GM = TR$.



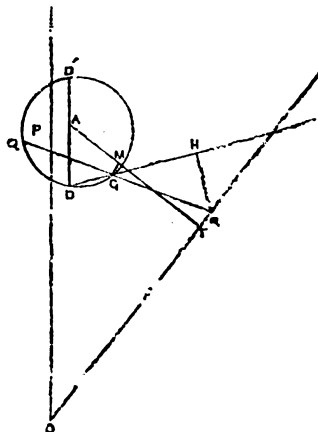
PROBLEM II.—Having given the generating Circle, the maximum Deviation, and the Magnetic Course to find B and C and to construct the Dygogram.

If POR be the maximum deviation OR will be a tangent to the dygogram. Hence the following construction:—

Draw AT perpendicular to OR .

From D , which is given, draw DGH such that $D'DG = \zeta$, which is given, and take R such that $TR = GM$.

Draw RH perpendicular to DH and RGQ . Then $GH = B$, $HR = C$, and Q is the pole of the dygogram which may be constructed.



cases we suppose the axes to be fixed relatively to the ship, and the horizon to turn round from right to left. APP. 2.

Construction.—Take a point O as the origin of co-ordinates.

Let axis of x or OX be directed to the ship's head.

„ y or OY „ the starboard side.

„ z or OZ „ vertically downwards or to nadir.

Let ζ be the magnetic azimuth of the ship's head measured from N to E.

Take a point R of which the co-ordinates are,—

$$x = a \cos \zeta - b \sin \zeta + \lambda \mathfrak{B}$$

$$y = d \cos \zeta - e \sin \zeta + \lambda \mathfrak{C}$$

$$z = g \cos \zeta - d \sin \zeta + \lambda \mathfrak{V}$$

PROBLEM III.—*Having given the generating Circle, the maximum Deviation, and the Pole of the Dygogram to find $\mathfrak{B}$ and $\mathfrak{C}$ and to construct the Dygogram.*

The construction is the same as in the last problem, except that we have now given Q instead of G, and have to find G tentatively by drawing QGR such that $GM = TR$. This is easily done by trial.

PROBLEM IV.—*Having given the generating Circle, the maximum Deviation, and the Course of no Deviation, to find $\mathfrak{B}$ and $\mathfrak{C}$ and to construct the Dygogram.*

Construction.—Let F be the course of no deviation ($D'DF$ its magnetic azimuth), and find by trial a point Q, from which we can draw QFU, QGR such that

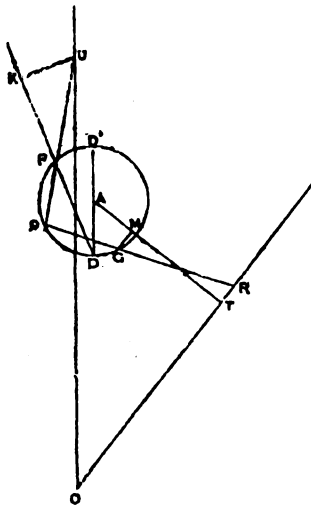
$$FU = GR \text{ and } GM = TR.$$

From U draw UK perpendicular to DF produced.

Then Q is the pole of the dygogram, and

$$FK = \mathfrak{B}. \quad KU = \mathfrak{C}.$$

Q.E.D.



This construction may occasionally be useful as the *amount* of maximum deviation and the *azimuth* of no deviation may be determined with great exactness, and, when no regular observations have been made, are the points most likely to have been noted.

APP. 2. Then we see from equations (4a), (5a), (6a), that x represents the force of the ship to head in terms of the earth's horizontal force as unit, y the force of the ship to starboard, and z the force of the ship vertically downwards in terms of the same unit. The line OR therefore represents in magnitude and direction the total force of a ship acting on a magnetic particle at O.

Equations of Ellipse.—Multiplying the first of these equations by $dh - eg$, the second by $gb - ah$, the third by $ae - bd$, and adding we get

$$(x - \lambda B)(dh - eg) + (y - \lambda C)(gb - ah) + (z - \lambda V)(ae - bd) = 0$$

which is the equation of a plane passing through a point the co-ordinates of which are λB , λC , and λV . The locus is therefore a plane curve.

Changing the origin of co-ordinates to the point λB , λC , λV , the equations become

$$x = a \cos \zeta - b \sin \zeta$$

$$y = d \cos \zeta - e \sin \zeta$$

$$z = g \cos \zeta - h \sin \zeta$$

From the two first we get

$$dx - ay = (ae - bd) \sin \zeta$$

$$ex - by = (ae - bd) \cos \zeta$$

Squaring and adding we get for the equation of the projection of the locus on the plane of xy

$$(dx - ay)^2 + (ex - by)^2 = (ae - bd)^2$$

So for the projection on the plane of xz

$$(gx - az)^2 + (hx - bz)^2 = (ah - gb)^2$$

and for the projection on the plane of yz

$$(gy - dz)^2 + (hy - ez)^2 = (dh - ge)^2$$

Each of these is the equation to an ellipse. The locus sought is therefore a plane ellipse.

If the soft iron be symmetrically distributed, b d f h are zero, and the equations to the three projections become

$$\frac{x^2}{a^2} + \frac{y^2}{e^2} = 1$$

$$\frac{x}{a} - \frac{z}{g} = 0$$

$$\frac{y^2}{e^2} + \frac{z^2}{g^2} = 1$$

The first shows that when the soft iron is symmetrically distributed the locus of the extremity of the line representing the horizontal force referred

to axes fixed in the ship is an ellipse of which the principal axes are fore and aft and transverse, the fore-and-aft semi-axis being $\pm a$, and the transverse semi-axis $-e$.* APP. 2.

The second equation shows that the projection of the locus on the fore-and-aft vertical plane is, under the same circumstances, a straight line.

If the soft iron is not symmetrically arranged the principal axes of the ellipse are not in the direction of the fore-and-aft and transverse lines, and the expressions for the direction and length of the principal axes are too complicated to be of any use. We may, however, in this case construct the ellipse which represents the horizontal force by points without difficulty. We may also approximate to its general character with considerable exactness in the following way :—

The equations of this ellipse being

$$\begin{aligned}x &= a \cos \zeta - b \sin \zeta \\ y &= d \cos \zeta - e \sin \zeta\end{aligned}$$

making $\zeta=0$, we get

$$\left. \begin{aligned}x &= a \\ y &= d\end{aligned} \right\} \text{ and } \frac{y}{x} = \frac{d}{a} \text{ for one point in the ellipse}$$

making $\zeta=90^\circ$, we get

$$\left. \begin{aligned}x &= -b \\ y &= -e\end{aligned} \right\} \text{ and } \frac{y}{x} = \frac{e}{b} \text{ for another point.}$$

Also by the differential calculus

$$\frac{dy}{dx} = \frac{d \sin \zeta + e \cos \zeta}{a \sin \zeta + b \cos \zeta}$$

$$\text{so that when } \zeta=0 \quad \frac{dy}{dx} = \frac{e}{b}$$

$$\text{when } \zeta=90^\circ \quad \frac{dy}{dx} = \frac{d}{a}$$

From which we see that the tangent to the ellipse at the first point is parallel to the diameter passing through the second point, and *vice versa*. The points so determined are therefore the extremities of a pair of conjugate diameters, and we thus draw the ellipse by hand without much trouble, and with considerable accuracy.

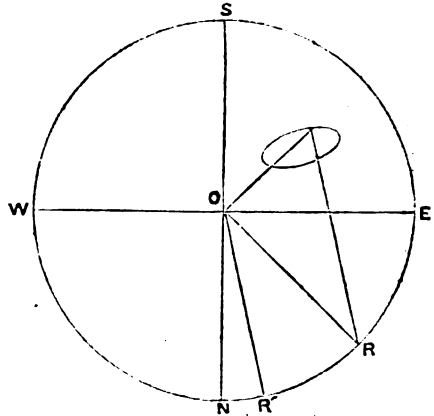
* In this case, to find the point in the ellipse corresponding to an angle ζ , draw a circle with radius a , and a circle with radius $-e$, both concentric with the ellipse, and draw a radius at an angle ζ from the vertical. Then draw from the intersection of the radius with the first circle a horizontal line, and from its intersection with the second circle a vertical line. The intersection of these two lines is the point in the ellipse corresponding to the correct magnetic azimuth ζ .

APP. 2.

To represent the Horizontal Force of Earth and Ship, and the Deviations.

Construct the ellipse as above. With O as centre and radius 1 describe a circle. Draw a vertical diameter marking it N at the lowest extremity, S at the upper; and a horizontal diameter marking it E at the right extremity, W at the left; and divide the circumference into degrees and points in the usual way, but of course in reverse order or "against the sun." Then if r be a point in the ellipse corresponding to any magnetic course ζ and if we take a point R on the circle such that $\text{NOR} = \zeta$ and join OR, O r , R r we have—

Fig. 20.



OR the earth's horizontal force.

O r the ship's horizontal force.

R r the horizontal force of earth and ship, or $\frac{H'}{H}$

OR r the deviation which is easterly if r lie to the right of RO looking from R to O.

The terminated line R r may be called the *chord* corresponding to the azimuth NOR.

If we lay the edge of a parallel ruler along R r and move it till it passes through O and cuts the circumference of the circle in R' then $\text{NOR}' = \zeta'$ is the compass course.

To draw the Ellipse and obtain the Co-efficients from Observations of Deviation and Horizontal Force made on a large number of Courses.

First Method.—Draw and divide the large circle as before.

Take on its circumference the points R_1, R_2, R_3 , &c., corresponding with courses ζ_1, ζ_2 , &c., and draw the chords $R_1 r_1, R_2 r_2, R_3 r_3$, &c., r_1, r_2, r_3 , &c., ought to lie on the ellipse. When the points r_1, r_2, r_3 , &c., are all laid down we may draw an ellipse as nearly as possible, passing through them all.

The co-ordinates of the centre of the ellipse will give λB and λC , and a, b, d, e , may be obtained from the position and length of the axes of the

ellipse. The expressions for doing this are, however, far too complicated to be of any use, except in the case of principal axes of the ellipse being horizontal and vertical; when this is the case b and d are zero, $-e$ is the horizontal semi-axis, and $+a$ or $-a$ is the vertical semi-axis according as the point on the ellipse corresponding to N is above or below the centre.

Second Method.—The co-efficients may, however, be obtained with great exactness in the following method :—

Having as above laid down the points R_1, r_1 , take a point R'_1 90° in advance of R . From R'_1 draw a vertical line, and from r_1 a horizontal line meeting in p_1 , and through R'_1 draw a horizontal line and through r_1 a vertical line meeting in q_1 .

Perform the same operation with r_2, r_3, r_4 , &c. The points p_1, p_2, p_3, p_4 , &c., will be found to lie either on a straight line, or if not, on a very elongated ellipse which may be drawn by hand without much difficulty.

The same will be the case with the points q_1, q_2, q_3 , &c.

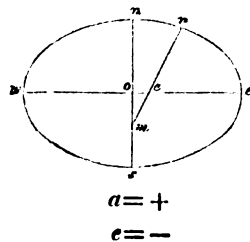
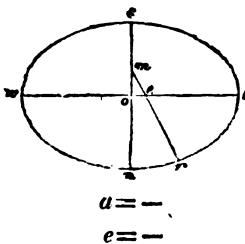
To find the co-efficients proceed as follows :—

Mark with the letters p_n, p_e, p_w, p_s , the points on the one elongated ellipse, and with the letters q_n, q_e, q_w, q_s , the points on the other elongated ellipse where they would be intersected by chords drawn from the points N, E, S, W , on the large circle. Then we have :—

$$\begin{aligned} \lambda A &= \frac{1}{2} \{E p_n + W p_e\} \\ &= \frac{1}{2} \{O p_e + O p_w\} \\ + a &= \frac{1}{2} \{E p_n - W p_e\} \\ + b &= \frac{1}{2} \{O p_w - O p_e\} \\ \lambda C &= \frac{1}{2} \{N q_w + S q_e\} \\ &= \frac{1}{2} \{O q_n + O q_s\} \\ - e &= \frac{1}{2} \{S q_e - N q_w\} \\ + d &= \frac{1}{2} \{O q_n - O q_s\} \end{aligned}$$

Simplification of Ellipse.—When A and C or b and d are zero the axes of the ellipse are horizontal or vertical, and if we mark the extremes n, e, w, s , then, as e is always negative, we shall find the ellipse has two forms according as a is $+$ or $-$.

Fig. 21.



APP. 2. In order to find the point r corresponding to any course ζ , take $nmr = \zeta$, $mr = -e$, and er will be equal $+a$ in the first case, and $-a$ in the second.

The following problems may be solved by the circle and ellipse.

PROBLEM I.—Given λ and $\mathcal{D}$ or a and e to find $\mathcal{B}$ and $\mathcal{C}$ from observations of deviation and horizontal force made on one course, $\mathcal{A}$ and $\mathcal{E}$ being zero.

Take on the large circle the point R corresponding to the given magnetic course and draw the chord $Rr = \frac{H'}{H}$ and from r draw rlm (fig. 21) parallel to OR if a be $-$, and parallel to OR' (R' being a point to which the point R would come if the circle were folded about the line EW or where the reflected image of R would be if EW were a mirror and which point is conveniently called the image of R) if a be $+$; and let $rl = a$, $rm = -e$ (l and m will always lie on the same side of r that O does of R or R' as the case may be); then through l draw an E. and W. line and through m a N. and S. line and let them intersect in O the co-ordinates of O give $\lambda\mathcal{B}$ and $\lambda\mathcal{C}$.

Corollary.—We may construct the ellipse by supposing rm to move, so that l moves on the E. and W. line, m on the N. and S. line.

PROBLEM 2.—Given the deviation and the horizontal force on two courses to find λ , $\mathcal{D}$, $\mathcal{B}$, and $\mathcal{C}$; $\mathcal{A}$ and $\mathcal{E}$ being zero.

Let ζ_1, ζ_2 , be the two magnetic courses; δ_1, δ_2 , the deviations.

$\frac{H'_1}{H}, \frac{H'_2}{H}$ the horizontal forces in terms of the earth's horizontal force.

Take R_1, R_2 , on the circle such that $NOR_1 = \zeta_1$, $NOR_2 = \zeta_2$ draw R_1r_1, R_2r_2 such that $OR_1r_1 = \delta_1$, $OR_2r_2 = \delta_2$.

Let $R_1r_1 = \frac{H'_1}{H}$; $R_2r_2 = \frac{H'_2}{H}$.

These are two cases which must be distinguished.

(a)—When the two Magnetic Courses are diametrically opposite.

r_1, r_2 , will then be at the end of a diameter of the ellipse. Bisect r_1r_2 in o , and through o draw nos and eow vertically and horizontally, there will be no difficulty in seeing whether n or s should be upwards, and draw $r_1l_1m_1$ such that $nm_1l_1 = \zeta_1$ cutting ew in e_1 and ns in m_1 , then $r_1l_1 = +a$, $r_1m_1 = -e$, and $\lambda = 1 + \frac{a+e}{2} \lambda\mathcal{D} = \frac{a-e}{2}$ and the co-ordinates of o are $\lambda\mathcal{B}$ and $\lambda\mathcal{C}$.

This construction it will be seen does not give e when the courses are N. and S., and does not give a when they are E. and W.

(β.)—When the two Magnetic Courses are not diametrically opposite. APP. 2.

The observations give us two points r_1, r_2 , and also the direction of the lines $r_1l_1m_1, r_2l_2m_2$; the positions of l_1l_2, m_1m_2 are then to be found as in the corollary to Prob. I. This may be done as follows:—Move a parallel ruler with its edge e and w till it cuts off $r_1l_1=r_2l_2$, and draw the line ew through l_1l_2 ; next move the ruler with its edge in the direction ns till it cuts off $r_1m_1=r_2m_2$, and draw the line ns through m_1m_2 intersecting ew in o , then $r_1l_1=r_2l_2=+a, r_1m_1=r_2m_2=-e$.

$$\lambda = 1 + \frac{a+e}{2} \quad \lambda \mathfrak{D} = \frac{a-e}{2}$$

and the co-ordinates of o are $\lambda \mathfrak{B}$ and $\lambda \mathfrak{C}$.

This construction it will be easily seen fails when the magnetic courses are at an equal distance on each side of N. and S., as we then only get the position of the line ns , the length $-e$, and $\lambda \mathfrak{C}$.

It also fails when the magnetic courses are at equal distances on each side of E. and W., as we only get the position of the line ew , the length a , and $\lambda \mathfrak{B}$.

DYGOGRAM No. II.

When $\mathfrak{A}$ and $\mathfrak{C}$ are zero the ellipse and circle are susceptible of a remarkable modification, which by converting the ellipse into a small circle greatly simplifies and facilitates the construction of the figure and the solution of the problems.

This is done as follows:—

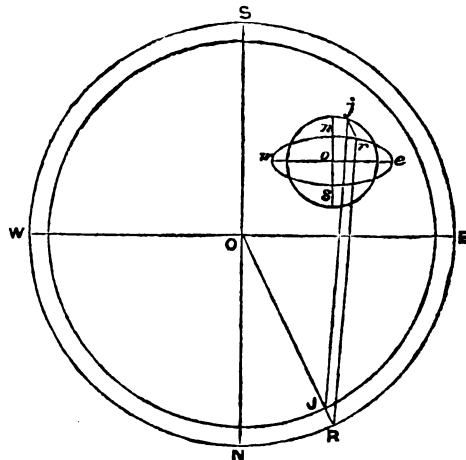
Having constructed as before the ellipse and circle, let R, r , be corresponding points in them for any given course.

With centre O draw a circle of which the radius is $\lambda = 1 + \frac{a+e}{2}$, cutting OR in J ,

then $RJ = -\frac{a+e}{2}$; and if we complete the parallelogram $JRrj$ it will be seen that $Jj = Rr$, and $OJj = ORr$, and

therefore the circle of radius λ and the curve which is the locus of j will give the same results as the circle of which the radius is 1 and the ellipse.

Fig. 22.



APP. 2. To find the locus of j we must add—

$$-\frac{a+e}{2} \cos \zeta \text{ to } x$$

$$+\frac{a+e}{2} \sin \zeta \text{ to } y$$

we therefore have for j , taking o as origin,—

$$x = a \cos \zeta - \frac{a+e}{2} \cos \zeta$$

$$= \frac{a-e}{2} \cos \zeta$$

$$= \lambda \mathfrak{D} \cos \zeta$$

$$y = -e \sin \zeta + \frac{a+e}{2} \sin \zeta$$

$$= \frac{a-e}{2} \sin \zeta$$

$$= \lambda \mathfrak{D} \sin \zeta$$

whence $x^2 + y^2 = \lambda^2 \mathfrak{D}^2$ or j lies on a circle of which the radius is $\lambda \mathfrak{D}$.*

Hence we get the same results as by the ellipse and circle if we take—

1. A circle radius = λ .
2. A point o of which the co-ordinates are $\lambda \mathfrak{B}$ and $\lambda \mathfrak{C}$.
3. A circle with o as centre of which the radius is $\lambda \mathfrak{D}$.

or as we may alter these quantities provided we alter them in the same proportion we get the following mode of constructing Dygogram No. 2, $\mathfrak{B} \in \mathfrak{D}$ being known, and $\mathfrak{A}$ and $\mathfrak{C}$ being zero.†

To construct Dygogram No. II.

With centre O and radius = 1 (this distance being any length conveniently divisible into 100 parts; as 5 inches, $3\frac{1}{8}$ inches, or 100 millimetres) describe a circle. Draw a vertical diameter, marking it N at the lower extremity, S at the upper; and a horizontal diameter, marking it E at the right extremity and W at the left, and suppose the circumference divided into degrees and points in the usual way, but in reverse order, or “against the sun.”

* It will be observed that in all ordinary cases $-e$ is + and numerically greater than a , so that $\mathfrak{D}$ is + whatever be the sign of a .

† We may see the connexion between Dygogram No. I. and Dygogram No. II. by supposing in the first $\mathfrak{A}=0$ and $\mathfrak{C}=0$, and then supposing the paper to turn to the left with the same velocity with which the ship turns to the right.

Then if the Dygogram No. I. be constructed in the 2nd method which we have described the line PC will remain fixed while CN will turn to the right, and PO to the left with the same velocity, and Dygogram No. I. is thus converted into Dygogram No. II. It was in fact this consideration which first suggested the construction of Dygogram No. II., and the mode of deriving it from the ellipse and circle.

The figure so drawn will represent a compass card looked at from behind, **APP. 2.**
or as reflected in a mirror.

From O draw $OB=B$, upwards if B be +, downwards if —
„ B „ $BC=C$, to the right if C be +, to the left if —

With centre C and radius $\mathfrak{D}$ describe a circle and draw a vertical diameter marking it n at the *upper* extremity and s at the *lower*, and a horizontal diameter, marking it e at the right extremity, w at the left, and suppose the circumference of the small circle divided in degrees and points in the usual way and in the usual order. By this dygogram we may solve the following problems:—

PROBLEM I.—*Having given any Magnetic Course, to find the Deviation and the Directive Force.*

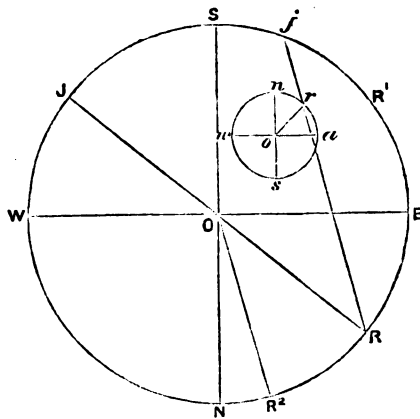
Let ζ be the given magnetic course.

Take R , a point on the circumference of the large circle, and r on the circumference of the small circle, such that $NOR=nor=\zeta$, and join OR, Rr . Then ORr is the deviation which is +, or easterly if r is to the right of O looking from R to O . If the large circle is graduated we may measure the angle ORr by producing RO, Rr , to intersect the circle in J and j . The arc Jj will then be twice the required angle.

Rr is the directive force in terms of the mean force to north as unit, $\frac{1}{\lambda}Rr$ is the directive force

in terms of the earth's horizontal force as unit. The line through Rr we may call the “chord corresponding to the course R ,” or to NOR .

Fig. 23.



PROBLEM II.—*To find the Compass Course corresponding to any given Magnetic Course.*

Lay a parallel ruler so as to pass through the given course on the large, and also on the small circle.

Move the ruler parallel to itself till it passes through the centre of the large circle. It will then intersect the large circle in the compass course.

APP. 2. PROBLEM III.—*To find the Magnetic Course corresponding to a given Compass Course.*

Lay a parallel ruler so as to pass through the centre of the large circle, and the compass course on the large circle. Move the ruler parallel to itself till it cuts the large and small circle in corresponding points. These give the magnetic course.

We may also solve the following—

PROBLEMS ON THE DETERMINATION OF CO-EFFICIENTS FROM OBSERVATIONS MADE ON A SMALL NUMBER OF COURSES.

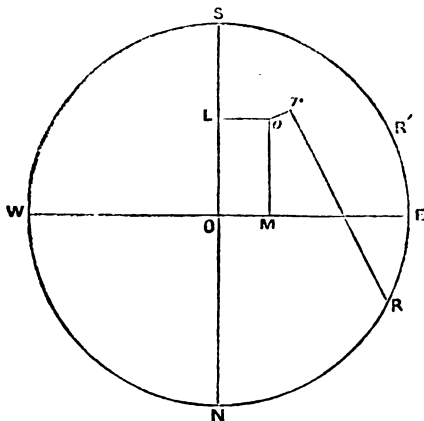
PROBLEM IV.—*Given λ and $\mathfrak{D}$ to find $\mathfrak{B}$ and $\mathfrak{C}$ from Observations of Deviation and Horizontal Force made on one Course.*

Take R the magnetic course on the large circle.

Draw the chord Rr making $Rr = \frac{1}{\lambda} \frac{H'}{H}$.

From r draw $ro = \mathfrak{D}$ parallel to OR' (R' being a point to which R would come if the large circle were folded about the line EW , or where the reflected image of R would be if EW were a mirror, which point R is conveniently called the image of R), and from o let fall oL a perpendicular on NS ; oM a perpendicular on EW .

Fig. 24.



Then o will be the centre of the small circle, and

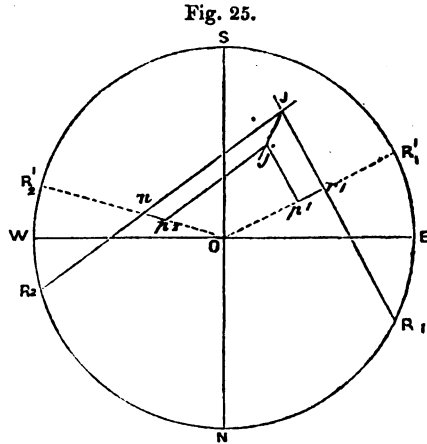
$$\mathfrak{B} = oL.$$

$$\mathfrak{C} = oM.$$

taking care to give to $\mathfrak{B}$ and $\mathfrak{C}$ their proper signs according to the position of C .

PROBLEM V.—Given $\mathfrak{D}$ to find $\mathfrak{B}$ and $\mathfrak{C}$ from Observations of Deviation APP. 2.
made on Two Courses.

Take R_1 one of the magnetic courses on the large circle, and R_1' its image, draw the chord corresponding to R_1 , and OR_1' cutting the chord in r , and on OR_1' lay off $rp = \mathfrak{D}$ (p being on the same side of r that o is of R_1'), and through p draw a line parallel to the chord. It will easily be seen that the centre of the small circle lies in that parallel. Proceed in the same way for the other magnetic course R_2 . The intersection of the two parallels gives j the centre of the small circle, and consequently the values of $\mathfrak{B}$ and $\mathfrak{C}$.



If we are uncertain of the value of $\mathfrak{D}$, then if we take J the intersection of the two chords and j the intersection of the two parallels, and draw a line Jj through these two intersections, the centre of the small circle will lie on the last drawn line.

PROBLEM VI.—Given, the Deviation on Three Courses to find $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$.

First Method.—Take R_1, R_2, R_3 the magnetic courses on the large circle, draw the chords, and assuming any value of $\mathfrak{D}$, draw, as in the last problem, the three parallels. If these intersect in a point it is the centre of the small circle; if they do not, then, through the intersection of each pair of chords and the intersection of the parallels, draw a line. The three lines so drawn will intersect in a point which will be the centre of the small circle, from which $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ may then be determined.

Second (Cayley's) Method.—Take R_1, R_2, R_3 the magnetic courses on the large circle, and draw the chords. Their intersection will form a small triangle. Take R_1', R_2', R_3' the images of R_1, R_2, R_3 on the large circle (or, if more convenient, on any other circle), and through them draw lines parallel to the chords. Their intersections will form a large triangle.

The small circle sought will occupy the same position relatively to the small triangle that the large circle does to the large triangle. The position of the centre and the radius of the small circle may therefore be found, and these give $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$.

This very elegant method is due to Professor Cayley.

APP. 2. The first method is, however, probably the safest, as it shows most distinctly whether the resulting value of $\mathfrak{D}$ is within the limits of probability, and if not what modifications ought to be made in the observed deviations.

PROBLEM VII.—*Given, the Deviation on the Four Quadrantal Compass Points to find the small Circle.*

Draw the four chords. Their intersections form a rectangle. Draw the diagonals of the rectangle. Their intersection is the centre of the small circle. The mean of the perpendiculars from this point on the sides of the rectangle is the radius of the small circle.

If the sides are unequal, a quarter the difference of the sides is $\mathfrak{A}$, which is + if the elongation is in the N.E. and S.W. lines, — if in the N.W. and S.W.

The deviations on the quadrantal points do not give the means of determining $\mathfrak{G}$, which must, as it safely may, be taken equal to zero. This method cannot be used without some modification if the deviations are very large.

Examples of the construction of Problem VI. by Professor Cayley's method for H.M.S. *Warrior*, Problem VII. for H.M.S. *Trident*, will be seen in Plate IV.

PROBLEM VIII.—*Given, the Deviations and Forces on Two Courses to find λ , $\mathfrak{D}$, $\mathfrak{B}$, $\mathfrak{C}$.*

Take R_1 and R_2 , the two magnetic courses on the large circle. Draw the chords R_1r_1 and R_2r_2 , and let $R_1r_1 = \frac{H'_1}{H}$; $R_2r_2 = \frac{H'_2}{H}$

Let R'_1, R'_2 be the "images" of R_1 and R_2 , and join R'_1, R'_2 .

Then if the line joining r_1, r_2 is parallel to the line joining R'_1, R'_2 , $\lambda = 1$ and r_1 and r_2 are points in the small circle, and we can find its centre o by drawing r_1o and r_2o parallel to R'_1o and R'_2o , and thus we find $\mathfrak{D}$, $\mathfrak{B}$ and $\mathfrak{C}$.

If the line joining r_1, r_2 be not parallel to R'_1, R'_2 , then we have to find in R_1r_1 produced a point r'_1 , and in R_2r_2 produced a point r'_2 , such that $R_1r'_1 : R_2r'_2 :: R_1r_1 : R_2r_2$, and that a line through r'_1, r'_2 is parallel to a line through R'_1, R'_2 . This is a purely geometrical problem, and is solved as follows:—

Through r_2 draw r_2p parallel to R_1, R_2 , and through r_1 draw r_1p parallel to R'_1, R'_2 and through R_1p draw R_1p, r'_2 cutting R_2, r_2 produced in r'_2 , and draw r'_2, r'_1 parallel to R'_1, R'_2 cutting R'_1, r_1 produced in r' .

Then r'_1, r'_2 are the points in the small circle required.

Next draw r'_1o parallel to R'_1O and r'_2o parallel to R'_2O intersecting in o we get the centre of the small circle, and therefore B and C . APP. 2.

$$\text{Also } \lambda = \frac{R_1 r_1}{R_1 r'_1} = \frac{R_2 r_2}{R_2 r'_2}$$

This construction fails when the magnetic courses are at equal distances from one of the cardinal points.

IV.—STEERING DYGOGRAM.

If we suppose dygogram No. 2. to be turned upside down about the line E. W. the large circle will then be divided exactly as, and may be represented by, the card of the standard compass. The small circle will then be represented by a small compass card turned upside down about the E. and W. line and pasted on the surface of the large card. The radius of the small circle being $\mathfrak{D}$ the position of its centre being $+B$ to the *South* and $+C$ to the *East* of the centre of the large card. A line drawn from any magnetic course on the card to the corresponding course on the small circle may as before be called the chord, and it will easily be seen that when the ship's head is on any *compass* course the chord drawn from the corresponding *magnetic* course will lie fore-and-aft. It is this property which gives this form its value as a steering dygogram.

I.—*To steer a given Magnetic Course.*

From the magnetic course on the compass card suppose a straight line drawn to the corresponding point on the small circle, bring this line fore-and-aft.

A line parallel to this through the centre of the card gives the compass course.

II.—*From the Compass Course to find the Magnetic Course.*

Find a fore-and-aft line, *i.e.* a line parallel to the compass course which will cut the compass card and the small circle in corresponding points. These points are the magnetic course.

ENVELOPE.

The process of steering by the steering dygogram is much facilitated by the following device.

Through the several points on the small circle draw pencil lines in the direction of the corresponding points on the large circle. The locus of the ultimate intersections or the envelope of these lines will form a figure of the shape of a diamond hollowed at the sides or spear head which may be drawn in ink and the pencil lines rubbed out.

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APP. 2.

It will be found convenient in order to indicate to which portion of the circumference each side of the diamond-shaped curve refers to draw four outline ships pointing in the direction of that portion.

Examples of these envelopes for H.M.S. *Warrior* and *Trident* will be seen in Plate V.

When the envelope is constructed on the compass card its use is very simple.

I.—*To steer a given Magnetic Course.*

From the magnetic course on the compass card suppose a straight line drawn, a tangent to that branch of the envelope which has an outline ship looking in the direction of the ship's head, bring that line fore-and-aft.

II.—*From the Compass Course to find the Magnetic Course.*

Suppose a fore-and-aft line drawn, a tangent to that branch of the envelope which has an outline ship looking in the direction of the ship's head, the point at which it cuts the margin of the compass card is the magnetic course required.

APPENDIX No. 3.

INSTRUCTIONS FOR THE USE OF THE ADMIRALTY STANDARD COMPASS.

This instrument is so constructed as to answer the purpose of a steering compass and an azimuth compass.

The chief points in its construction are the following:—

1. The *bowl* is of stout copper, with the view to calm the vibrations of the needle, and the intersecting point of the axis of its gimbals is made to coincide with the point of suspension of the card, and also with the centre of the azimuth circle.

2. In observing amplitudes and azimuths, the bearings are read from the card without reference to the azimuth circle, the card being graduated to twenty minutes of arc. The *azimuth circle* is accurately graduated to minutes of arc, and may be used on shore for surveying purposes. When so used and when accurate magnetic bearings are required, the zero of the circle may be adjusted to the magnetic north, shown by the card, then clamped, and any number of magnetic bearings may then be obtained round the circle. Or by adjusting the zero of the circle to any given object, and clamping the compass to its stand, the angles of objects round the horizon may be observed and read off to the nearest minute.

3. The *magnetic needles* employed are compound bars, formed of laminæ of that kind of steel (clock-spring) which has been ascertained by numerous

experiments to be capable of receiving the greatest magnetic power. Each compass has two cards, A and J; the former is used at all times, except in stormy weather with much motion in the ship, when the heavy or J card is substituted.

Each *card* is fitted with four needles fixed vertically and equidistant on a light framework of brass screwed to the card; the pair of central needles are 7·3 inches long and the pair of external ones 5·3 inches, the whole weight of the A card being 1525 grains.

The extremities of the needles are 15° and 45° from the extremities of the diameter of the card, which is parallel to them.

Two of the pivots for card A are pointed with "native alloy," a material which is harder than steel, and which does not corrode by exposure to the atmosphere; and the ruby and agate caps are worked to a form to suit these points.

Two spare pivots pointed with hardened steel are likewise supplied, and these are gilded by the electrical process.

The two ruby headed pivots are made exclusively for the heavier card J, and no other kind of pivot is to be used with this card.

4. The *impressions of the cards* are taken after the paper has been cemented to the mica plate forming the basis; distortions from shrinking are thus prevented and a more perfect centering attained.

The zero points of the cards and the magnetic axes of the needles are adjusted to each other with great care at the Compass observatory for Her Majesty's navy at Woolwich. The various adjustments for centering, and the elimination of errors due to the displacement of sight vanes and prism, are always made at this observatory, the compass being afterwards supplied to H.M. ships free from error.

1. *When used as an Azimuth Compass.*

This compass may be immediately converted to this purpose by removing the glass or steering cover, and fixing the azimuth circle with its glass on its upper margin.

In observing amplitudes and azimuths at sea, the bearings are read from the card, without reference to the external graduated circle.

2. *When used either as an Azimuth or a Steering Compass.*

Be very careful to preserve the *pivot point* from injury, when screwing the pivot into the bowl; and place the card gently upon it; and never move the compass without first having lifted the card (by means of the side screw) against the centre-pin. This pin is so contrived as to leave the card entirely free when in its level or proper position, and it cannot touch any part of the collar, except under some violent motions of the ship, for which the gimbals do not compensate.

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APP. 3. The pivots, caps, and margins of the cards should be examined occasionally to see that their free working is not impeded by dust or fibres from the paper. When card A is not sufficiently steady, the heavy card J is to be used with one of its own ruby headed pivots.

Whenever the card works sluggishly, or injury by accident or long wear be suspected, a new cap or pivot should be screwed in, or (as the case may be) the second card with its appropriate pivot may be used. The central cap-screw, or nut on the face of the A card, must be taken off before the ruby cap is attempted to be unscrewed.

The card should be adjusted for dip by the balancing slides, when necessary.

When the bowl does not work freely on its gimbals, the axes and their bushes should be examined and slightly rubbed with plumbago.

Caution.

Experience having proved that both the pivots and caps of the A card, as also the ruby headed pivot of the J card, have received injury by the concussion arising both from the firing and exercise of heavy guns in the vicinity of the compass, it is especially enjoined that the card should on these occasions (unless it is very inconvenient to do so) be lifted off the pivot by the apparatus at the side of the bowl.

Should it be necessary to remove the prism for the purpose of cleaning it from dust or moisture the cover is arranged to slide upwards on guides, and can easily be removed and replaced by hand, taking care in replacing the prism that the lens or rounded part is placed below. All the screws attached to the prism plate have received special adjustment, and must not therefore be altered.

TABLE OF THE PRODUCTS OF EVERY
FIFTH MINUTE OF ARC,

From $0^{\circ} 0'$ to $34^{\circ} 55'$,

MULTIPLIED BY THE SINES OF THE RHUMBS,

OR

$S_1, S_2, S_3, S_4, S_5, S_6,$ and $S_7,$

$$S_1 = \text{Nat. sin. } 11^{\circ} 15' = \cdot 19509$$

$$S_2 = \text{Nat. sin. } 22^{\circ} 30' = \cdot 38268$$

$$S_3 = \text{Nat. sin. } 33^{\circ} 45' = \cdot 55557$$

$$S_4 = \text{Nat. sin. } 45^{\circ} 0' = \cdot 70710$$

$$S_5 = \text{Nat. sin. } 56^{\circ} 15' = \cdot 83147$$

$$S_6 = \text{Nat. sin. } 67^{\circ} 30' = \cdot 92388$$

$$S_7 = \text{Nat. sin. } 78^{\circ} 45' = \cdot 98078$$

AND

TABLE FOR COMPUTING THE PRODUCTS OF THE NUMBERS FROM $\cdot 001$ to $1\cdot 000$ BY
THE SINES OF THE RHUMBS.

TABLE OF NATURAL SINES, TANGENTS, &c.

PRODUCTS of ARCS

ARCS. 0° 0' - 3° 35'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 0° 0' - 3° 35'.
0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
0 5	0 1	0 2	0 3	0 4	0 4	0 5	0 5	0 5
0 10	0 2	0 4	0 6	0 7	0 8	0 9	0 10	0 10
0 15	0 3	0 6	0 8	0 11	0 12	0 14	0 15	0 15
0 20	0 4	0 8	0 11	0 14	0 17	0 18	0 20	0 20
0 25	0 5	0 10	0 14	0 18	0 21	0 23	0 24	0 25
0 30	0 6	0 11	0 17	0 21	0 25	0 28	0 29	0 30
0 35	0 7	0 13	0 19	0 25	0 29	0 32	0 34	0 35
0 40	0 8	0 15	0 22	0 28	0 33	0 37	0 39	0 40
0 45	0 9	0 17	0 25	0 32	0 37	0 42	0 44	0 45
0 50	0 10	0 19	0 28	0 35	0 42	0 46	0 50	0 50
0 55	0 11	0 21	0 31	0 39	0 46	0 51	0 54	0 55
1 0	0 12	0 23	0 33	0 42	0 50	0 55	0 59	1 0
1 5	0 13	0 25	0 36	0 46	0 54	1 0	1 4	1 5
1 10	0 14	0 27	0 39	0 49	0 58	1 5	1 9	1 10
1 15	0 15	0 29	0 42	0 53	1 2	1 9	1 14	1 15
1 20	0 16	0 31	0 44	0 57	1 7	1 14	1 18	1 20
1 25	0 17	0 33	0 47	1 0	1 11	1 19	1 23	1 25
1 30	0 18	0 34	0 50	1 4	1 15	1 23	1 28	1 30
1 35	0 19	0 36	0 53	1 7	1 19	1 28	1 33	1 35
1 40	0 20	0 38	0 56	1 11	1 23	1 32	1 38	1 40
1 45	0 21	0 40	0 58	1 14	1 27	1 37	1 43	1 45
1 50	0 22	0 42	1 1	1 18	1 32	1 42	1 48	1 50
1 55	0 22	0 44	1 4	1 21	1 36	1 46	1 53	1 55
2 0	0 23	0 46	1 7	1 25	1 40	1 51	1 58	2 0
2 5	0 24	0 48	1 9	1 28	1 44	1 56	2 3	2 5
2 10	0 25	0 50	1 12	1 32	1 48	2 0	2 8	2 10
2 15	0 26	0 52	1 15	1 36	1 52	2 5	2 12	2 15
2 20	0 27	0 54	1 18	1 39	1 56	2 9	2 17	2 20
2 25	0 28	0 56	1 21	1 43	2 1	2 14	2 22	2 25
2 30	0 29	0 57	1 23	1 46	2 5	2 19	2 27	2 30
2 35	0 30	0 59	1 26	1 50	2 9	2 23	2 32	2 35
2 40	0 31	1 1	1 29	1 53	2 13	2 28	2 37	2 40
2 45	0 32	1 3	1 32	1 57	2 17	2 32	2 42	2 45
2 50	0 33	1 5	1 34	2 0	2 21	2 37	2 47	2 50
2 55	0 34	1 7	1 37	2 4	2 26	2 42	2 52	2 55
3 0	0 35	1 9	1 40	2 7	2 30	2 46	2 57	3 0
3 5	0 36	1 11	1 43	2 11	2 34	2 51	3 1	3 5
3 10	0 37	1 13	1 46	2 14	2 38	2 56	3 6	3 10
3 15	0 38	1 15	1 48	2 18	2 42	3 0	3 11	3 15
3 20	0 39	1 17	1 51	2 21	2 46	3 5	3 16	3 20
3 25	0 40	1 19	1 54	2 25	2 50	3 9	3 21	3 25
3 30	0 41	1 20	1 57	2 29	2 55	3 14	3 26	3 30
3 35	0 42	1 22	1 59	2 32	2 59	3 19	3 31	3 35

Multiplied by the SINES of the RHUMBS.

ARCS. 3° 40' 7" 15'	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 3° 40' 7" 15'
3 40	0 43	1 24	2 2	2 36	3 3	3 23	3 36	3 40
3 45	0 44	1 26	2 5	2 39	3 7	3 28	3 41	3 45
3 50	0 45	1 28	2 8	2 43	3 11	3 33	3 46	3 50
3 55	0 46	1 30	2 11	2 46	3 15	3 37	3 51	3 55
4 0	0 47	1 32	2 13	2 50	3 20	3 42	3 55	4 0
4 5	0 48	1 34	2 16	2 53	3 24	3 46	4 0	4 5
4 10	0 49	1 36	2 19	2 57	3 28	3 51	4 5	4 10
4 15	0 50	1 38	2 22	3 0	3 32	3 56	4 10	4 15
4 20	0 51	1 40	2 24	3 4	3 36	4 0	4 15	4 20
4 25	0 52	1 41	2 27	3 7	3 40	4 5	4 20	4 25
4 30	0 53	1 43	2 30	3 11	3 45	4 10	4 25	4 30
4 35	0 54	1 45	2 33	3 15	3 49	4 14	4 30	4 35
4 40	0 55	1 47	2 36	3 18	3 53	4 19	4 35	4 40
4 45	0 56	1 49	2 38	3 22	3 57	4 23	4 40	4 45
4 50	0 57	1 51	2 41	3 25	4 1	4 28	4 44	4 50
4 55	0 58	1 53	2 44	3 29	4 5	4 33	4 49	4 55
5 0	0 59	1 55	2 47	3 32	4 9	4 37	4 54	5 0
5 5	1 0	1 57	2 49	3 36	4 14	4 42	4 59	5 5
5 10	1 1	1 59	2 52	3 39	4 18	4 46	5 4	5 10
5 15	1 2	2 1	2 55	3 43	4 22	4 51	5 9	5 15
5 20	1 2	2 3	2 58	3 46	4 26	4 56	5 14	5 20
5 25	1 3	2 4	3 1	3 50	4 30	5 0	5 19	5 25
5 30	1 4	2 6	3 3	3 53	4 34	5 5	5 24	5 30
5 35	1 5	2 8	3 6	3 57	4 39	5 10	5 29	5 35
5 40	1 6	2 10	3 9	4 0	4 43	5 14	5 34	5 40
5 45	1 7	2 12	3 12	4 4	4 47	5 19	5 38	5 45
5 50	1 8	2 14	3 14	4 8	4 51	5 23	5 43	5 50
5 55	1 9	2 16	3 17	4 11	4 55	5 28	5 48	5 55
6 0	1 10	2 18	3 20	4 15	4 59	5 33	5 53	6 0
6 5	1 11	2 20	3 23	4 18	5 4	5 37	5 58	6 5
6 10	1 12	2 22	3 26	4 22	5 8	5 42	6 3	6 10
6 15	1 13	2 24	3 28	4 25	5 12	5 46	6 8	6 15
6 20	1 14	2 25	3 31	4 29	5 16	5 51	6 13	6 20
6 25	1 15	2 27	3 34	4 32	5 20	5 56	6 18	6 25
6 30	1 16	2 29	3 37	4 36	5 24	6 0	6 23	6 30
6 35	1 17	2 31	3 39	4 39	5 28	6 5	6 27	6 35
6 40	1 18	2 33	3 42	4 43	5 33	6 10	6 32	6 40
6 45	1 19	2 35	3 45	4 46	5 37	6 14	6 37	6 45
6 50	1 20	2 37	3 48	4 50	5 41	6 19	6 42	6 50
6 55	1 21	2 39	3 51	4 54	5 45	6 23	6 47	6 55
7 0	1 22	2 41	3 53	4 57	5 49	6 28	6 52	7 0
7 5	1 23	2 43	3 56	5 1	5 53	6 33	6 57	7 5
7 10	1 24	2 45	3 59	5 4	5 58	6 37	7 2	7 10
7 15	1 25	2 47	4 2	5 8	6 2	6 42	7 7	7 15

Products of Arcs

ARCS. 7°20'-10°55'	S (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 7°20'-10°55'
7 20	1 26	2 49	4 4	5 11	6 6	6 47	7 12	7 20
7 25	1 27	2 51	4 7	5 15	6 10	6 51	7 17	7 25
7 30	1 28	2 52	4 10	5 18	6 14	6 56	7 21	7 30
7 35	1 29	2 54	4 13	5 22	6 18	7 0	7 26	7 35
7 40	1 30	2 56	4 16	5 25	6 23	7 5	7 31	7 40
7 45	1 31	2 58	4 18	5 29	6 27	7 10	7 36	7 45
7 50	1 32	3 0	4 21	5 32	6 31	7 14	7 41	7 50
7 55	1 33	3 2	4 24	5 36	6 35	7 19	7 46	7 55
8 0	1 34	3 4	4 27	5 39	6 39	7 24	7 51	8 0
8 5	1 35	3 6	4 29	5 43	6 43	7 28	7 56	8 5
8 10	1 36	3 8	4 32	5 47	6 47	7 33	8 1	8 10
8 15	1 37	3 10	4 35	5 50	6 52	7 37	8 6	8 15
8 20	1 38	3 11	4 38	5 54	6 56	7 42	8 10	8 20
8 25	1 39	3 13	4 40	5 57	7 0	7 46	8 15	8 25
8 30	1 40	3 15	4 43	6 1	7 4	7 51	8 20	8 30
8 35	1 40	3 17	4 46	6 4	7 8	7 56	8 25	8 35
8 40	1 41	3 19	4 49	6 8	7 12	8 0	8 30	8 40
8 45	1 42	3 21	4 52	6 11	7 16	8 5	8 35	8 45
8 50	1 43	3 23	4 54	6 15	7 21	8 10	8 40	8 50
8 55	1 44	3 25	4 57	6 18	7 25	8 14	8 45	8 55
9 0	1 45	3 27	5 0	6 22	7 29	8 19	8 50	9 0
9 5	1 46	3 28	5 3	6 25	7 33	8 23	8 55	9 5
9 10	1 47	3 30	5 6	6 29	7 37	8 28	8 59	9 10
9 15	1 48	3 32	5 8	6 32	7 41	8 33	9 4	9 15
9 20	1 49	3 34	5 11	6 36	7 46	8 37	9 9	9 20
9 25	1 50	3 36	5 14	6 39	7 50	8 42	9 14	9 25
9 30	1 51	3 38	5 17	6 43	7 54	8 47	9 19	9 30
9 35	1 52	3 40	5 20	6 47	7 58	8 51	9 24	9 35
9 40	1 53	3 42	5 22	6 50	8 2	8 56	9 29	9 40
9 45	1 54	3 44	5 25	6 54	8 6	9 0	9 34	9 45
9 50	1 55	3 46	5 28	6 57	8 11	9 5	9 39	9 50
9 55	1 56	3 48	5 30	7 1	8 15	9 10	9 44	9 55
10 0	1 57	3 50	5 33	7 4	8 19	9 14	9 48	10 0
10 5	1 58	3 52	5 36	7 8	8 23	9 19	9 53	10 5
10 10	1 59	3 53	5 39	7 11	8 27	9 24	9 58	10 10
10 15	2 0	3 55	5 42	7 15	8 31	9 28	10 3	10 15
10 20	2 1	3 57	5 44	7 18	8 36	9 33	10 8	10 20
10 25	2 2	3 59	5 47	7 22	8 40	9 38	10 13	10 25
10 30	2 3	4 1	5 50	7 25	8 44	9 42	10 18	10 30
10 35	2 4	4 3	5 53	7 29	8 48	9 47	10 23	10 35
10 40	2 5	4 5	5 56	7 33	8 52	9 51	10 28	10 40
10 45	2 6	4 7	5 58	7 36	8 56	9 56	10 33	10 45
10 50	2 7	4 9	6 1	7 40	9 0	10 1	10 38	10 50
10 55	2 8	4 11	6 4	7 43	9 4	10 5	10 42	10 55

Multiplied by the Sines of the Rhumbs—*continued.*

ARCS. 11°-14° 35'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 11°-14° 35'.
11 0	2 9	4 13	6 7	7 47	9 9	10 10	10 47	11 0
11 5	2 10	4 14	6 10	7 50	9 13	10 15	10 52	11 5
11 10	2 11	4 16	6 12	7 54	9 17	10 19	10 57	11 10
11 15	2 12	4 18	6 15	7 57	9 21	10 24	11 2	11 15
11 20	2 13	4 20	6 18	8 1	9 25	10 28	11 7	11 20
11 25	2 14	4 22	6 20	8 5	9 30	10 33	11 12	11 25
11 30	2 15	4 24	6 23	8 8	9 34	10 37	11 17	11 30
11 35	2 16	4 26	6 26	8 12	9 38	10 42	11 22	11 35
11 40	2 17	4 28	6 29	8 15	9 42	10 47	11 27	11 40
11 45	2 18	4 30	6 32	8 19	9 46	10 51	11 32	11 45
11 50	2 19	4 32	6 34	8 22	9 50	10 56	11 36	11 50
11 55	2 19	4 34	6 37	8 26	9 55	11 0	11 41	11 55
12 0	2 20	4 36	6 40	8 29	9 59	11 5	11 46	12 0
12 5	2 21	4 38	6 43	8 33	10 3	11 10	11 51	12 5
12 10	2 22	4 39	6 46	8 36	10 7	11 14	11 56	12 10
12 15	2 23	4 41	6 48	8 40	10 11	11 19	12 1	12 15
12 20	2 24	4 43	6 51	8 43	10 15	11 24	12 6	12 20
12 25	2 25	4 45	6 54	8 47	10 20	11 28	12 11	12 25
12 30	2 26	4 47	6 57	8 50	10 24	11 33	12 16	12 30
12 35	2 27	4 49	7 0	8 54	10 28	11 38	12 20	12 35
12 40	2 28	4 51	7 2	8 57	10 32	11 42	12 25	12 40
12 45	2 29	4 53	7 5	9 1	10 36	11 47	12 30	12 45
12 50	2 30	4 55	7 8	9 4	10 40	11 51	12 35	12 50
12 55	2 31	4 56	7 10	9 8	10 45	11 56	12 40	12 55
13 0	2 32	4 58	7 13	9 12	10 49	12 1	12 45	13 0
13 5	2 33	5 0	7 16	9 15	10 53	12 5	12 50	13 5
13 10	2 34	5 2	7 19	9 19	10 57	12 10	12 55	13 10
13 15	2 35	5 4	7 22	9 22	11 1	12 14	13 0	13 15
13 20	2 36	5 6	7 24	9 26	11 5	12 19	13 5	13 20
13 25	2 37	5 8	7 27	9 29	11 9	12 24	13 10	13 25
13 30	2 38	5 10	7 30	9 33	11 13	12 28	13 14	13 30
13 35	2 39	5 12	7 33	9 36	11 18	12 33	13 19	13 35
13 40	2 40	5 14	7 36	9 40	11 22	12 38	13 24	13 40
13 45	2 41	5 16	7 38	9 43	11 26	12 42	13 29	13 45
13 50	2 42	5 18	7 41	9 47	11 30	12 47	13 34	13 50
13 55	2 43	5 20	7 44	9 50	11 34	12 52	13 39	13 55
14 0	2 44	5 21	7 47	9 54	11 38	12 56	13 44	14 0
14 5	2 45	5 23	7 49	9 57	11 42	13 1	13 49	14 5
14 10	2 46	5 25	7 52	10 1	11 47	13 5	13 54	14 10
14 15	2 47	5 27	7 55	10 5	11 51	13 10	13 59	14 15
14 20	2 48	5 29	7 58	10 8	11 55	13 15	14 3	14 20
14 25	2 49	5 31	8 0	10 12	11 59	13 19	14 8	14 25
14 30	2 50	5 33	8 3	10 15	12 3	13 24	14 13	14 30
14 35	2 51	5 35	8 6	10 19	12 7	13 28	14 18	14 35

Products of Arcs

ARCS. 14° 40' - 18° 15'	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 14° 40' - 18° 15'
14 40	2 52	5 37	8 9	10 22	12 12	13 33	14 23	14 40
14 45	2 53	5 39	8 12	10 26	12 16	13 37	14 28	14 45
14 50	2 54	5 41	8 14	10 29	12 20	13 42	14 33	14 50
14 55	2 55	5 42	8 17	10 33	12 24	13 47	14 38	14 55
15 0	2 56	5 44	8 20	10 36	12 28	13 51	14 43	15 0
15 5	2 57	5 46	8 23	10 40	12 32	13 56	14 48	15 5
15 10	2 58	5 48	8 26	10 43	12 37	14 1	14 53	15 10
15 15	2 59	5 50	8 28	10 47	12 41	14 5	14 58	15 15
15 20	2 59	5 52	8 31	10 51	12 45	14 10	15 2	15 20
15 25	3 0	5 54	8 34	10 54	12 49	14 14	15 7	15 25
15 30	3 1	5 56	8 37	10 58	12 53	14 19	15 12	15 30
15 35	3 2	5 58	8 40	11 1	12 58	14 24	15 17	15 35
15 40	3 3	6 0	8 42	11 5	13 2	14 28	15 22	15 40
15 45	3 4	6 2	8 45	11 8	13 6	14 33	15 27	15 45
15 50	3 5	6 4	8 48	11 12	13 10	14 38	15 32	15 50
15 55	3 6	6 6	8 50	11 15	13 14	14 42	15 37	15 55
16 0	3 7	6 7	8 53	11 19	13 18	14 47	15 42	16 0
16 5	3 8	6 9	8 56	11 22	13 22	14 51	15 47	16 5
16 10	3 9	6 11	8 59	11 26	13 27	14 56	15 51	16 10
16 15	3 10	6 13	9 2	11 29	13 31	15 1	15 56	16 15
16 20	3 11	6 15	9 4	11 33	13 35	15 5	16 1	16 20
16 25	3 12	6 17	9 7	11 36	13 39	15 10	16 6	16 25
16 30	3 13	6 19	9 10	11 40	13 43	15 15	16 11	16 30
16 35	3 14	6 21	9 13	11 44	13 47	15 19	16 16	16 35
16 40	3 15	6 23	9 16	11 47	13 51	15 24	16 21	16 40
16 45	3 16	6 25	9 18	11 51	13 55	15 29	16 26	16 45
16 50	3 17	6 27	9 21	11 54	14 0	15 33	16 31	16 50
16 55	3 18	6 28	9 24	11 58	14 4	15 38	16 35	16 55
17 0	3 19	6 30	9 27	12 1	14 8	15 42	16 40	17 0
17 5	3 20	6 32	9 30	12 5	14 12	15 47	16 45	17 5
17 10	3 21	6 34	9 32	12 8	14 16	15 52	16 50	17 10
17 15	3 22	6 36	9 35	12 12	14 20	15 56	16 55	17 15
17 20	3 23	6 38	9 38	12 15	14 25	16 1	17 0	17 20
17 25	3 24	6 40	9 40	12 19	14 29	16 5	17 5	17 25
17 30	3 25	6 42	9 43	12 22	14 33	16 10	17 10	17 30
17 35	3 26	6 44	9 46	12 26	14 37	16 15	17 15	17 35
17 40	3 27	6 46	9 49	12 30	14 41	16 19	17 20	17 40
17 45	3 28	6 48	9 52	12 33	14 45	16 24	17 25	17 45
17 50	3 29	6 50	9 54	12 37	14 50	16 29	17 29	17 50
17 55	3 30	6 52	9 57	12 40	14 54	16 33	17 34	17 55
18 0	3 31	6 53	10 0	12 44	14 58	16 38	17 39	18 0
18 5	3 32	6 55	10 3	12 47	15 2	16 42	17 44	18 5
18 10	3 33	6 57	10 6	12 51	15 6	16 47	17 49	18 10
18 15	3 34	6 59	10 8	12 54	15 10	16 52	17 54	18 15

Multiplied by the Sines of the Rhumbs—*continued.*

ARCS. 18°20'–21°55'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 18°20'–21°55'.
18 20	8 35	7 1	10 11	12 58	15 15	16 56	17 59	18 20
18 25	8 36	7 3	10 14	13 1	15 19	17 1	18 4	18 25
18 30	8 37	7 5	10 17	13 5	15 23	17 6	18 9	18 30
18 35	8 38	7 7	10 20	13 8	15 27	17 10	18 14	18 35
18 40	8 39	7 9	10 22	13 12	15 31	17 15	18 18	18 40
18 45	8 40	7 10	10 25	13 16	15 36	17 20	18 23	18 45
18 50	8 40	7 12	10 28	13 19	15 40	17 24	18 28	18 50
18 55	8 41	7 14	10 30	13 23	15 44	17 29	18 33	18 55
19 0	8 42	7 16	10 33	13 26	15 48	17 33	18 38	19 0
19 5	8 43	7 18	10 36	13 30	15 52	17 38	18 43	19 5
19 10	8 44	7 20	10 39	13 33	15 56	17 43	18 48	19 10
19 15	8 45	7 22	10 41	13 37	16 0	17 47	18 53	19 15
19 20	8 46	7 24	10 44	13 40	16 4	17 52	18 58	19 20
19 25	8 47	7 26	10 47	13 44	16 8	17 56	19 3	19 25
19 30	8 48	7 28	10 50	13 47	16 13	18 1	19 7	19 30
19 35	8 49	7 30	10 53	13 51	16 17	18 5	19 12	19 35
19 40	8 50	7 32	10 56	13 54	16 21	18 10	19 17	19 40
19 45	8 51	7 34	10 58	13 58	16 25	18 15	19 22	19 45
19 50	8 52	7 35	11 1	14 1	16 29	18 19	19 27	19 50
19 55	8 53	7 37	11 4	14 5	16 33	18 24	19 32	19 55
20 0	8 54	7 39	11 7	14 8	16 38	18 29	19 37	20 0
20 5	8 55	7 41	11 10	14 12	16 42	18 33	19 42	20 5
20 10	8 56	7 43	11 12	14 16	16 46	18 38	19 47	20 10
20 15	8 57	7 45	11 15	14 19	16 50	18 42	19 52	20 15
20 20	8 58	7 47	11 18	14 23	16 54	18 47	19 57	20 20
20 25	8 59	7 49	11 20	14 26	16 58	18 52	20 2	20 25
20 30	9 0	7 51	11 23	14 30	17 3	18 56	20 6	20 30
20 35	9 1	7 53	11 26	14 33	17 7	19 1	20 11	20 35
20 40	9 2	7 55	11 29	14 37	17 11	19 6	20 16	20 40
20 45	9 3	7 56	11 32	14 40	17 15	19 10	20 21	20 45
20 50	9 4	7 58	11 34	14 44	17 19	19 15	20 26	20 50
20 55	9 5	8 0	11 37	14 47	17 23	19 20	20 31	20 55
21 0	9 6	8 2	11 40	14 51	17 28	19 24	20 36	21 0
21 5	9 7	8 4	11 43	14 54	17 32	19 29	20 41	21 5
21 10	9 8	8 6	11 46	14 58	17 36	19 33	20 46	21 10
21 15	9 9	8 8	11 48	15 1	17 40	19 38	20 51	21 15
21 20	9 10	8 10	11 51	15 5	17 44	19 43	20 55	21 20
21 25	9 11	8 12	11 54	15 8	17 48	19 47	21 0	21 25
21 30	9 12	8 14	11 57	15 12	17 53	19 52	21 5	21 30
21 35	9 13	8 16	12 0	15 16	17 57	19 56	21 10	21 35
21 40	9 14	8 18	12 2	15 19	18 1	20 1	21 15	21 40
21 45	9 15	8 20	12 5	15 23	18 5	20 6	21 20	21 45
21 50	9 16	8 21	12 8	15 26	18 9	20 10	21 25	21 50
21 55	9 17	8 23	12 10	15 30	18 13	20 15	21 30	21 55

Products of Arcs

ARCS. 22°-25° 35'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 22°-25° 35'.
22 0	4 18	8 25	12 13	15 33	18 17	20 20	21 35	22 0
22 5	4 19	8 27	12 16	15 37	18 21	20 24	21 40	22 5
22 10	4 20	8 29	12 19	15 40	18 26	20 29	21 44	22 10
22 15	4 20	8 31	12 22	15 44	18 30	20 33	21 49	22 15
22 20	4 21	8 33	12 24	15 48	18 34	20 38	21 54	22 20
22 25	4 22	8 35	12 27	15 51	18 38	20 43	21 59	22 25
22 30	4 23	8 37	12 30	15 55	18 42	20 47	22 4	22 30
22 35	4 24	8 38	12 33	15 58	18 46	20 52	22 9	22 35
22 40	4 25	8 40	12 36	16 2	18 51	20 57	22 14	22 40
22 45	4 26	8 42	12 38	16 5	18 55	21 1	22 19	22 45
22 50	4 27	8 44	12 41	16 9	18 59	21 6	22 24	22 50
22 55	4 28	8 46	12 44	16 12	19 3	21 10	22 29	22 55
23 0	4 29	8 48	12 47	16 16	19 7	21 15	22 33	23 0
23 5	4 30	8 50	12 50	16 19	19 11	21 20	22 38	23 5
23 10	4 31	8 52	12 52	16 23	19 16	21 24	22 43	23 10
23 15	4 32	8 54	12 55	16 26	19 20	21 29	22 48	23 15
23 20	4 33	8 56	12 58	16 30	19 24	21 33	22 53	23 20
23 25	4 34	8 58	13 0	16 34	19 28	21 38	22 58	23 25
23 30	4 35	9 0	13 3	16 37	19 32	21 43	23 3	23 30
23 35	4 36	9 2	13 6	16 41	19 36	21 47	23 8	23 35
23 40	4 37	9 3	13 9	16 44	19 41	21 52	23 13	23 40
23 45	4 38	9 5	13 12	16 48	19 45	21 56	23 18	23 45
23 50	4 39	9 7	13 14	16 51	19 49	22 1	23 22	23 50
23 55	4 40	9 9	13 17	16 55	19 53	22 6	23 27	23 55
24 0	4 41	9 11	13 20	16 58	19 58	22 10	23 32	24 0
24 5	4 42	9 13	13 23	17 2	20 2	22 15	23 37	24 5
24 10	4 42	9 15	13 26	17 6	20 6	22 20	23 42	24 10
24 15	4 43	9 16	13 29	17 9	20 10	22 24	23 47	24 15
24 20	4 44	9 18	13 32	17 12	20 14	22 28	23 52	24 20
24 25	4 45	9 20	13 34	17 16	20 18	22 33	23 57	24 25
24 30	4 46	9 22	13 36	17 20	20 22	22 38	24 2	24 30
24 35	4 47	9 24	13 39	17 23	20 26	22 43	24 7	24 35
24 40	4 48	9 26	13 42	17 26	20 30	22 48	24 12	24 40
24 45	4 49	9 28	13 45	17 30	20 35	22 52	24 17	24 45
24 50	4 50	9 30	13 48	17 34	20 40	22 56	24 22	24 50
24 55	4 51	9 32	13 51	17 37	20 44	23 1	24 27	24 55
25 0	4 52	9 34	13 54	17 40	20 48	23 6	24 32	25 0
25 5	4 53	9 36	13 57	17 44	20 52	23 11	24 36	25 5
25 10	4 54	9 38	14 0	17 48	20 56	23 16	24 40	25 10
25 15	4 55	9 40	14 2	17 51	21 0	23 20	24 45	25 15
25 20	4 56	9 42	14 4	17 54	21 4	23 24	24 50	25 20
25 25	4 57	9 44	14 7	17 58	21 8	23 29	24 55	25 25
25 30	4 58	9 46	14 10	18 2	21 12	23 34	25 0	25 30
25 35	4 59	9 48	14 13	18 5	21 16	23 38	25 5	25 35

Multiplied by the Sines of the Rhumbs—*continued.*

ARCS. 25°40'–29°15'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67- 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 25°40'–29°15'.
25 40	5 0	9 50	14 16	18 8	21 20	23 42	25 10	25 40
25 45	5 1	9 51	14 18	18 12	21 25	23 47	25 15	25 45
25 50	5 2	9 53	14 20	18 16	21 30	23 52	25 20	25 50
25 55	5 3	9 55	14 23	18 20	21 34	23 57	25 25	25 55
26 0	5 4	9 56	14 26	18 24	21 38	24 2	25 30	26 0
26 5	5 5	9 58	14 29	18 27	21 42	24 6	25 35	26 5
26 10	5 6	10 0	14 32	18 30	21 46	24 10	25 40	26 10
26 15	5 7	10 2	14 35	18 34	21 50	24 15	25 45	26 15
26 20	5 8	10 4	14 38	18 38	21 54	24 20	25 50	26 20
26 25	5 9	10 6	14 41	18 41	21 58	25 24	25 55	26 25
26 30	5 10	10 8	14 44	18 44	22 2	24 28	26 0	26 30
26 35	5 11	10 10	14 46	18 48	22 6	24 33	26 5	26 35
26 40	5 12	10 12	14 48	18 52	22 10	24 38	26 10	26 40
26 45	5 13	10 14	14 51	18 55	22 14	24 43	26 15	26 45
26 50	5 14	10 16	14 54	18 58	22 18	24 48	26 20	26 50
26 55	5 15	10 18	14 57	19 2	22 22	24 52	26 24	26 55
27 0	5 16	10 20	15 0	19 6	22 26	24 56	26 28	27 0
27 5	5 17	10 22	15 3	19 9	22 31	25 1	26 33	27 5
27 10	5 18	10 24	15 6	19 12	22 36	25 6	26 38	27 10
27 15	5 19	10 26	15 9	19 16	22 40	25 11	26 43	27 15
27 20	5 20	10 28	15 12	19 20	22 44	25 16	26 48	27 20
27 25	5 21	10 30	15 14	19 23	22 48	25 20	26 53	27 25
27 30	5 22	10 32	15 16	19 26	22 52	25 24	26 58	27 30
27 35	5 23	10 34	15 19	19 30	22 56	25 29	27 3	27 35
27 40	5 24	10 36	15 22	19 34	23 0	25 34	27 8	27 40
27 45	5 25	10 38	15 25	19 37	23 4	25 39	27 13	27 45
27 50	5 26	10 40	15 28	19 40	23 8	25 44	27 18	27 50
27 55	5 27	10 41	15 31	19 44	23 12	25 48	27 23	27 55
28 0	5 28	10 42	15 34	19 48	23 16	25 52	27 28	28 0
28 5	5 29	10 44	15 36	19 51	23 20	25 57	27 33	28 5
28 10	5 30	10 46	15 38	19 54	23 24	26 2	27 38	28 10
28 15	5 31	10 48	15 41	19 58	23 29	26 6	27 43	28 15
28 20	5 32	10 50	15 44	20 2	23 34	26 10	27 48	28 20
28 25	5 33	10 52	15 47	20 6	23 38	26 15	27 53	28 25
28 30	5 34	10 54	15 50	20 10	23 42	26 20	27 58	28 30
28 35	5 35	10 56	15 53	20 13	23 46	26 25	28 2	28 35
28 40	5 36	10 58	15 56	20 16	23 50	26 30	28 6	28 40
28 45	5 37	11 0	15 58	20 20	23 54	26 34	28 11	28 45
28 50	5 38	11 2	16 0	20 24	23 58	26 38	28 16	28 50
28 55	5 39	11 4	16 3	20 27	24 2	26 43	28 21	28 55
29 0	5 40	11 6	16 6	20 30	24 6	26 48	28 26	29 0
29 5	5 41	11 8	16 9	20 34	24 10	26 52	28 31	29 5
29 10	5 42	11 10	16 12	20 38	24 14	26 56	28 36	29 10
29 15	5 43	11 12	16 15	20 41	24 18	27 1	28 41	29 15

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Products of Arcs

ARCS. 29°20'-32°55'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 29°20'-32°55'.
29 20	5 44	11 14	16 18	20 44	24 26	27 6	28 46	29 20
29 25	5 45	11 16	16 21	20 48	24 28	27 10	28 51	29 25
29 30	5 46	11 18	16 24	20 52	24 32	27 14	28 56	29 30
29 35	5 47	11 20	16 26	20 55	24 36	27 19	29 1	29 35
29 40	5 48	11 22	16 28	20 58	24 40	27 24	29 6	29 40
29 45	5 49	11 23	16 31	21 2	24 44	27 29	29 11	29 45
29 50	5 50	11 24	16 34	21 6	24 48	27 34	29 16	29 50
29 55	5 51	11 26	16 37	21 9	24 52	27 39	29 21	29 55
30 0	5 52	11 28	16 40	21 12	24 56	27 42	29 26	30 0
30 5	5 53	11 30	16 43	21 16	25 0	27 47	29 31	30 5
30 10	5 54	11 32	16 46	21 20	25 4	27 52	29 36	30 10
30 15	5 55	11 34	16 49	21 23	25 9	27 57	29 41	30 15
30 20	5 56	11 36	16 52	21 26	25 14	28 2	29 46	30 20
30 25	5 57	11 38	16 54	21 30	25 18	28 6	29 51	30 25
30 30	5 58	11 40	16 56	21 34	25 22	28 10	29 56	30 30
30 35	5 58	11 42	16 59	21 38	25 26	28 15	30 0	30 35
30 40	5 59	11 44	17 2	21 42	25 30	28 20	30 4	30 40
30 45	5 59	11 46	17 5	21 45	25 34	28 24	30 9	30 45
30 50	6 0	11 48	17 8	21 48	25 38	28 28	30 14	30 50
30 55	6 1	11 50	17 11	21 52	25 42	28 33	30 19	30 55
31 0	6 2	11 52	17 14	21 56	25 46	28 38	30 24	31 0
31 5	6 3	11 54	17 17	21 59	25 51	28 43	30 29	31 5
31 10	6 4	11 56	17 20	22 2	25 56	28 48	30 34	31 10
31 15	6 5	11 58	17 22	22 6	26 0	28 52	30 39	31 15
31 20	6 6	12 0	17 24	22 10	26 4	28 56	30 44	31 20
31 25	6 7	12 2	17 27	22 13	26 8	29 1	30 49	31 25
31 30	6 8	12 4	17 30	22 16	26 12	29 6	30 54	31 30
31 35	6 9	12 6	17 33	22 20	26 16	29 11	30 59	31 35
31 40	6 10	12 8	17 36	22 24	26 20	29 16	31 4	31 40
31 45	6 11	12 10	17 38	22 27	26 24	29 20	31 9	31 45
31 50	6 12	12 12	17 40	22 30	26 28	29 24	31 14	31 50
31 55	6 13	12 13	17 43	22 34	26 32	29 29	31 19	31 55
32 0	6 14	12 14	17 46	22 38	26 36	29 34	31 24	32 0
32 5	6 15	12 16	17 49	22 41	26 40	29 38	31 29	32 5
32 10	6 16	12 18	17 52	22 44	26 44	29 42	31 34	32 10
32 15	6 17	12 20	17 55	22 48	26 48	29 47	31 38	32 15
32 20	6 18	12 22	17 58	22 52	26 52	29 52	31 42	32 20
32 25	6 19	12 24	18 1	22 55	26 57	29 57	31 47	32 25
32 30	6 20	12 26	18 4	22 58	27 2	30 2	31 52	32 30
32 35	6 21	12 28	18 6	23 2	27 6	30 6	31 57	32 35
32 40	6 22	12 30	18 8	23 6	27 10	30 10	32 2	32 40
32 45	6 23	12 32	18 11	23 9	27 14	30 15	32 7	32 45
32 50	6 24	12 34	18 14	23 12	27 18	30 20	32 12	32 50
32 55	6 25	12 36	18 17	23 15	27 22	30 25	32 17	32 55

Multiplied by the Sines of the Rhumbs—concluded.

ARCS. 33°-34°55'.	S ₁ (Sin. 11° 15'.)	S ₂ (Sin. 22° 30'.)	S ₃ (Sin. 33° 45'.)	S ₄ (Sin. 45°.)	S ₅ (Sin. 56° 15'.)	S ₆ (Sin. 67° 30'.)	S ₇ (Sin. 78° 45'.)	ARCS. 33°-34°55'.
33° 0'	6 26	12 38	18 20	23 20	27 26	30 30	32 22	33° 0'
33 5	6 27	12 40	18 23	23 24	27 30	30 34	32 27	33 5
33 10	6 28	12 42	18 26	23 28	27 34	30 38	32 32	33 10
33 15	6 29	12 44	18 29	23 31	27 38	30 43	32 37	33 15
33 20	6 30	12 46	18 32	23 34	27 42	30 48	32 42	33 20
33 25	6 31	12 48	18 34	23 38	27 46	30 53	32 47	33 25
33 30	6 32	12 50	18 36	23 42	27 50	30 58	32 52	33 30
33 35	6 33	12 52	18 39	23 45	27 55	31 2	32 57	33 35
33 40	6 34	12 54	18 42	23 48	28 0	31 6	33 2	33 40
33 45	6 35	12 55	18 45	23 52	28 4	31 11	33 6	33 45
33 50	6 36	12 56	18 48	23 56	28 8	31 16	33 10	33 50
33 55	6 37	12 58	18 51	23 59	28 12	31 20	33 15	33 55
34 0	6 38	13 0	18 54	24 2	28 16	31 24	33 20	34 0
34 5	6 39	13 2	18 57	24 6	28 20	31 29	33 25	34 5
34 10	6 40	13 4	19 0	24 10	28 24	31 34	33 30	34 10
34 15	6 41	13 6	19 2	24 13	28 28	31 39	33 35	34 15
34 20	6 42	13 8	19 4	24 16	28 32	31 44	33 40	34 20
34 25	6 43	13 10	19 7	24 20	28 36	31 48	33 45	34 25
34 30	6 44	13 12	19 10	24 24	28 40	31 52	33 50	34 30
34 35	6 45	13 14	19 13	24 27	28 45	31 56	33 55	34 35
34 40	6 46	13 16	19 16	24 30	28 50	32 1	34 0	34 40
34 45	6 47	13 18	19 18	24 34	28 54	32 5	34 5	34 45
34 50	6 48	13 20	19 20	24 38	28 58	32 10	34 10	34 50
34 55	6 49	13 22	19 23	24 41	29 2	32 15	34 15	34 55

TABLE for COMPUTING the PRODUCTS of the NUMBERS from .001 to 1.000 by
the SINES of the RHUMBS.

NUMBERS.	S ₁ .19509 (Sin. 11° 15'.)	S ₂ .38268 (Sin. 22° 30'.)	S ₃ .55557 (Sin. 33° 45'.)	S ₄ .70710 (Sin. 45°.)	S ₅ .83147 (Sin. 56° 15'.)	S ₆ .92388 (Sin. 67° 30'.)	S ₇ .98078 (Sin. 78° 45'.)	NUMBERS.
.001	.000	.000	.001	.001	.001	.001	.001	.001
.002	.000	.001	.001	.001	.002	.002	.002	.002
.003	.001	.001	.002	.002	.003	.003	.003	.003
.004	.001	.002	.002	.003	.003	.004	.004	.004
.005	.001	.002	.003	.004	.004	.005	.005	.005
.006	.001	.002	.003	.004	.005	.006	.006	.006
.007	.001	.003	.004	.005	.006	.007	.007	.007
.008	.002	.003	.004	.006	.007	.007	.008	.008
.009	.002	.003	.006	.006	.007	.008	.009	.009
.01	.002	.004	.006	.007	.008	.009	.010	.01
.02	.004	.007	.011	.014	.017	.018	.020	.02
.03	.006	.011	.017	.021	.025	.028	.030	.03
.04	.008	.015	.022	.028	.033	.037	.039	.04
.05	.010	.019	.028	.035	.042	.046	.049	.05
.06	.012	.023	.033	.042	.050	.055	.059	.06
.07	.014	.027	.039	.049	.058	.065	.069	.07
.08	.016	.031	.044	.056	.066	.074	.078	.08
.09	.018	.034	.050	.063	.075	.083	.088	.09
.10	.020	.038	.055	.071	.083	.092	.098	.10
.11	.021	.042	.061	.078	.091	.102	.108	.11
.12	.023	.046	.067	.085	.100	.111	.118	.12
.13	.025	.050	.072	.092	.108	.120	.128	.13
.14	.027	.054	.078	.099	.116	.129	.137	.14
.15	.029	.057	.083	.106	.125	.139	.147	.15
.16	.031	.061	.089	.113	.133	.148	.157	.16
.17	.033	.065	.094	.120	.141	.157	.167	.17
.18	.035	.069	.100	.127	.150	.166	.177	.18
.19	.037	.073	.106	.134	.158	.176	.186	.19
.20	.039	.077	.111	.141	.166	.185	.196	.20
.21	.041	.080	.117	.148	.175	.194	.206	.21
.22	.043	.084	.122	.156	.183	.203	.216	.22
.23	.045	.088	.128	.163	.191	.212	.226	.23
.24	.047	.092	.133	.170	.200	.222	.235	.24
.25	.049	.096	.139	.177	.208	.231	.245	.25
.26	.051	.099	.144	.184	.216	.240	.255	.26
.27	.053	.103	.150	.191	.224	.249	.265	.27
.28	.055	.107	.156	.198	.233	.259	.275	.28
.29	.057	.111	.161	.205	.241	.268	.284	.29
.30	.059	.115	.167	.212	.250	.277	.294	.30
.31	.060	.119	.172	.219	.258	.286	.304	.31

Table for Computing the Products of the Numbers from .001 to 1.000—*cont.*

NUMBERS.	S ₁ ·19509 (Sin. 11° 15'.)	S ₂ ·38268 (Sin. 22° 30'.)	S ₃ ·55537 (Sin. 33° 45'.)	S ₄ ·70710 (Sin. 45°.)	S ₅ ·83147 (Sin. 56° 15'.)	S ₆ ·92388 (Sin. 67° 30'.)	S ₇ ·98078 (Sin. 78° 45'.)	NUMBERS.
·32	·062	·122	·178	·226	·266	·296	·314	·32
·33	·064	·126	·183	·233	·274	·305	·324	·33
·34	·066	·130	·189	·240	·282	·314	·333	·34
·35	·068	·134	·194	·248	·291	·323	·343	·35
·36	·070	·138	·200	·255	·299	·333	·353	·36
·37	·072	·142	·206	·262	·308	·342	·363	·37
·38	·074	·145	·211	·269	·316	·351	·373	·38
·39	·076	·149	·217	·276	·324	·360	·383	·39
·40	·078	·153	·222	·283	·333	·370	·392	·40
·41	·080	·157	·228	·290	·341	·379	·402	·41
·42	·082	·161	·233	·297	·349	·388	·412	·42
·43	·084	·165	·239	·304	·358	·397	·422	·43
·44	·086	·168	·244	·311	·366	·407	·432	·44
·45	·088	·172	·250	·318	·374	·416	·441	·45
·46	·090	·176	·256	·325	·382	·425	·451	·46
·47	·092	·180	·261	·332	·391	·434	·461	·47
·48	·094	·184	·267	·339	·399	·443	·471	·48
·49	·096	·188	·272	·346	·407	·453	·481	·49
·50	·098	·191	·278	·354	·416	·462	·490	·50
·51	·100	·195	·283	·361	·424	·471	·500	·51
·52	·101	·199	·289	·368	·432	·480	·510	·52
·53	·103	·203	·294	·375	·441	·490	·520	·53
·54	·105	·207	·300	·382	·449	·499	·530	·54
·55	·107	·210	·305	·389	·457	·508	·539	·55
·56	·109	·214	·311	·396	·466	·517	·549	·56
·57	·111	·218	·317	·403	·474	·527	·559	·57
·58	·113	·222	·322	·410	·482	·536	·569	·58
·59	·115	·226	·328	·417	·491	·545	·579	·59
·60	·117	·230	·333	·424	·499	·554	·588	·60
·61	·119	·233	·339	·431	·507	·564	·598	·61
·62	·121	·237	·344	·438	·516	·573	·608	·62
·63	·123	·241	·350	·445	·524	·582	·618	·63
·64	·125	·245	·356	·453	·532	·591	·628	·64
·65	·127	·249	·361	·460	·540	·601	·638	·65
·66	·129	·253	·367	·467	·549	·610	·647	·66
·67	·131	·256	·372	·474	·557	·619	·657	·67
·68	·133	·260	·378	·481	·565	·628	·667	·68
·69	·135	·264	·383	·488	·574	·637	·677	·69
·70	·137	·268	·389	·495	·582	·647	·687	·70
·71	·139	·272	·394	·502	·590	·656	·696	·71
·72	·140	·276	·400	·509	·599	·665	·706	·72
·73	·142	·279	·406	·516	·607	·674	·716	·73
·74	·144	·283	·411	·523	·615	·684	·726	·74
·75	·146	·287	·417	·530	·624	·693	·736	·75

Table for Computing the Products of the Numbers from .001 to 1.000—cont.

NUMBERS.	S ₁ .19509 (Sin. 11° 15'.)	S ₂ .38208 (Sin. 22° 30'.)	S ₃ .55557 (Sin. 33° 45'.)	S ₄ .70710 (Sin. 45°.)	S ₅ .83147 (Sin. 56° 15'.)	S ₆ .92388 (Sin. 67° 30'.)	S ₇ .98078 (Sin. 78° 45'.)	NUMBERS.
.76	.148	.291	.422	.537	.632	.702	.745	.76
.77	.150	.295	.428	.544	.640	.711	.755	.77
.78	.152	.298	.433	.552	.649	.721	.765	.78
.79	.154	.302	.439	.559	.657	.730	.775	.79
.80	.156	.306	.444	.566	.665	.739	.785	.80
.81	.158	.310	.450	.573	.673	.748	.794	.81
.82	.160	.314	.456	.580	.682	.758	.804	.82
.83	.162	.318	.461	.587	.690	.767	.814	.83
.84	.164	.321	.467	.594	.698	.776	.824	.84
.85	.166	.325	.472	.601	.707	.785	.834	.85
.86	.168	.329	.478	.608	.715	.795	.843	.86
.87	.170	.333	.483	.615	.723	.804	.853	.87
.88	.172	.337	.489	.622	.732	.813	.863	.88
.89	.174	.341	.494	.629	.740	.822	.873	.89
.90	.176	.344	.500	.636	.748	.831	.883	.90
.91	.178	.348	.506	.643	.757	.840	.893	.91
.92	.179	.352	.511	.651	.765	.850	.902	.92
.93	.181	.356	.517	.658	.773	.859	.912	.93
.94	.183	.360	.522	.665	.782	.868	.922	.94
.95	.185	.364	.528	.672	.790	.878	.932	.95
.96	.187	.367	.533	.679	.798	.887	.942	.96
.97	.189	.371	.539	.686	.807	.896	.951	.97
.98	.191	.375	.544	.693	.815	.905	.961	.98
.99	.193	.379	.550	.700	.823	.915	.971	.99
1.00	.195	.383	.556	.707	.831	.924	.981	1.00

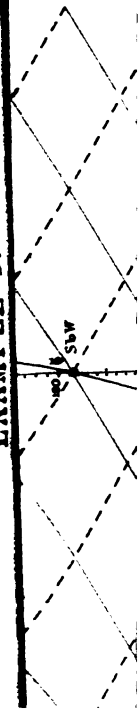
NATURAL SINES, TANGENTS, &c.

Angle.	Sino.	Diff. for 10'.	Arc.	Tangent.	Cotan.	Versin.	Cosine.	—
0°	·0000		·0000	·0000	infinite.	·0000	1·0000	90
1	·0175		·0175	·0175	57·290	·0002	·9998	89
2	·0349		·0349	·0349	28·636	·0006	·9994	88
3	·0523		·0524	·0524	19·081	·0014	·9986	87
4	·0698		·0698	·0700	14·301	·0024	·9976	86
5	·0872	·0029	·0873	·0875	11·430	·0038	·9962	85
6	·1045		·1047	·1051	9·514	·0055	·9945	84
7	·1219		·1222	·1228	8·144	·0075	·9925	83
8	·1392		·1396	·1405	7·115	·0097	·9903	82
9	·1564		·1571	·1584	6·314	·0123	·9877	81
10	·1736		·1745	·1763	5·671	·0152	·9848	80
11	·1908		·1920	·1944	5·145	·0184	·9816	79
12	·2079		·2094	·2126	4·705	·0219	·9781	78
13	·2250		·2269	·2309	4·331	·0256	·9744	77
14	·2419		·2443	·2493	4·011	·0297	·9703	76
15	·2588		·2618	·2679	3·732	·0341	·9659	75
16	·2756	·0028	·2793	·2867	3·487	·0387	·9613	74
17	·2924		·2967	·3057	3·271	·0437	·9563	73
18	·3090		·3142	·3249	3·078	·0489	·9511	72
19	·3256		·3316	·3443	2·904	·0545	·9455	71
20	·3420		·3491	·3640	2·747	·0603	·9397	70
21	·3584		·3665	·3839	2·605	·0664	·9336	69
22	·3746	·0027	·3840	·4040	2·475	·0728	·9272	68
23	·3907		·4014	·4245	2·356	·0795	·9205	67
24	·4067		·4189	·4452	2·246	·0865	·9135	66
25	·4226		·4363	·4663	2·145	·0937	·9063	65
26	·4384		·4538	·4877	2·050	·1012	·8988	64
27	·4540	·0026	·4712	·5095	1·963	·1090	·8910	63
28	·4695		·4887	·5317	1·881	·1171	·8829	62
29	·4848		·5061	·5543	1·804	·1254	·8746	61
30	·5000		·5236	·5774	1·732	·1340	·8660	60
31	·5150	·0025	·5411	·6009	1·664	·1428	·8572	59
32	·5299		·5585	·6249	1·600	·1520	·8480	58
33	·5446		·5760	·6494	1·540	·1613	·8387	57
34	·5592	·0024	·5934	·6745	1·483	·1710	·8290	56
35	·5736		·6109	·7002	1·428	·1808	·8192	55
36	·5878		·6283	·7265	1·376	·1910	·8090	54
37	·6018	·0023	·6458	·7536	1·327	·2014	·7986	53
38	·6157		·6632	·7813	1·280	·2120	·7880	52
39	·6293		·6807	·8098	1·235	·2229	·7771	51
40	·6428	·0022	·6981	·8391	1·192	·2340	·7660	50
41	·6561		·7156	·8693	1·150	·2453	·7547	49
42	·6691		·7330	·9004	1·111	·2569	·7431	48
43	·6820	·0021	·7505	·9325	1·072	·2686	·7314	47
44	·6947		·7679	·9657	1·036	·2807	·7193	46
45	·7071		·7854	1·0000	1·000	·2929	·7071	45
—	Cosine.	Diff. for 10'.	—	Cotan.	Tangent.	—	Sine.	Angle.

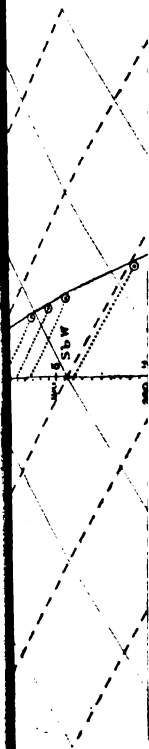
LONDON:
Printed by GEORGE E. EYRE and WILLIAM SPOTTISWOODE,
Printers to the Queen's most Excellent Majesty.
For Her Majesty's Stationery Office.
[3064.—250.—7/69.]
[P. 556.—750.—7/69.]



EXAMPLE 1.



EXAMPLE 2.



West Variation

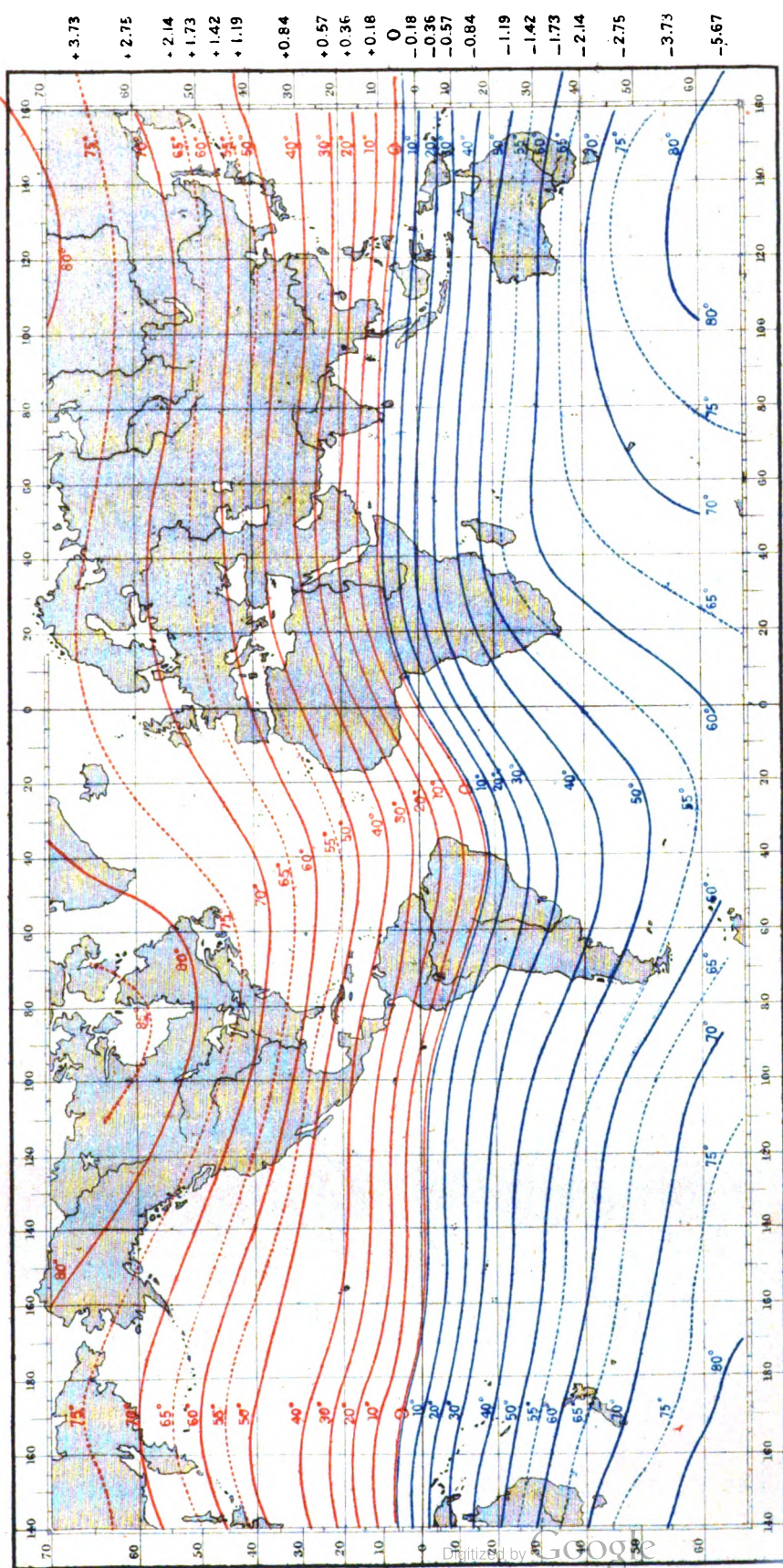
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Lines of Equal Magnetic DIP. 1860.

Natural
of
DIP.

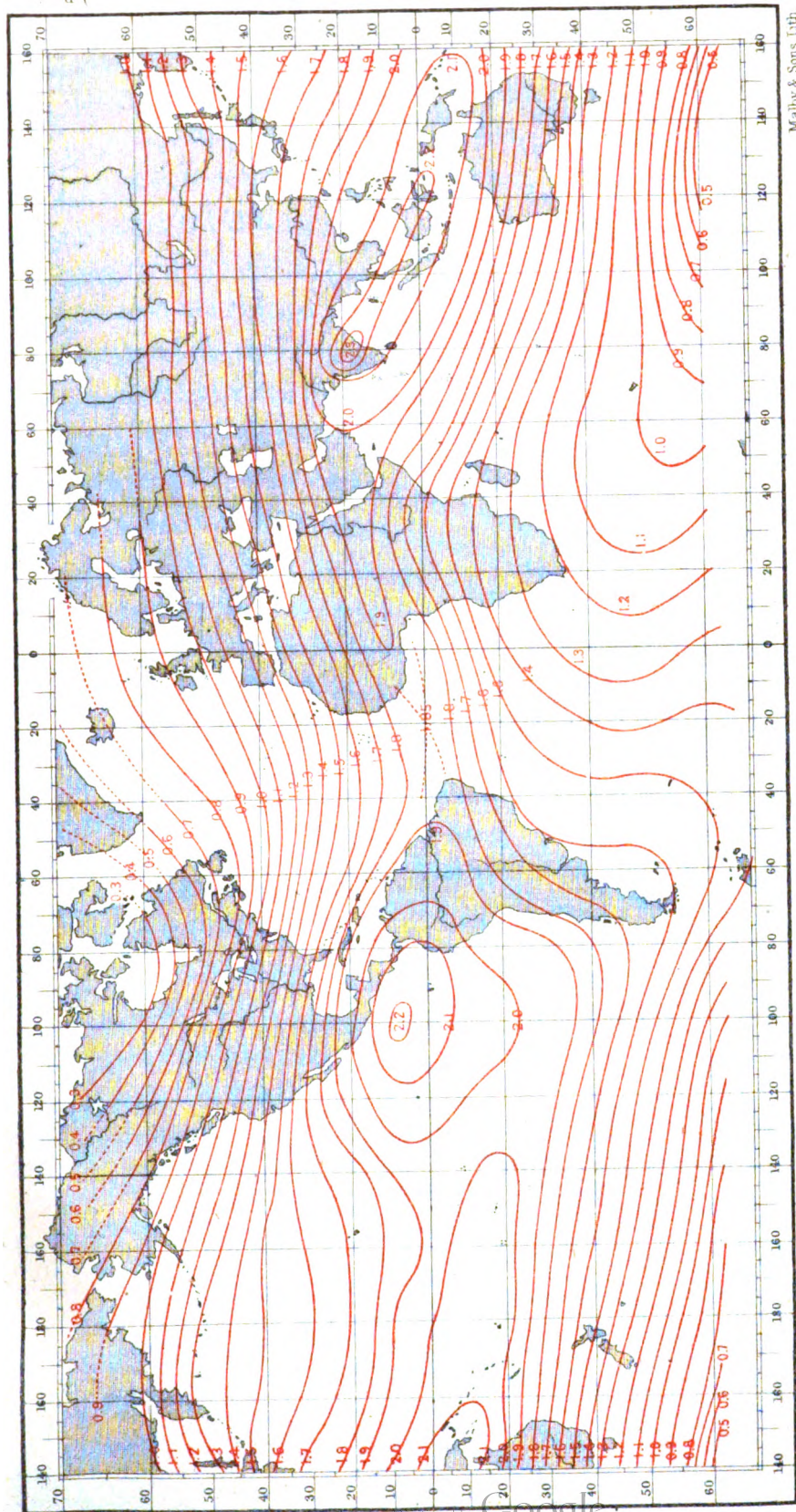
+ 5.67



Milby & Sons, Lith.

No. 2.

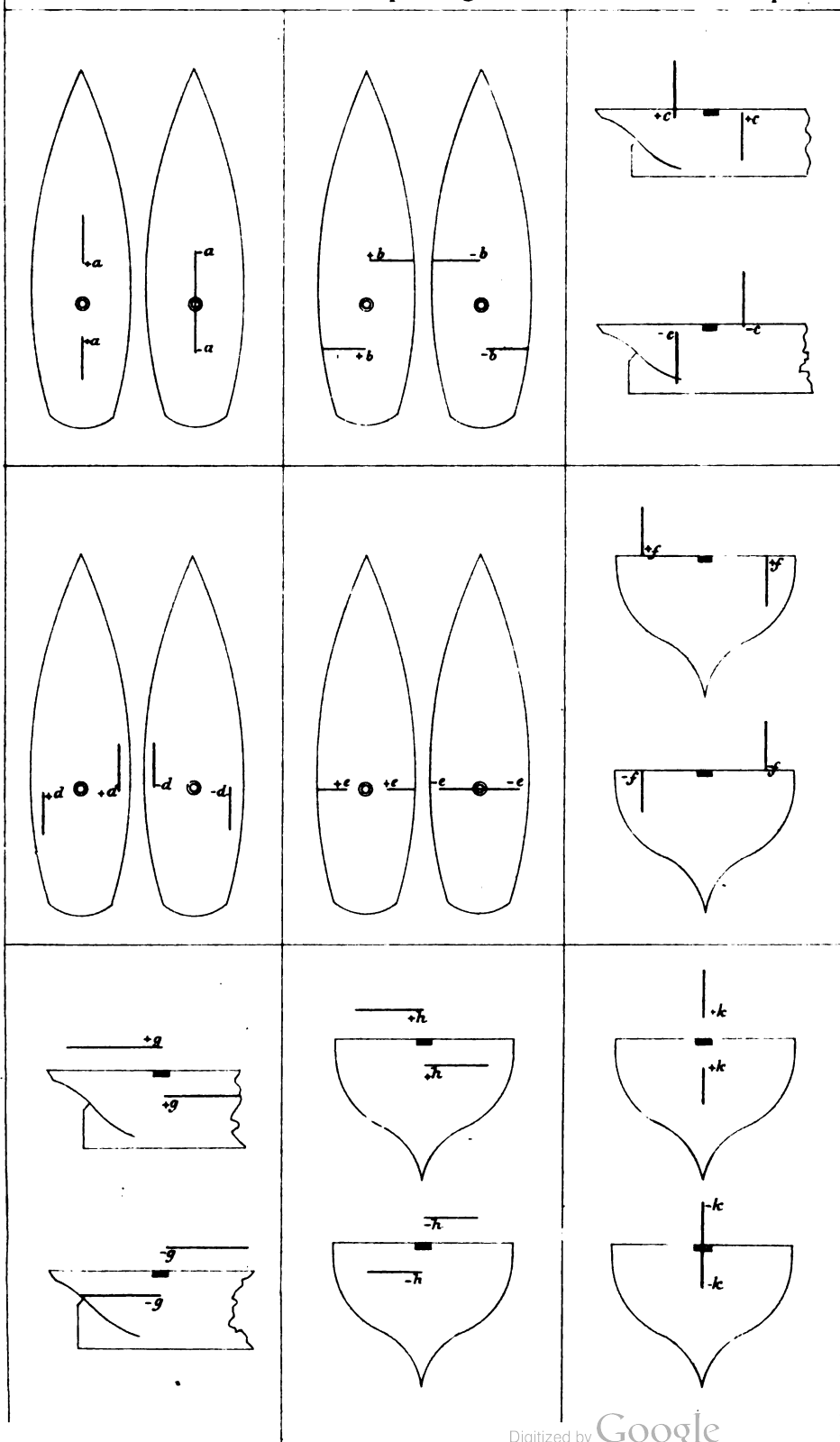
Lines of Equal Horizontal Force. 1860.



Malby & Sons, Lith.

No 3.

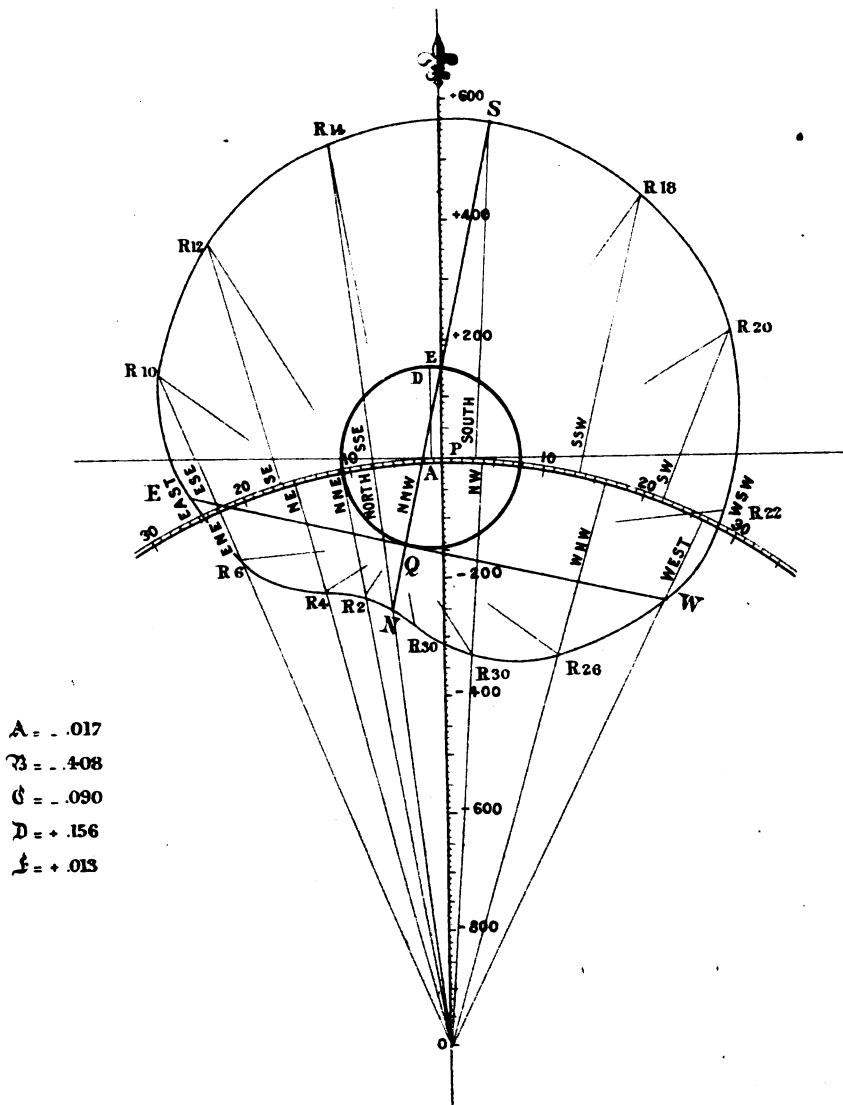
Diagram showing the positions of the nine soft iron rods which represent the whole of the soft iron of a ship as regards its action on the compass.



DYGOGRAM N° 1.

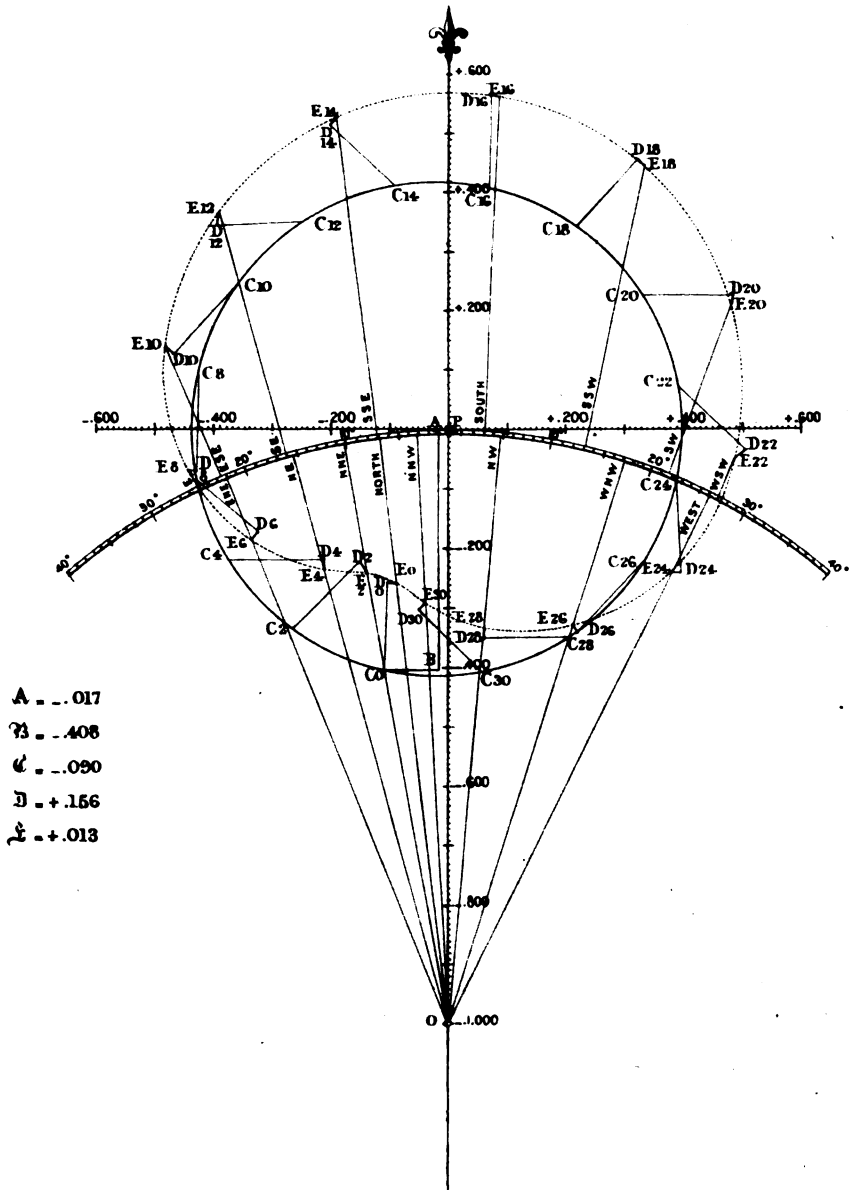
H.M. S. Warrior. Spithead, October 1861.

(Colongue's Method.)

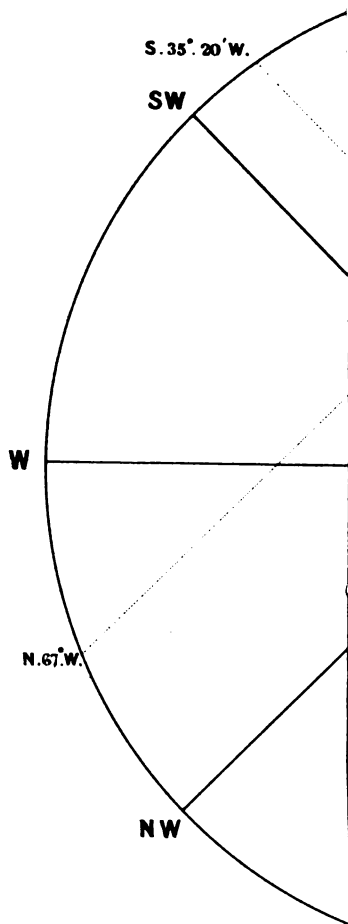


DYGOGRAM.

H.M.S. Warrior. Spithead. October 1861.



H.M.S. WARRIOR Cor
H.M.S.V. TRIDENT



Warrior

A.

B.

C.

Trident

STEERING DYGOGRAM.

(See Appendix IV.)

WARRIOR.

$$\mathcal{B} = - .408$$

$$\mathcal{C} = - .090$$

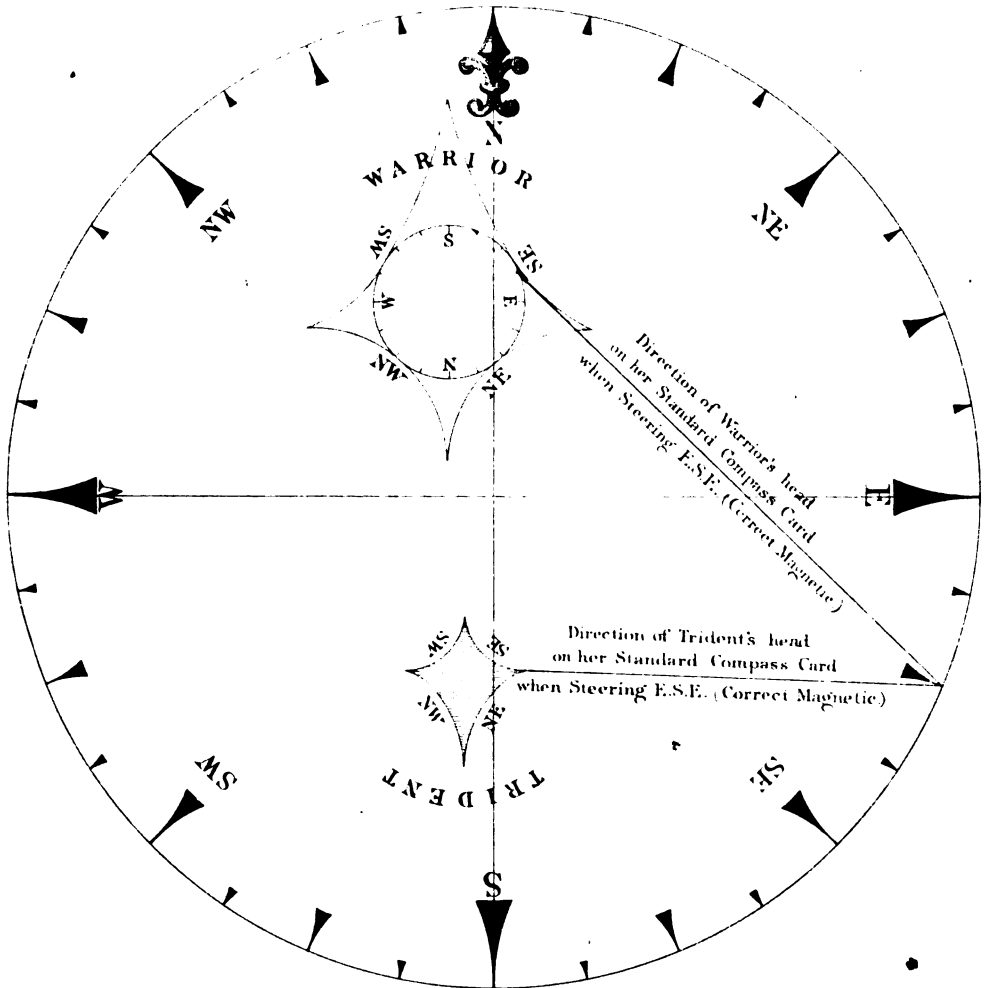
$$\mathcal{D} = + .156$$

TRIDENT.

$$\mathcal{B} = + .383$$

$$\mathcal{C} = - .057$$

$$\mathcal{D} = + .065$$



Note.—Every tangent to the envelope is a "directrix" and cuts the Margin of the Card at the "Correct Magnetic" course, which it indicates. A parallel diameter cuts the Margin at the corresponding "compass" course.

To Steer a given correct Magnetic Course .

Keep the Ship's head in the line of the directrix which cuts the Margin of the Card at the given correct Magnetic course.



